Restoration in WDM Mesh Networks by Finite Differences

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***Abstract*-Fully optical fiber networks that transmit large quantities of information due to multiple applications or because they serve large urban areas need to be maintained fault-free at all times of their operation. The protection and restoration strategies are critical because they provide optical network survivability from a variety of failure scenarios that can occur in this environment. The topology with inherent capability to survive by failures is the mesh one. It is valid because there are usually at least two paths between end nodes. In this paper the approximate finite difference method and the linear one are used to study the problem of the restoration that is executed by shortest path algorithm dynamically.**

Key words: Planning, designing, WDM, mesh, protection, dynamic restoration

I. INTRODUCTION

Fully optical fiber WDM (Wavelength Division Multiplex) mesh networks are high capacity networks (potentially tens of Terabits per second) that transmit large quantities of information due to multiple applications or because they serve large urban areas, need to be maintained fault-free at all times of their operation. Such protection from faults extends to both service providers and end-users. To ensure service continuity, service providers plan and design suitable solutions to alleviate such disruptions. These solutions include protection systems and restoration paths. Planning depends on the demands or needs that the network satisfies as well as on the percentage of protection and fulfillment of the network systems. The protection is provided on optical fiber layer or on wavelength layer or combining both of them. In this paper the linear and approximate polynomial finite differences methods are used to study the problem of the restoration.

The term "restoration" is usually defined as the method by which backup or spare capacity for the connection is dynamically discovered after connection has failed due to a link or node failure. Restoration is not the primary mechanism to deal with a failure. Instead, it is used to provide either more efficient routes and additional resilience against further failures before the first failure is fixed. As a result, it provides a slower alternative to protection. Complex algorithms can be used to reduce the excess bandwidth required. In [1] and [2] the authors write about the evolution of the OTN. In [3] and [5] address issues in designing a survivable optical layer. There are several approaches to ensure fiber network survivability [4] and [9]. In [6] deals with protection cycles in WDM Mesh Networks. In [7] a mesh based hybrid OMS / OCh protection /restoration scheme is suggested. In [8], the authors suggest a dynamical bandwidth distribution for protection in IP over WDM networks. In [10], a new shared path protection is suggested supported by dynamic provisioning of restorable bandwidth. In [11], the paper provides an overview of multilayer recovery issues and solutions in an Internet protocol (IP) over optical network environment, which is illustrated by quantitative case studies. In [12] there are finite difference issues. In [13] deals with the modeling and simulation and gives practical advices for network designers and developers. In [14] the author writes about the shared protection method, examines the linear function method and the accurate polynomial function one of the finite differences. Then the problem of the unused available network capacity minimization is approached.

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The following analysis presents the solution of the below problem associated with the survival optical networks on the basis of the finite differences.

The network topology and other parameters are known as WDM and optical fiber capacity, one optical fiber per link with an extension to a 1+1 fiber protection system. So this network is characterized by one working fiber per link, edges of two links, links of two optical fibers, one for working and one for protection. In this paper we are usually referred at the working optical fiber. The connections are lightpaths originating in the source nodes and terminating at the destination nodes proceeding from preplanned optical working paths. The connections that have been set up are protected dynamically by the shortest path algorithm that uses the available capacity (no busy capacity) of each link. The connections of the same node pair by same preplanned optical paths form a connection group along the network. The nodes have wavelength conversion capability. The problem solution is to calculate the final available capacity of the network for a given traffic table first without any failure, second after a single failure when the restoration is done by shortest path algorithm. The traffic table contains the number of the node pairs, the node pairs and the number of the connections of each node pair when their working paths are preplanned. The role of the Difference Calculus is in the study of the Numerical Methods. Computer solves these Numerical Methods. The subject of the Difference Equations is in the treatment of discontinuous processes.

The network final available capacity is revealed as a difference equation because the final available capacity of each individual working optical fiber is also a difference equation. The reduction of the available capacity of each working optical fiber is a discontinuous process when connection groups of several sizes pass through it. Two methods, the linear function method and the approximate polynomial function method reveal each difference equation. These methods include arithmetical methods to study the optical networks and their problems.

