Detection Performance of an M-Sequence-Based MIMO Radar System Considering Jitter Influence

Idnin Pasya, *Member, IEEE*, and Takehiko Kobayashi, *Member, IEEE*

***Abstract*—This paper studies the jitter effects on the detection performance of M-sequence-based multiple-input multiple-output (MIMO) radar systems through time domain simulations. Jitter was modeled to be present in the system as timing jitter in the receivers' clock, and as phase jitter in the receivers' local oscillators. The system performance was evaluated in terms of probability of detection versus signal to noise ratio (*SNR*). Three types of MIMO radar processing schemes are investigated: a non-coherent MIMO, a decentralized radar network (DRN), and a re-phased netted radar (RPNR) processing. It was found that the presence of time jitter degraded the detection performance of the RPNR; however, it did not impose significant effects on the non-coherent MIMO and DRN processing schemes. Furthermore, it was demonstrated that the phase jitter at the MIMO radar receivers directly influenced the *SNR* yielding the same probability of detection, regardless the type of processing scheme chosen. Limiting the phase jitter terms to below 0.1 radian suppresses the performance degradation.**

***Index Terms*—MIMO radar, phase jitter, probability of detection, time jitter**

I. Introduction

Multiple-input multiple-output (MIMO) radars have been receiving extensive research interests due to its potential in improving numerous aspects of the conventional single-input single-output (SISO) radar systems. The MIMO radar concept originated from the idea to utilize spatial diversity in combating radar cross section (RCS) fluctuations, which can be achieved by employing orthogonal signals using widely separated transmitting antenna elements. This approach has been proven viable in producing improved detection performance compared to the conventional phased array antenna methods [1, 2]. In parallel, MIMO radar scheme with co-located antennas was also being investigated, focusing on waveform design to increase the accuracy of direction estimation and target localization [3-5].

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Later research works indicated that the usage of wideband MIMO radar improves the radar ambiguity function characteristics and the resolution of imaging and synthetic aperture radar results [6, 7].

Nevertheless, the initial works on MIMO radar assumed ideal conditions of the transmitting waveforms, target mobility, antenna orientat ion, and perfect synchronization. However, these assumptions are unlikely to hold in practical systems. Moreover, a MIMO radar consists of multiple transmitters and receivers distributed in the system, consequently enhancing the system complexity compared to the SISO systems. This will increase the effects of hardware imperfections and design errors on the system performance, particularly in MIMO radars utilizing widely distributed antennas. From the implementation point of view, it is essential to consider these effects in evaluating MIMO radar performance.

Several researches have been done to evaluate the performance of MIMO radar while considering system imperfections: for example, Chen et al. reported that calibration accuracy of antenna position in a MIMO array is essential to achieve good target location estimation [8]. Furthermore, Akcakaya et al. studied the performance of a coherent MIMO radar system under phase synchronization error effects due to modeling errors [9]. It was shown that synchronization mismatch degraded the detection performance severely, and an adaptive energy distribution technique to compensate the losses in signal to noise ratio (*SNR*) was proposed.

In this present paper, the effects of jitter on the detection performance of a MIMO radar system with widely separated antennas were investigated through time domain simulations. The motivation of this work was to evaluate the system performance under the influence of hardware imperfections (presence of jitter), even in the absence of modeling errors. Despite the fact that radar systems suffer performance degradation in various aspects due to jitter [10, 11], to the authors knowledge, the evaluation of jitter effects on the detection performance of MIMO radar systems have not been quantitatively evaluated. Furthermore, other works mainly took the approach of using Monte Carlo simulations, where random data were used as the MIMO radar signal, while assuming that the orthogonality condition was achieved in some way. The present paper used a deterministic simulation in the time domain, adopting pre-determined M-sequences as the MIMO radar

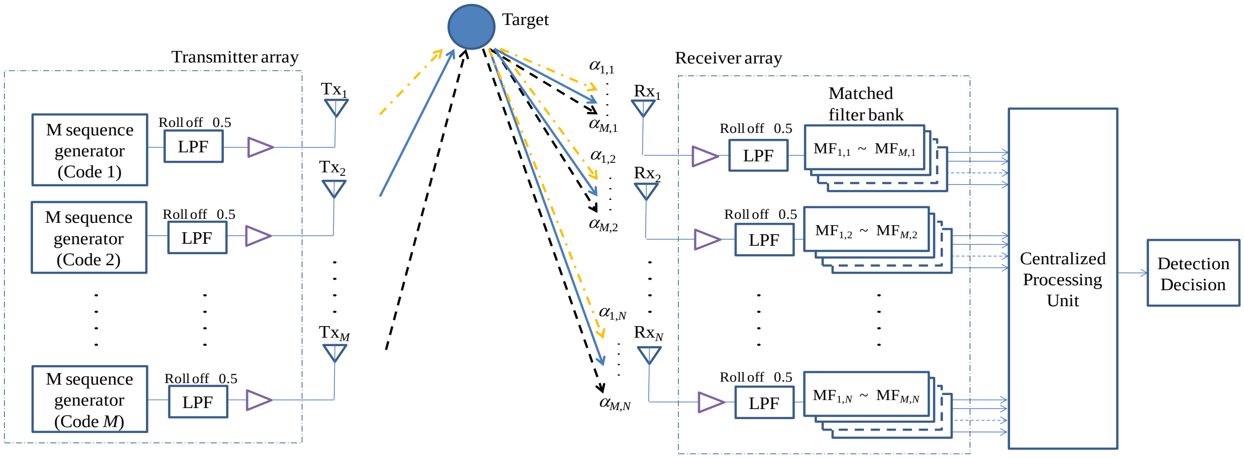


Fig. 1. MIMO radar system diagram.

signal. Previously, the authors investigated the effects of sampling jitter on a MIMO radar system detecting a stationary target providing a constant RCS [12]. This present study further the investigation, firstly by considering a fluctuating target model, and secondly, by taking into account the effects of phase jitter at the local oscillators of the radar receivers.