This paper is broken down in the following sections: Section II shows how the finite differences are used for each optical fiber and illustrates the optical fiber residual available capacity; Section III describes the problem and provides a solution, detail description of algorithm, an example and a discussion; Section IV draws conclusions and finally ends with the references.

II. THE OPTICAL FIBER AND THE FINITE DIFFERENCES

In [14], there is a more analytical presentation of finite differences. In this paper I write them to connect it with finite differences. Let's assume that $y_1, y_2, \ldots, y_n$ is a sequence of numbers in which the order is determined by the index n. The number n is an integer and the y_n can be regarded as a function of n, an independent variable with function domain the natural numbers and it is discontinuous. Such a sequence shows the available capacity reduction of a telecommunication fibre network link between two nodes when the telecommunication traffic of 1, 2, …,n source-destination node pairs pass through. It is assumed that the telecommunication traffic unit is the optical channel that is one wavelength (1λ). The telecommunications traffic includes optical connections. The total connections of a node pair form its connection group. The first order finite differences represent symbolically the connection group of each node pair that passes through a fiber. This connection group occupies the corresponding number of optical channels and it is the bandwidth that is consumed by connections of a node pair through this fiber. The first order finite differences are used to represent the connection groups in optical channels of the node pairs that pass through an optical fiber .An equation of the first order finite differences gives the available capacity of an optical fiber network link when a connection group passes through it. When the first connection group of Δy_1 connections passes through an optical fiber network link with installed capacity of y_1 optical channels the first order finite difference equation gives the available capacity y_2 (y_{1+1}) which is written as following

$$y_{1+1} = y_1 - \Delta y_1 \qquad (1)$$

The sequence Δy_1, Δy_2, $\Delta y_3, \ldots$, Δy_n represents the connection groups that pass through this optical fiber network link. When Δy_1 subtracted from y_1, creates y_2, when Δy_2 subtracted from y_2, creates $y_{3,\ldots,}$ when Δy_n subtracted from y_n creates y_{n+1} which is the total unused available capacity of this optical fiber. The total unused available capacity of each network optical fiber is calculated after n connections groups pass through it. This could be written with two methods, the linear function method and the polynomial function method. So these methods could be used to check each other. Table 1 gives a short comprehensive presentation of the computation of the finite differences for a given link that can be arranged quite simply.

The unused available capacity in the linear function method of an optical fiber link *(i)* after *n(i)* connection groups have passed through it corresponding to the communication between *n* source destination node pairs is the following.

$$y_{i,\,n(i)+1} = y_{i,1} - \sum_{j=1}^{n(i)} \Delta y_{i,j} - dy_i \qquad (2)$$

$y_{i,\,n(i)+1}$ is the unused available capacity in optical channels (wavelengths) of the optical fiber network link (i) for the n(i)+1 node pairs.

$y_{i,1}$ is the available working capacity in optical channels (wavelengths) of the optical fiber (i) for the first node pair. It gives the installed capacity and it is a boundary condition.

The mid term of the right part is the sum of *n* first order finite differences and it is the sum of all optical channels of *n* nodes pairs that pass through this optical fiber.

dy_i is the restoration capacity in optical channels (wavelengths) of the optical fiber network link. When there is not any failure and no restoration happen this quantity is zero.