Timing jitter is defined as uncertainty of sampling points at the receiver, attributed to jitters in the universal clock of the system, affecting all signal processing operations that are driven by the system's clock. On the other hand, the phase jitter exists as the instability of the frequency generated by the local oscillators at the radar receivers. The effects of time and phase jitter were evaluated separately to provide a deeper insight on their effects on the system's performance.

The remainder of the paper is organized as follows. The next section discusses the simulation method used to evaluate timing jitter effects on the performance of MIMO radar systems. Section III describes the effects of phase jitter on the detection performance of MIMO radar systems, and finally, the concluding remarks are presented in Section IV.

II. Simulation of Timing Jitter Effects on the Detection Performance of MIMO Radar System

A. Simulation Model

The block diagram of the simulation model is depicted in Fig. 1. A time domain model of a MIMO radar system with M transmitters and N receivers is modeled in the equivalent baseband region. The system employs M-sequences as the transmit signals generated from 7-stage shift registers. Sequences with low cross-correlation codes were used in order for them to be orthogonal to each other. The signals are sampled at 2.5 GS/s, and filtered to occupy 500 MHz of bandwidth. Major parameters of the simulation model are summarized in Table I. Mathematically, the baseband representation of the

TABLE I
SIMULATION PARAMETERS.

Parameters	Description
Sampling rate	2.5 GS/s
Filter roll-off factor	0.5
Oversampling	5
Bandwidth	500 MHz
M-Sequence (Code length)	Order of 7 (127)
Number of points in each iteration	100 000

signal arriving at the k^{th} receiver can be expressed by

$$r_k(t) = H_{0/1}\sum_{m=1}^{M}\alpha_{m,k}(\sigma)s_m(t-\tau_{m,k})+z_k(t), \qquad (1)$$

where $H_{0/1}$ is 0 or 1 depending on the absence or presence of target, respectively; s_m is the m^{th} transmitted signal, $\tau_{m,k}$ is the delay occurring during the path between the m^{th} transmitter, target and the k^{th} receiver, z_k is the thermal noise, and $\alpha_{m,k}(\sigma)$ is the transmission coefficient that accounts for the RCS distribution of the target. In this study, a fluctuating RCS corresponding to Swerling I model [13] was considered, where $\alpha_{m,k}$ is represented by random variables following Chi-squared probability density function (PDF) with two degrees of freedom derived by

$$p(\sigma)=\frac{1}{\sigma_{av}}\exp\left(-\frac{\sigma}{\sigma_{av}}\right), \qquad (2)$$

where σ_{av} is the average RCS over all target fluctuations. The Swerling I model was chosen since it is well known to approximate the RCS distribution of a moving airplane. At every receiver, a matched filter corresponding to each transmitting sequence was implemented. Taking into account all receiving signals entering the receiver, the output of the h^{th} matched filter can be derived by

$$
\begin{aligned}
x_{h,k} &= r_k(t) \otimes s_h(t) \\
&= H_{0/1} \sum_{m=1}^{M} \alpha_{m,k}(\sigma) s_m(t - \tau_{m,k}) \otimes s_h(t) \\
&\quad + z_k(t) \otimes s_h(t) \\
&= H_{0/1} \alpha_{h,k}(\sigma) R_h(t - \tau_{h,k}) \\
&\quad + H_{0/1} \sum_{\substack{m=1 \\ m \neq h}}^{M} \alpha_{m,k}(\sigma) R_{m,h}(t - \tau_{h,k}) + n_{h,k}(t),
\end{aligned} \tag{3}
$$

where $\otimes$ denotes a convolutional operation, s_h is the reference signal at the h^{th} matched filter, R_h is the autocorrelation function of s_m, $R_{m,h}$ is the cross-correlation function of s_m with s_h, and $n_{h,k}$ is the resulting noise after the h^{th} matched filter at the k^{th} receiver. Consequently, considering a MIMO radar system with M transmitters and N receivers, a total of M matched filters are adopted at each receiver, providing MN outputs, arranged as

$$
X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1N} \\ x_{21} & x_{22} & \cdots & x_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ x_{M1} & x_{M2} & \cdots & x_{MN} \end{bmatrix}, \tag{4}
$$

and can be re-arranged into a vector as

$$
\mathbf{X} = [x_{11}, \ldots, x_{1M}, x_{21}, \ldots, x_{2M}, \ldots x_{N1}, \ldots, x_{MN}]^T, \tag{5}
$$

where T is the transpose operator. The resulting signal matrix is then jointly processed at a centralized processing unit, returning a detection decision based on an appropriate threshold. The detection performance of the MIMO radar systems depend on the type of processing scheme applied to the receiving signal matrix, which will be explained in the subsequent sub-section.

B. MIMO Radar Signal Processing Scheme used in this Study

This paper considers three types of joint signal processing schemes which have been proposed in MIMO radar through previous literatures, namely the non-coherent MIMO, the re-phased netted radar (RPNR), and the decentralized radar network (DRN) processing. The non-coherent MIMO processing (afterward termed "MIMO") has been proposed in many initial literatures on MIMO radar as a general scheme to process the receiving signals in a non-coherent way, where a generalized likelihood ratio test is utilized [1, 2]. The RPNR and DRN were discussed here due to the fact that these types of processing can be applied to the same radar network while keeping its geometry fixed.

The MIMO processing scheme is derived from the optimal Neyman-Pearson detector, where a likelihood ratio test is formulated between two hypotheses H_1 and H_0, corresponds to the presence and absence of the target, respectively. The likelihood ratio test is therefore given by

$$
\log \frac{p(\mathbf{r}(\mathrm{t})|H_1)}{p(\mathbf{r}(t)|H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda, \tag{6}
$$

where $p(\mathbf{r}(t))|H_1)$ and $p(\mathbf{r}(t)|H_0)$ are the probability density functions of the observation vector ($\mathbf{r}$ = [$\mathrm{r}_1, \ldots, \mathrm{r}_N$]^T) under the respective hypotheses. Following similar steps described by Fishler et al. [1], when the detector has full knowledge of the energy contained in $\mathbf{X}$ and the noise variance, it was shown that the detector can be given by

$$
\|\mathbf{X}\|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \lambda, \tag{7}
$$

where $\| \ \|$ denotes the vector norm, and λ is an appropriate threshold to keep a desired probability of false alarm (P_{fa}). This processing scheme is non-coherent since the detection is based on the received signal power alone.

On the other hand, the RPNR performs a coherent summation of the signals after they are processed through the bank of matched filters. The signals phases were re-aligned in such a way so that the summation maximizes the SNR. The detection in this type of processing is determined by

$$
\left| \sum_{k=1}^{N} \sum_{h=1}^{M} x_{h,k} \exp(-j\phi_k) \right|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \lambda, \tag{8}
$$

where the ϕ_k is the phase of the desired signal arriving at the k^{th} recevier, when target is present. This type of processing is challenging to be implemented, since it requires the prior knowledge of the signal path lengths. This may be obtained for example through utilization of target localization algorithm in pre-detection stage. The RPRN provides the upper bound of detection performance, and is used as a benchmark with the other processing schemes.

The DRN scheme works in a decentralized way where it implements a two-stage approach of detection. All the transmitter-receiver pairs works separately to produce its own detection decision in the initial stage using non-coherent processing, and these results are subsequently fused at the central processing unit. The output of the first stage of threshold processing at the k^{th} receiver is given by

$$
v[k] = \begin{cases} 1, & \left| \sum_{h=1}^{M} x_{h,k} \right|^2 \geq \lambda \\ 0, & \text{otherwise} \end{cases} \tag{9}
$$

The second stage applies another threshold processing on the output of all receivers, hence giving the detection by

$$
\sum_{k=1}^{N} v[k] \geq L, \tag{10}
$$

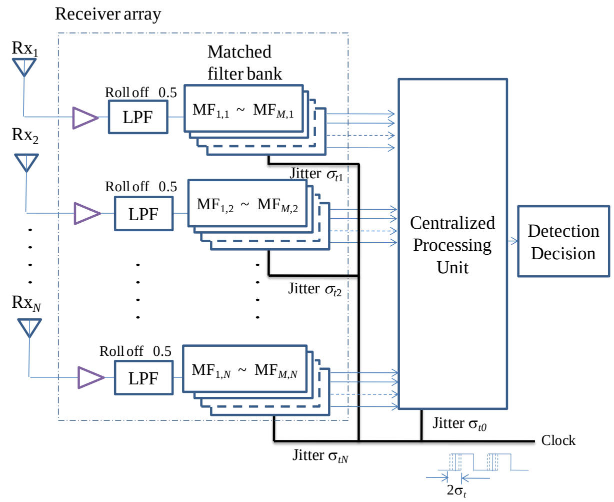


Fig. 2. Timing jitter in the MIMO radar receiver.

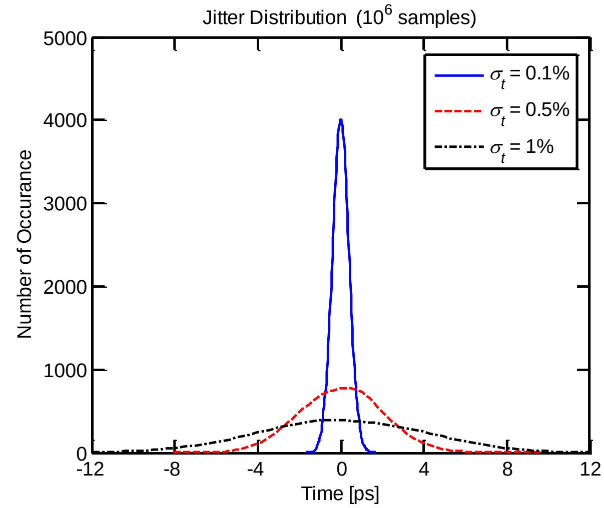


Fig. 3. Modeled jitter distribution with different values of σ_t.

where L is an appropriate threshold to achieve the chosen P_{fa}

It can be observed that all processing schemes utilize MN number of receiving signals for detection. This implies that the MIMO radar system's performance is a function of MN (refer to Appendix I for a general approximation of the MIMO radar probability of detection (P_d)). The value of MN has a direct impact on the total receiving signal power, hence the signal to noise ratio (SNR). Therefore, when M is not equal to N, the P_d will yield similar trend to the case of M equals to N, however, with a shift in SNR, depending on the value of MN. For example, a 2×4 configuration would yield a poorer P_d compared to a 4×4 system, shifting the P_d curve to the right due to loss in SNR. For the sake of simplicity, simulations in this study were conducted in cases of M equals to N. The number of antenna will also have a direct impact on the P_d of the MIMO radar system, since it governs the total number of receiving signal, MN.

C. *Timing Jitter Specification*

The timing jitter was modeled in the time domain as a random jitter. The system's clock samples at $T + \varepsilon_k$, where T was the actual clock edge, and ε_k was the variation in spacing due to time jitter. The ε_k was modeled to yield random values following a Gaussian distribution, with zero mean and standard deviation σ_t of 0.1, to 1% of the ideal clock period. The Gaussian assumption of the jitter distribution was supported by the fact that the primary source of the random jitter was the thermal noise; and according to the central limit theorem, the summation of many uncorrelated noise sources, regardless of their distributions, approaches a Gaussian distribution [14, 15]. Considering that the MIMO radar system consisted of multiple receivers in a widely separated location, independent timing jitter was modeled in each of the receiver, as depicted in Fig. 2.