TABLE 1
A SHORT PRESENTATION OF THE FINITE DIFFERENCE COMPUTATION

Node Pair	Node pair index	Available capacity connection group	Number of the optical connection group	Variation of the optical differences number	Higher order differences number
(S_1,D_1)	1	y_1			
			Δy_1		
(S_2,D_2)	2	y_2		$\Delta^2 y_1$	
			Δy_2		$\Delta^3 y_1$
(S_3,D_3)	3	y_3		$\Delta^2 y_2$	
			Δy_3		
(S_4,D_4)	4	y_4			
………					
(S_{n-1},D_{n-1})	n-1	y_{n-1}			
			$\Delta y_{,n-1}$		
(S_n,D_n)	n	y_n		$\Delta^2 y_{n-1}$	
			Δy_n		
(S_{n+1},D_{n+1})	n+1	y_{n+1}			

The previous equation for optical fiber available capacity (linear function method) of the link (i) gives the following equation and it is written

$$y_{i,\,n(i)+1} = y_{i,1} - \sum_{l=1}^{n(i)} a_{i,l} * x_l - dy_i \qquad (3)$$

whereby $a_{i,l}$ is a coefficient that takes the value one (1) if the node pair (l) passes all its working connections from this fiber (i) and zero (0) if no passes. The x_l is the total of the connections of the node pair (l) that is the connections groups and it is called connection group size.

The n the total number of the node pairs and $n >= n(i)$ and dy_i is the restoration optical channels (wavelengths) of the link i.

The assessment of the polynomial function coefficients is done with the values that the polynomial function represents for 1,2,…, n, n +1 .The values of the function y_{n+1} for each n must be integral because each value represents optical channels.

The optical fiber available capacity for connections (approximate polynomial function method) of the link (i) is given by the general form of a polynomial function after the serving n(i) working connection groups and it written is as follows

$$y_{i,\,n(i)+1} = \sum_{r=0}^{n(i)} \alpha_r * (n(i)+1)^r \tag{4}$$

Some notes on the polynomial function are presented.

-When no connection group passes through an optical fiber, the polynomial function is constant and independent of n.

-When only one connection group passes through an optical fiber the polynomial function is of the first degree.

-When only a two-connection group passes through an optical fiber the polynomial function is of the second degree, etc.

-The degree of the polynomial function of an optical fiber depends on the connection group number that passes through it.

-The polynomial function of an optical fiber is different when there is full or partial servicing of the connection groups that pass through it.

-Two polynomial functions with the same available capacity have different coefficients when the order and the size of the same number connection groups are different.

-If the equation (4) is written analytically as follows

$$y_{0+1}= \alpha_0*(0+1)^0 +\alpha_1*(0+1)^1+\alpha_2*(0+1)^2 +\ldots+\alpha_n*(0+1)^n$$
$$y_{1+1}= \alpha_0*(1+1)^0 +\alpha_1*(1+1)^1+\alpha_2*(1+1)^2 +\ldots+\alpha_n*(1+1)^n$$
$$y_{2+1}= \alpha_0*(2+1)^0 +\alpha_1*(2+1)^1+\alpha_2*(2+1)^2 +\ldots+\alpha_n*(2+1)^n$$
…………………………………………………………
$$Y_{n+1}= \alpha_0*(n+1)^0 +\alpha_1*(n+1)^1+\alpha_2*(n+1)^2 +\ldots+\alpha_n*(n+1)^n$$

The value of the function has reduced accuracy and depends of the value of *n*. This method is *an approximated* one. They are systems of n+1, equations with n+1, unknown coefficients. The values of the coefficients depend of the number of the connection groups and the connections of each connection group. It is showed to the Table I column 4. This method is an approximate one because one factor is added for all equations.

The checking and studying of the optical fiber available capacity are also presented. The equations, (3) and (4) must be greater or equal to zero, for full servicing all connection groups that pass through an optical fiber.

The following cases present the polynomials that calculate the available capacity of each optical link for this method for all possible values up to four, WDM capacity 30 Och. The number, the order and the size of $\Delta y_{i,j}$ are critical.

-If no one-connection group passes through an optical link the polynomial function is constant.

$y_{j,0+1}= \alpha_0*(0+1)^0$

$y_{j,0+1}=30.$

-If only one-connection group passes through an optical link the polynomial function is of the first degree.