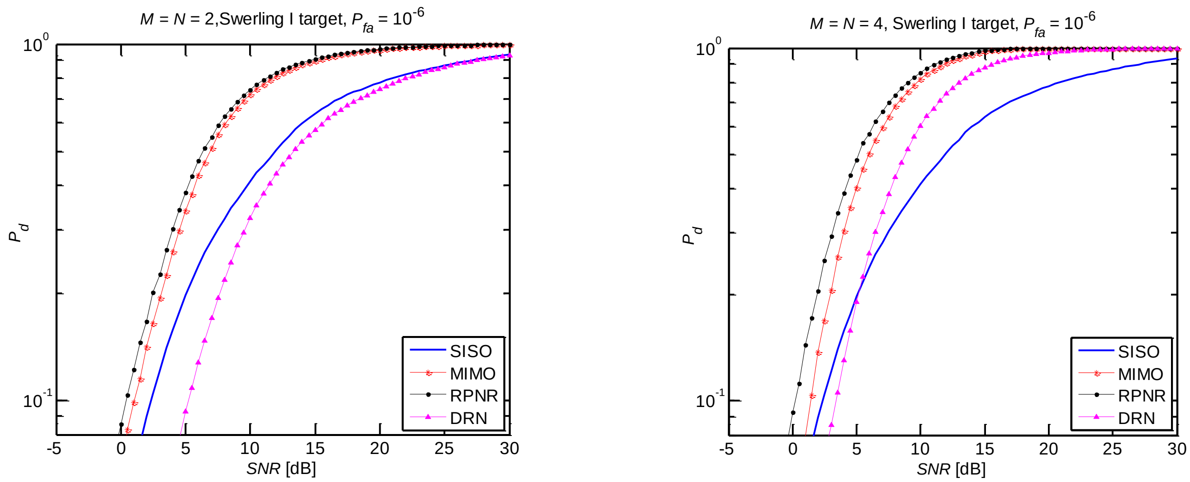
Figure 3 illustrates the probability density function of the modeled time jitter with different values of standard deviation. In can be observed in the figure that at the sampling rate used in the simulation model, σ_t of 0.1 and 1 % corresponded to standard deviations of 0.4 and 4 ps in the time domain, respectively. These values of σ_t were chosen in similar order to standard jitter values in most high speed applications such as in [16]. It was assumed that the transmitter side was not subject to time jitter, since it can be represented by the jitter at the receivers.

D. *Results and Analyses*

The detection performance of the simulated MIMO radar system under the presence of jitter was presented in terms of probability of detection (P_d) versus SNR, for all type of processing schemes. The SNR here was defined by the ratio of the mean of the total signal power to the total noise power at the receiver. In the simulations, the P_{fa} was fixed at 10^{-6} for all cases.

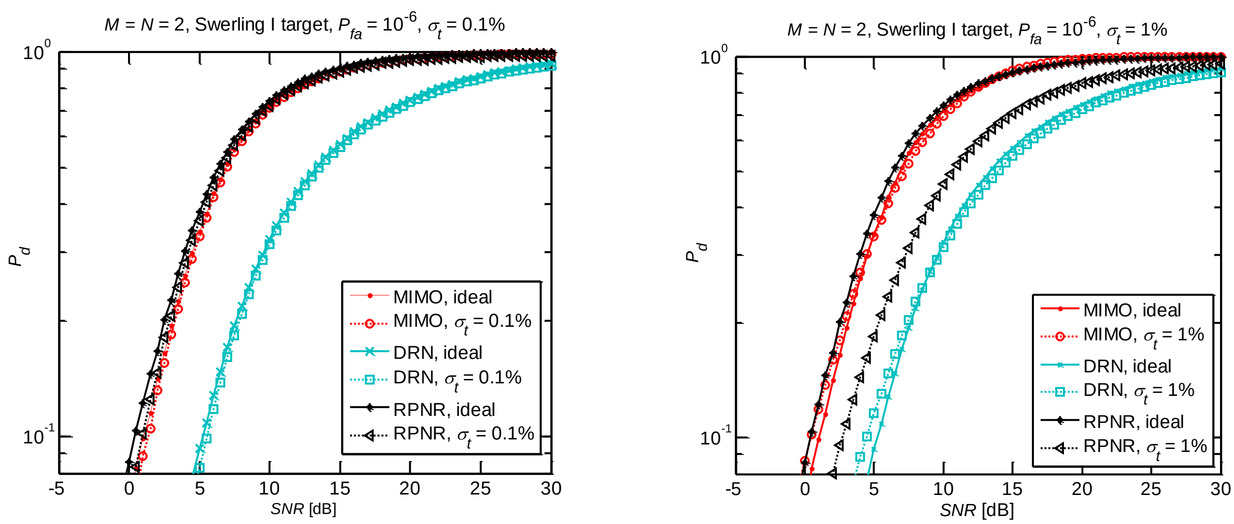
In Fig. 4 (a), we plot the P_d curve for each type of processing in a 2×2 configuration, in ideal cases, where jitter is not present (σ_t = 0). It is shown that all processing schemes yielded improvements in detection probability compared to the SISO case. The RPNR yielded the best performance due to the processing gain obtained from the phase information of the signals. The MIMO processing scheme performed almost equally with RPNR, with not much difference at high SNR region, for example at SNR = 15 dB which corresponded to P_d = 0.8. The DRN yielded the worst performance, however, its P_d improved with increasing transmitter and receiver pairs, as shown in the 4×4 configuration in Fig. 4 (b). These results agreed well with numerical and experimental results reported in [17].

The P_d curves in the presence of jitter are presented in Figs. 5 and 6, while varying σ_t from 0.1 to 1% of the ideal clock. In the 2×2 case shown in Fig. 5 (a), negligible performance degradation was observed when σ_t = 0.1%. However, when significant amount of jitter is present, as in the case of σ_t = 1%, the RPNR performance degraded significantly. This is evident in Fig. 5 (b), where a loss of 6 dB in SNR can be observed at P_d = 0.8. The degradation was attributable to inaccurate sampling during the re-phasing of the receiving signal due to timing jitter.



(a) (b)

Fig. 4. Probability of detection of the simulated MIMO radar system in ideal cases: (a) 2×2 and (b) 4×4 configurations.



(a) (b)

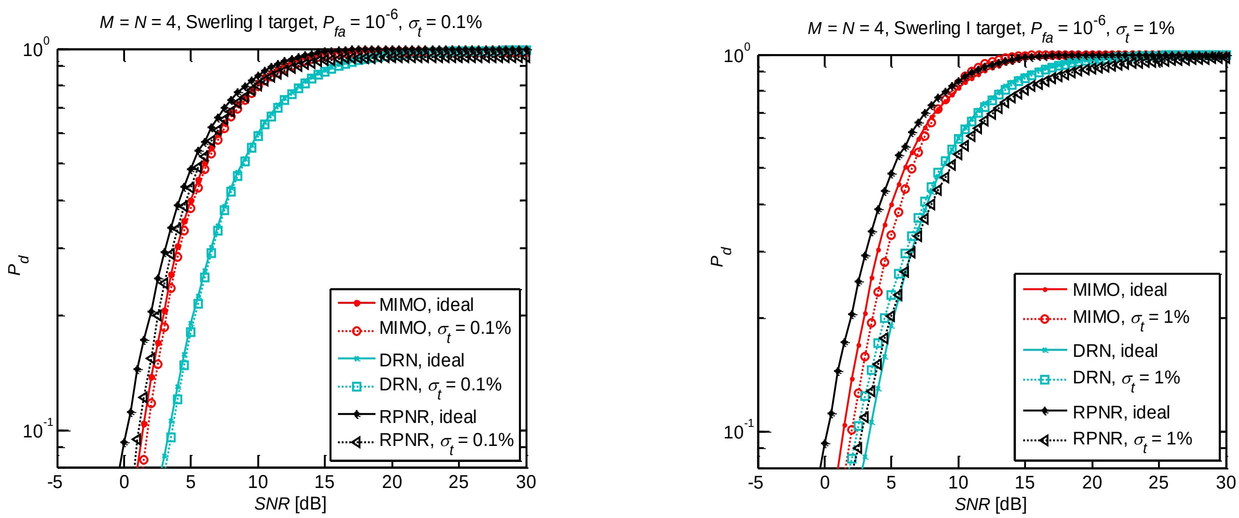
Fig. 5. Probability of detection under jitter influence in a 2×2 configuration: (a) σ_t = 0.1 % and (b) σ_t = 1 %.

In this particular case, the MIMO processing only suffered negligible degradation, and as a result, it performed better than the RPNR. Fig. 5 (b) depicted similar trends in the 4×4 case, where RPNR requires 5 dB of extra *SNR* to perform equally with MIMO processing at P_d = 0.8. Here, it is also shown that the DRN outperformed the RPNR, since it did not exhibit significant degradation. The performance degradation of RPNR however will be bounded by the performance of the scheme without implementation of signal re-phasing at the receivers.

It can be concluded that the presence of timing jitter had no significant impact on MIMO radar systems using the MIMO and DRN, which utilized a non-coherent detection in a joint, and de-centralized processing, respectively. On the other hand, the RPNR processing scheme suffered significant degradation when large amount of jitter was present, since it required precise alignment of the receiving signals phase. This result demonstrated the robustness of the MIMO processing against the presence of jitter, since they are based on statistical processing and were not timing-wise critical. However, in either way, suppressing the timing jitter's standard deviation below 0.1% of the ideal clock (0.4 ps in this case) was sufficient to prevent performance loss in either case of processing scheme used.

III. Simulation of Phase Jitter Effects on the Performance of MIMO Radar System

In this section, the detection performance of the simulated MIMO radar systems is evaluated while considering the effects of phase jitter at the local oscillator of each receiver. We modified the previous simulation model into a passband model, so that the effects of phase jitter at each receiver can be simulated. The details of the model will be described in the next sub-section.



(a) (b)

Fig. 6. Probability of detection under jitter influence in a 4×4 configuration: (a) σ_t = 0.1 % and (b) σ_t = 1 %.

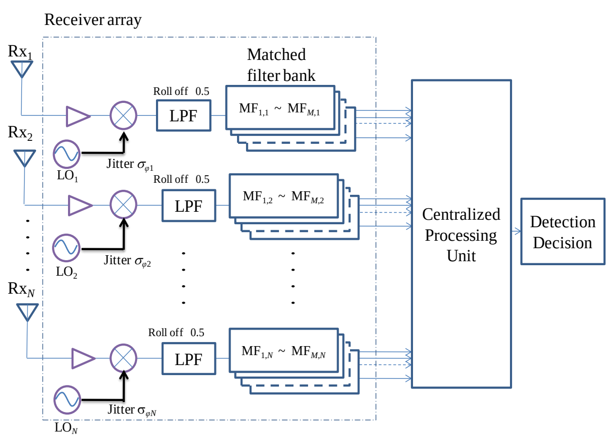


Fig. 7. Phase jitter in the MIMO radar receiver array.