$\Delta y_{i,1}$,	$y_{i,1+1}= \alpha_0*(1+1)^0 +\alpha_1*(1+1)^1$
1	$y_{i,1+1}= (30+1)*(1+1)^0 -1*(1+1)^1 =29$
2	$y_{i,1+1}= (30+2)*(1+1)^0 -2*(1+1)^1=28$
3	$y_{i,1+1}= (30+3)*(1+1)^0 -3*(1+1)^1=27$
4	$y_{i,1+1}= (30+4)*(1+1)^0 -4*(1+1)^1=26$

-If only two-connection groups pass through an optical link the polynomial function is of the second degree.

$\Delta y_{i,1}$,	$\Delta y_{i,2}$,	$y_{i,1+1}= \alpha_0*(2+1)^0 +\alpha_1*(2+1)^1+\alpha_2*(2+1)^2$
1 ,	1,	$y_{i,2+1}= 31*(2+1)^0 -1.0*(2+1)^1+0.000*(2+1)^2 =28$
2 ,	1,	$y_{i,2+1}= 33*(2+1)^0 -3.5*(2+1)^1+0.500*(2+1)^2=27$
1 ,	2,	$y_{i,2+1}= 30*(2+1)^0 +0.5*(2+1)^1-0.500*(2+1)^2 =27$
3 ,	1,	$y_{i,2+1}= 35*(2+1)^0 -6.0*(2+1)^1+1.000*(2+1)^2 =26$
2 ,	2,	$y_{i,2+1}= 32*(2+1)^0 -2.0*(2+1)^1-0.000*(2+1)^2=26$
1 ,	3,	$y_{i,2+1}= 29*(2+1)^0 +2.0*(2+1)^1-1.000*(2+1)^2=26$
4 ,	1,	$y_{i,2+1}= 37*(2+1)^0 -8.5*(2+1)^1+1.500*(2+1)^2=25$
3 ,	2,	$y_{i,2+1}= 34*(2+1)^0 -4.5*(2+1)^1+0.500*(2+1)^2 =25$
2 ,	3,	$y_{i,2+1}= 31*(2+1)^0 -0.5*(2+1)^1-0.500*(2+1)^2=25$
1 ,	4,	$y_{i,2+1}= 28*(2+1)^0 +3.5*(2+1)^1-1.500*(2+1)^2=25$
4 ,	2,	$y_{i,2+1}= 36*(2+1)^0 -7.0*(2+1)^1+1.000*(2+1)^2=24$
3 ,	3,	$y_{i,2+1}= 33*(2+1)^0 -3.0*(2+1)^1+0.000*(2+1)^2 =24$
2 ,	4,	$y_{i,2+1}= 30*(2+1)^0 +1.0*(2+1)^1-1.000*(2+1)^2=24$

e.t.c

III. THE PROBLEM AND ITS SOLUTION

A. The problem

The network topology and other parameters are known as WDM and optical fiber capacity, one optical fiber per link with an extension to a 1+1 fiber protection system. So this network is characterized by one working fiber per link, edges of two links, links of two optical fibers, one for working and one for protection. The connections are lightpaths originating in the source nodes and terminating at the destination nodes proceeding from preplanned optical working paths. The nodes have wavelength conversion capability. The problem solution is to calculate the final available capacity of the network for a given traffic table first without any failure, second after a single failure of a cut link when the restoration is done by shortest algorithm with the approximate polynomial finite difference method and the linear finite difference one. The traffic table contains the number of the node pairs, the node pairs and the number of the connections of each node pair when their working paths are preplanned and the connections that have been set up, are protected dynamically by the shortest path algorithm that uses the available capacity (no busy capacity) of each link.

B. The formulation

A difference table (table 1) is calculated for each optical fiber and the problem is solved using two methods, the first is the linear function and the second the approximate polynomial function.

The formulation of the *linear function* method is presented below. The available capacity of all optical fibers is written as a column matrix. The equation of the linear function method is written as

$$Y_2=Y_1-A*Xn-Dy \qquad (5)$$

A is a matrix that shows the active network links Y_2,Y_1 ,Xn , *Dy* are column matrices consisting of elements related to the available capacity of each fiber, capacity that offered for connections groups that pass through each, connection group size, restoration optical channels in wavelengths of each fiber respectively. When all connections have been done then each element of Y_2 must be greater than or equal to zero. In other cases some connections are not set up.

The equation of the polynomial function method is similar to that of the equation (4) but for all network fibers there are two column matrices, the left one that is equals with the right one. When all connections have been set up then each element of the column matrix must be greater or equal to zero. In other cases some connections are not possible. The total final available capacity of the network for the linear function method is the following

$$\sum_{i=1}^{2p} y_{i,\,n(i)+1} = \sum_{i=1}^{2p} y_{1,i} - \sum_{i=1}^{2p}\sum_{j=1}^{n(i)} a_{i,j}*x_j - \sum_{i=1}^{2p} dy_i \qquad (6)$$

$$y_{i,\,n(i)+1} >=0 \;,\; y_{1,i} >=0,\; a_{i,j} >0 \;,\; dy_i >0 \;,\; x_j>0$$

The total final available capacity of the network for the polynomial function method is the following

$$\sum_{i=1}^{2p} y_{i,\,n(i)+1} = \sum_{i=1}^{2p}\sum_{r=0}^{n(i)} \alpha_r * (n(i)+1)^r \qquad (7)$$

$$y_{i,\,n(i)+1} >=0 \;,\; n(i)>=0$$

C. Detailed description of the algorithm

The algorithm describes the operation of the WDM optical fiber mesh network with 1+1 optical fiber protection, the working connections passed through preplanned optical paths but the restoration is done dynamically with the shortest path algorithm. The algorithm is driven by suitable data and then simulates the actual dynamic behavior of the network. Simulation language is critical to the economic feasibility of the entire investigation. TURBO PASCAL is used to program the model. In this section the detailed presentation of this algorithm is showed. The algorithm is as follows.

First step (Network parameters)

Initially the following data are known: network topology, node number, edge number, link number per edge , working optical fiber number per link , protection optical fiber number per link , wavelength number per optical fiber, optical fiber numbering. This information allows the computer to draw a graph and an OXC is on the vertex of the graph. Each edge corresponds to two links with opposite direction to each other. All fibers have the same wavelength number and all links the same fiber number. The computer reads the adjacent matrix and is informed about the network topology.

Second step (Connection selections)

In this step, the connection node pair number, the connection node pair selection for connections and the desired connection group size are done. The preplanned working optical paths for connections of every node pair are also provided.

TABLE 2
THE SYMBOLS OF THIS PAPER

SN	Symbol	Comments
1	q	**The node number**
2	p	**The edge number**
3	G(V,E)	The network graph
4	V(G)	**The network node set**
5	E(G)	**The network edge set**
6	2p	The number of working and backup fiber for 1+1 line protection
7	n	**The number of source – destination nodes pairs of the network**
8	Xn	Column matrix with dimension (nx1) and elements the connection group sizes of the corresponding source-destination nodes pairs
9	n(i)	**The number of the connection groups that passes through the fiber (*i*) and means that each fiber has different number of connection groups pass through it**
10	k	**The number of the wavelengths channels on each fiber that is the WDM system capacity**
11	Y_1	**Column matrix (2px1) with elements the installed capacity of fiber network links of the linear function method**
12	Y_2	Column matrix (2px1) with elements the unused available capacity of each fiber network link of the linear function method
13	A	**Matrix (2p x n) which shows the network active links that corresponding to working fibers**
14	$a_{i,j}$	Element of the matrix A and takes the value one if the node pair (j) passes all its working connections from the fiber (i) and zero (0) if no passes
15	$\Delta y_{i,j}$	First order of finite difference that corresponds to a group of optical connections that pass through the optical fiber *i* with serial number *j* and valid 1<=i<=2p and 1<=j<=n(i)
16	$y_{i,j}$	Unused available capacity of the optical fiber i that it is offered for optical connections group with serial number j and valid 1<=i<=2p and 1<=j<=n(i)
17	Dy	**Column matrix (2px1) and elements the restoration capacity of each optical fiber that is used**
18	dy_i	Element of the Dy column matrix and shows restoration capacity of the optical fiber *(i)*
19	d_j	**Demand of each node pairs**
20	$a_{i,r}$	Real number coefficient for the polynomial function method for the link *(i).*
21	Cav	The total network available capacity
22	Cb	The total network busy capacity
23	Cinst	The total network installed capacity