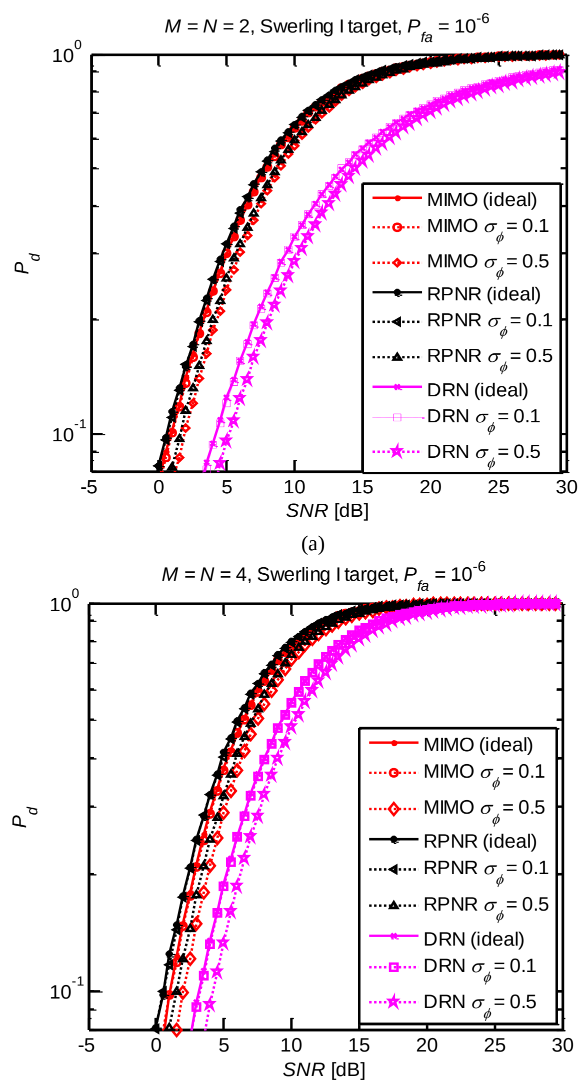
A. Simulation Model Considering Phase Jitter.

Figure 7 depicts the simulation model considering phase jitter in the receiver side of the MIMO radar system. Similar simulation parameters as listed in Table I were used, and each transmitting signal were up-converted to a center frequency of 1 GHz. At the receivers, the arriving signals were passed through mixers for down-converting operation, prior to matched filtering and signal processing. The mixers were fed with carrier signals from the receiver's local oscillators, which were modeled to have specific phase jitter characteristics. Ideally, the carrier signal can be expressed by

$$y(t) = A\sin(2\pi f_c t), \tag{11}$$

where A is the peak amplitude and f_c is the center frequency. The phase jitter was modeled in such a way that the signal in Eq. (11) consisted of variation of phase characterized by

$$y(t) = A(1+\psi(t))\sin(2\pi f_c t + \phi(t)), \tag{12}$$



(a)

(b)

Fig. 8. Simulated probability of detection of MIMO radar system under phase jitter influence: (a) 2×2, and (c) 4×4 configurations.

where $\phi(t)$ was the phase variation in radian, taking values from $-\pi$ to π, and $\psi(t)$ was termed as the amplitude error which corresponded to small amplitude variations due to other noises in the system. The phase jitter was represented by $\phi(t)$, which followed Gaussian distribution, with standard deviation σ_ϕ as the controlling parameter. In this study, independent phase jitter was modeled at the each receiver. The amplitude error $\psi(t)$ was assumed to be relatively small and thus not considered in this work.

B. Results and Analyses

The simulation results considering phase jitter is presented in Fig. 8. Similar to the previous simulations, the P_{fa} was fixed at 10^{-6} for all cases. It was shown that when only phase jitter was taken into consideration, the performance of the simulated MIMO radar degraded with increasing value of σ_ϕ. Interestingly, the P_d of all types of processing schemes exhibited similar trend of degradation versus *SNR*, as shown in Fig. 8. This was evident in cases of σ_ϕ taking a large value of 0.5 radian, where the P_d resulted in 2 dB loss at low *SNR* region. Nevertheless, no significant degradation was observed when σ_ϕ took values of 0.1 radian or lower (not shown here due to space consideration), indicating that the performance losses were negligible. Furthermore, referring to Fig. 8 (c), 4×4 configuration marked similar trend of performance degradation to the 2×2 case.

Figure 9 shows the receiver operating curves (ROC) of the MIMO radar systems, where the P_d is plotted while varying the P_{fa} from 10^{-1} to 10^{-6}. The ROC is plotted with fixed *SNR* of 10 dB, and cases where the phase jitters were $\sigma_\phi = 0.1$ and 0.5 radian. This is to show that at a fixed *SNR*, the performance losses in P_d of all type of processing schemes were similar, even when choosing different P_{fa}. It can be observed in Fig. 9 (a) that at $P_{fa} = 10^{-6}$, all processing schemes marked approximately 5% loss of P_d, when σ_ϕ took values of 0.5 radian. The degradation amount gradually decreased with increasing P_{fa}, for example, to 2% at $P_{fa} = 10^{-2}$. Figure 9 (b) shows that the 4×4 case yielded similar trend, however, with slightly larger P_d degradation. The P_d degradation of all processing schemes at $P_{fa} = 10^{-6}$ and $\sigma_\phi = 0.5$ were calculated and summarized in Table II. From the table, it was evident that the DRN processing indicated significant P_d degradation with increasing numbers of antenna. The P_d degraded approximately 2% when scaling up the MIMO configuration from 2×2 to 3×3, and another 2% to 4×4. This result implied that when a large MIMO configuration was used (e.g. 10 to 100 elements), the phase jitter could significantly degrade the P_d when σ_ϕ was sufficiently large, especially for the DRN.