Failure-free Network Phase

Third step (Wavelength allocation)

In this step, wavelength allocation is initiated. A connection starts from the source node and progresses through the network occupying one wavelength on each optical fiber and switch to another fiber on the same or other wavelength by OXC, according to its preplanned optical path up to arrive at the destination node. The number of connections of each node pair is equal to its connection group size. After a connection has been established, the available capacity is also calculated under two methods of the finite differences. Thus the available capacity of the one method is compared to available capacity of the other method for one connection.

Forth step (Finite difference presentation)

The total available capacity of each optical fiber is calculated and represented under two finite difference methods, i.e. the linear function and the approximate polynomial function ones. Thus the total fiber capacity available under the first method is compared to the total fiber capacity available of the other method, for all connections.

Fifth step (Results and comparisons thereof)

Having the desired connection group size the total results are computed under each finite difference method that are the total network available capacity, the total network busy capacity and the total network installed capacity. These results of the linear function methods are compared to the results of the approximate polynomial function method. The results of the linear method must to be equal (or their difference is an accepted error) to the results of the approximate polynomial method.

Otherwise, the program does not operate properly and needs a general checking so that any problems are identified. If there is no failure, the program is terminated.

Network with failure Phase

When a failure occurs and a link cuts, the optical fibers of this link also cut that means the optical fiber protection 1+1 and the network topology change. The connection groups that passed through the cut link are also cut and the restoration is carried out dynamically by shortest path algorithm of other links. The computer is informed of the cut link and modifies suitably the network parameters. The cut optical fiber sets its capacity and wavelengths to zero, the connection groups that passing through the cut link set their using wavelengths to zero and through the others to free, the matrix A changes as well as the number of the group size that passing through optical fibers and the algorithm is repeated from the begin carrying out the restoration dynamically by shortest path algorithm calculating new results and after them the algorithm ends.

The complexity of this algorithm for the node number q depends on the square of the node number and the total number of the requests for connection (s) so it is written as O $(s*q^2)$. The time complexity of that algorithm is 'order q2, O$(s*q^2)$. Thus, on a 133MHz computer, q=6 and s=12, the time is 4 hundredths of second. It means that worst time consuming depends by the network size for the same computer. The consuming time of each method is about equal and their time differences are negligent.

The shortest path algorithm finds the shortest path between two given vertices in an undirected graph G=(V,E).The shortest path connects the two vertices and its length is minimum .Shortest path means that the sum of the individuals edge paths is minimum. The path length of each individual link presents an optical hop and the shortest path is the minimum sum of the optical hops.

D. Example

The network here below is studied for the best presentation of the results. It is because for larger networks are difficult to present the results as well as the number, the size of tables are larger, the dimensions of the matrices are also larger as well as the degree of the polynomial higher. It is assumed that the topology of the network is presented by the graph G (V, E). This mesh topology is used because it is a simple, palpable and an analytical example of the finite differences and it is easy to expand to any mesh topology. The vertex set has q=6 elements which are V={v_1, v_2, v_3, v_4, v_5, v_6} and the edge set has p=9 elements which are E={e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9}. Connection groups transverse the mesh networks. They correspond to n source-destination node pairs. The capacity of WDM system is 30 OCh. Table 3 presents the network parameters. The problem is solved for an instance with n=12 of 30 possible connection groups. These have their order and sizes for each source-destination node pair, their preplanned working paths as shown in table 4 and it is studied for two cases. In the first case, the finite difference methods are investigated and presented when the network is operating under normal conditions and in the second case, when the network is operating under failure conditions and the restoration is executed dynamically with the shortest path algorithm.

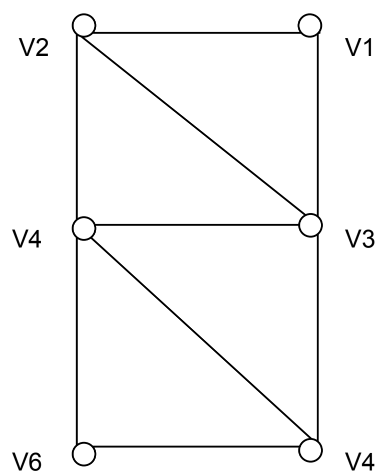

Fig. 1.The mesh topology of the network.

TABLE 3
THE NETWORK PARAMETERS

Network parameters	Amount
Node number	6
Edge number	9
Working fiber per edge	2
Working fiber per link	1
Network working fiber	18
Protection fiber per edge	2
Protection fiber per link	1
Network protection fiber	18
WDM system capacity	30

In the *first case*, the table 1 (finite difference table) of each fiber is not presented because the number of these tables is twelve (12). The higher order finite differences and the number of connection groups that pass through each optical fiber are showed in the following table 5. (Fiber, i) shows the optical fiber numbering. Optical fiber link means the corresponded link of this fiber. “Without failure” means the network phase without failure and “With failure” means the corresponded network phase.

TABLE 4

THE NODE PAIRS WITH PREPLANNED PATHS AND THE CONNECTION GROUP SIZE

Node Pair [Si,Di]	Node pair $[v_i, v_j]$	Working Path	Group size
$[S_1, D_1]$	$[v_1, v_2]$	v_1, v_2	2
$[S_2, D_2]$	$[v_1, v_3]$	v_1, v_3	3
$[S_3, D_3]$	$[v_1, v_5]$	v_1, v_3, v_5	5
$[S_4, D_4]$	$[v_2, v_3]$	v_2, v_3	3
$[S_5, D_5]$	$[v_2, v_4]$	v_2, v_4	2
$[S_6, D_6]$	$[v_2, v_5]$	V_2, v_3, v_5	2
$[S_7, D_7]$	$[v_3, v_4]$	v_3, v_4	1
$[S_8, D_8]$	$[v_3, v_6]$	V_3, v_4, v_6	2
$[S_9, D_9]$	$[v_4, v_1]$	v_4, v_3, v_1	5
$[S_{10}, D_{10}]$	$[v_4, v_5]$	V_4, v_5	1
$[S_{11}, D_{11}]$	$[v_5, v_4]$	v_5, v_3, v_4	2
$[S_{12}, D_{12}]$	$[v_6, v_1]$	V_6, v_5, v_3, v_1	5

The n(i) shows the number of the connection groups that pass through each optical fiber. The m(i)=1, 2, 3, 4, 5 of the fiber (i) is the order finite differences with $\Delta^{m(i)} y_i$ the value of this order which now is not represented. The intermediate order finite differences are not showed. Fiber 9 has 3rd order finite difference. It is obvious that optical fiber nine (9) has the larger difference table.

For the linear function (3) that provides the final available capacity of all optical fibers are written below, Y_2, Y_1, A, Dy and X_n have dimensions of (18x1), (18x1), (18x12), (18x1) and (12x1) respectively. For this case matrix A is a known matrix (18x12) which is always

28		30		2		0
30		30		0		0
22		30		8		0
20		30		10		0
25		30		5		0
30		30		0		0
28		30		2		0
30		30		0		0
25	=	30	-	5	-	0
25		30		5		0
23		30		7		0
23		30		7		0
29		30		1		0
30		30		0		0
28		30		2		0
30		30		0		0
30		30		0		0
25		30		5		0

constant because it depends on the optical paths that are preplanned and are constant. The final available capacity for each optical fiber is positive or zero so there is no problem with their connections. The total network available, busy and installed capacities of all cases are written in the following table 6.