Summarizing the above results, it can be concluded that phase jitters directly affects the *SNR* yielding equivalent performance of the MIMO radar systems, regardless of the types of processing scheme used. Although the degradation was negligible when phase jitter was insignificant, it is worth to consider the degradation amount, especially when the system does not have sufficient *SNR* margin, and when the MIMO configuration is large. This is because the performance degradation scaled up with increasing antenna elements in the MIMO radar system. Implementing a large MIMO configuration up to 100 elements and above as described in [18] could suffer significant performance degradation due to phase jitter. Therefore, in severe cases, it is recommended to utilize adaptive techniques to compensate the *SNR* loss, for example, an optimal energy distribution method proposed in [9].

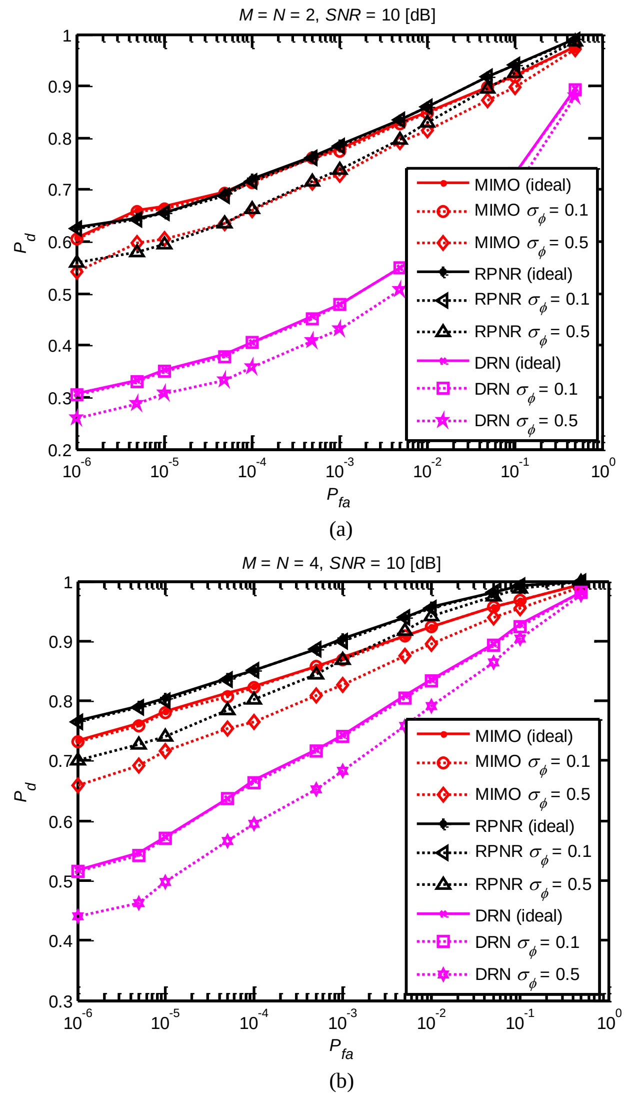


Fig. 9. Receiver operating curves of the simulated MIMO radar systems with presence of severe phase jitter: (a) 2×2 and (b) 4×4 configurations.

IV. Concluding Remarks

The detection performance of M-sequence-based MIMO radar systems considering the presence of jitter was evaluated through time domain simulations. Firstly, it was observed that the effects of timing jitter on the detection performance of MIMO radar were negligible when a non-coherent MIMO and DRN processing scheme were used. However, the detection performance of RPNR scheme degraded significantly when the standard deviation of the timing jitter was 1 % of the ideal clock.

TABLE II
PROBABILITY OF DETECTION DEGRADATION FOR INCREASING TRANSMIT-RECEIVE ELEMENTS IN MIMO RADAR

MIMO configuration	**P_d Degradation[a]**		
	MIMO	***RPNR***	***DRN***
2×2	0.061	0.063	0.046
3×3	0.061	0.064	0.066
4×4	0.067	0.065	0.084

[a] $P_{fa} = 10^{-6}$, $\sigma_\phi = 0.5$, $SNR = 10$ dB.

Secondly, it was shown that phase jitter at the local oscillator of each receiver directly degraded the *SNR* yielding similar probability of detection of the MIMO radar system, regardless of processing scheme chosen. In can be concluded from the results that the performance degradation due to phase jitter were negligible if the amount of jitter was limited to below 0.1 radian. Nevertheless, the effects of phase jitter may become significant when using a large MIMO array in conditions with low *SNR* margin.

In conclusion, the effects of timing jitters were significant only in the case of a coherent processing (i.e. RPNR scheme), therefore consideration of timing jitter should be given when using this type of processing. On the other hand, significant phase jitters were found to give a direct impact on the P_d, regardless of the type of processing scheme, especially when using a large MIMO configuration. The author recommends that the phase jitter is to be considered when dealing with large MIMO array, *e.g.*, consisting of 10 to 100 antenna elements.

APPENDIX I
APPROXIMATION OF THE MIMO RADAR PROBABILITY OF DETECTION

Statistical description of the probability of detection of the MIMO radar system is presented. Since the simulation model used pre-determined M-sequence-based signals, it is difficult to derive a close-form solution of the system. However, by assuming that the transmitted signals are random data with Gaussian distribution, we can derive an approximation of the probability of detection statistically. For a SISO radar system detecting a Swerling I target, the PDF of the receive signal in the presence noise is derived as

$$P_{SISO}(\zeta) = \frac{1}{\sigma_s^2 + \sigma_n^2} \exp\left\{-\frac{\zeta}{\sigma_s^2 + \sigma_n^2}\right\}, \qquad (13)$$

where σ_s and σ_n are the variance of the receive signal and noise, respectively. The probability of detection hence can be expressed by

$$\begin{aligned} P_{d\_SISO}(\lambda) &= \int_{\lambda_{SISO}}^{+\infty} \frac{1}{\sigma_s^2 + \sigma_n^2} \exp\left\{-\frac{\zeta}{\sigma_s^2 + \sigma_n^2}\right\} d\zeta \\ &= \exp\left\{-\frac{\lambda_{SISO}}{\sigma_n^2 + \sigma_n^2}\right\}, \end{aligned} \qquad (14)$$

where is the threshold to achieve a chosen P_{fa} for the SISO case. When a MIMO processing is applied, the overall PDF of the receive signal follows a gamma distribution [19], derived as

$$p_{MIMO}(\zeta) = \frac{1}{\sigma_s + \sigma_n} \frac{1}{\Gamma(MN)} \left(\frac{\zeta}{\sigma_s^2 + \sigma_n^2}\right) \exp\left(-\frac{\zeta}{\sigma_s^2 + \sigma_n^2}\right). \quad (15)$$

Hence, P_d of the system can be expressed by

$$\begin{aligned} P_{d\_MIMO}(\lambda) &= \int_{\lambda_{MIMO}}^{+\infty} \frac{1}{\sigma_s^2 + \sigma_n^2} \frac{1}{\Gamma(MN)} \left(\frac{\zeta}{\sigma_s^2 + \sigma_n^2}\right) \exp\left(-\frac{\zeta}{\sigma_s^2 + \sigma_n^2}\right) d\zeta \\ &= \exp\left\{-\frac{\lambda_{MIMO}}{\sigma_n^2 + \sigma_n^2}\right\} \sum_{k=0}^{MN-1} \frac{1}{k!} \left(\frac{\lambda_{MIMO}}{\sigma_s^2 + \sigma_n^2}\right). \end{aligned} \qquad (16)$$

Here, by choosing the same threshold and P_{fa}, it can be observed the improvement of P_d in a MIMO radar system compared to a SISO system depends on the resulting number of *MN* matched filter outputs, as expressed in the final form of Eq. (16).

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REFERENCES

[1] E. Fishler, A. Haimovich, R. S. Blum, L. J. Cimini, Jr., D. Chizhik, and R. A. Valenzuela, "Spatial diversity in radars-models and detection performance," *IEEE Trans. on Signal Process.*, vol. 54, pp. 823-838, 2006.

[2] A. M. Haimovich, R. S. Blum, and L. J. Cimini, "MIMO radar with widely separated antennas," *IEEE Signal Process. Magazine*, vol. 25, pp. 116-129, 2008.

[3] L. Jian and P. Stoica, "MIMO radar with colocated antennas," *IEEE Signal Processing Magazine*, vol. 24, pp. 106-114, 2007.

[4] X. H. Wu, A. A. Kishk, and A. W. Glisson, "MIMO-OFDM radar for direction estimation," *IET Radar, Sonar & Navigation*, vol. 4, pp. 28-36, 2010.

[5] A. Hassanien and S. A. Vorobyov, "Transmit energy focusing for DOA estimation in MIMO radar with colocated antennas," *IEEE Trans. on Signal Proces.*, vol. 59, pp. 2669-2682, 2011.

[6] Y. Honghui, S. Guowei, R. Zetik, O. Hirsch, and R. S. Thoma, "Ultra-wideband MIMO ambiguity function and its factorability," *IEEE Trans. on Geoscience and Remote Sensing*, vol. 51, pp. 504-519, 2013.

[7] Z. Xiaodong and A. G. Yarovoy, "Study on two-dimensional sparse MIMO UWB arrays for high resolution near-field imaging," *IEEE Trans. on Antennas and Propagation*, vol. 60, pp. 4173-4182, 2012.

[8] C. Haowen, L. Xiang, J. Weidong, and Z. Zhaowen, "MIMO radar sensitivity analysis of antenna position for direction finding," *IEEE Trans. on Signal Process.*, vol. 60, pp. 5201-5216, 2012.

[9] M. Akcakaya and A. Nehorai, "MIMO radar detection under phase synchronization errors," in *2010 IEEE Int. Conf. on Acoustics Speech and Signal Process. (ICASSP)*, 2010, pp. 2578-2581.

[10] R. Zemmari, "Time jitter influence on GSM passive radar," in *2011 Int. Radar Symposium (IRS)*, pp. 292-300, 2011.

[11] H. Q. Nguyen, N. V. Vu, J. Whittington, E. Custovic, B. Bienvenu, and J. Devlin, "Noise immunity enhancement for a distributed clock system in digital HF radar," in *2011 6th Int. Conf. on Broadband and Biomedical Communications (IB2Com)*, pp. 227-231, 2011.

[12] I. Pasya and T. Kobayashi, "Simulation of time jitter effects on the detection performance of M-sequence based MIMO radar systems," in *2013 IEEE Radio and Wireless Symposium*, Austin, TX, 2013.
[13] M. I. Skolnik, *Introduction to Radar Systems*, 2nd ed.: McGraw-Hill College, 1980.
[14] C. Daniel, *Jitter Visualization, Part 1: Random Jitter*, www.designconcommunity.com/author.asp?section_id=3017&id=261545.
[15] Tektronix, *Understanding and Characterizing Timing Jitte-Primer*, *www*.tek.com/application/jitter-measurement-and-timing-analysis.
[16] USB Implementers Forum, *Universal Serial Bus Revision 3.0 Specification*, www.usb.org/developers/docs/.
[17] T. Otsuki, I. Pasya, and T. Kobayashi, "Experimental evaluation of detection performance of a MIMO radar testbed," in *2013 UNSC-URSI National Radio Science Meeting*, C1-6, 2013.
[18] P. V. Brennan, A. Narayanan, and R. Benjamin, "Grating lobe control in randomised, sparsely populated MIMO radar arrays," *IET Radar, Sonar & Navigation*, vol. 6, pp. 587-594, 2012.
[19] J. Li and P. Stoica, *MIMO Radar Signal Processing*. New Jersey: Wiley, 2009.

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