The matrix of equation (4) that provides the final available capacity of all optical fibers has dimension is (18x1). If the degree of polynomials increases then the above writing of the polynomial numerical coefficients has error because they are difficult to be represented and introduce error in the result. This method is an approximate one and there is an inherent error in the result (fiber 9). The errors of coefficients add in the errors of the results. In the table 6 (Without failure) the network available, busy and installed capacities are written. The error of the available capacity is compensated by error of the busy capacity so that the installed capacity is constant. The values of the approximate polynomial method close the corresponded ones of the linear method. In the same table the absolute errors are also represented.

$y_{1,2}$		32 – 2*2		28
$y_{2,0}$		30		30
$y_{3,3}$		31 – 1*9		22
$y_{4,3}$		35 – 5*3		20
$y_{5,3}$		34-4.5*3 +0.5*9		25
$y_{6,1}$		30		30
$y_{7,2}$		32 - 2 *2		28
$y_{8,1}$		30		30
$y_{9,4}$	=	29 + 2.3425*4 -1.5*16+0.1665*64	=	25.026
$y_{10,2}$		35 - 5*2		25
$y_{11,3}$		38 - 9.5*3+1.5*9		23
$y_{12,3}$		29 + 2.5*3-1.5*9		23
$y_{13,2}$		31 - 1*2		29
$y_{14,1}$		30		30
$y_{15,2}$		32-2*2		28
$y_{16,1}$		30		30
$y_{17,1}$		30		30
$y_{18,2}$		35 - 5*2		25

TABLE 5

THE HIGH ORDER FINITE DIFFERENCES AND THE CONNECTION GROUPS OF EACH FIBER

Fiber,i	Optical fiber link	Without Failure n(i)	Without Failure m(i)	With Failure n(i)	With Failure m(i)
1	$<v_1, v_2>$	1	1	1	1
2	$<v_2, v_1>$	0	0	0	0
3	$<v_1, v_3>$	2	2	2	2
4	$<v_3, v_1>$	2	2	2	2
5	$<v_2, v_3>$	2	2	2	2
6	$<v_3, v_2>$	0	0	1	1
7	$<v_2, v_4>$	1	1	2	2
8	$<v_4, v_2>$	0	0	0	0
9	$<v_3, v_4>$	3	3	0	0
10	$<v_4, v_3>$	1	1	1	1
11	$<v_3, v_5>$	2	2	3	3
12	$<v_5, v_3>$	2	2	1	1
13	$<v_4, v_5>$	1	1	1	1
14	$<v_5, v_4>$	0	0	1	1
15	$<v_4, v_6>$	1	1	0	0
16	$<v_6, v_4>$	0	0	0	0
17	$<v_5, v_6>$	0	0	1	1
18	$<v_6, v_5>$	1	1	1	1

In *the second case*, the network is operating under failure conditions with the link $< v_3, v_4>$ cut that corresponds to the fiber 9. For this case matrix A changed because the link $< v_3, v_4>$ cut. When this link cuts then the connections groups of the node pair $[v_3, v_4]$, $[v_3, v_6]$ and $[v_5, v_4]$ are also cut (table 4). The restoration is executed dynamically by the shortest path algorithm and connections are done by new optical paths. So for the node pair [v_3, v_4] the restoration light path is v_3, v_2, v_4 (links 6 and 7), for the node pair [v_3, v_6] the restoration light path is v_3, v_5, v_6 (links 11 and 17) and for the node pair [v_5, v_4] the restoration light path is v_5, v_4 (link 14). The light path lengths of these node pairs when the network works under normal conditions are 1+2+2=5 and the corresponded connections lengths are 1*1+2*2+2*2 =9 by links 9, 12 and 15 but when the network works under failure condition with the link < v_3, v_4> cuts the restoration light path lengths are 2+2+1=5 and the corresponded connections lengths are 1*2+2*2+2*1=8. So for this example, the restoration connections lengths are shorter than the working connections lengths. As in the first case, I don't present table 1 (finite difference table) of each fiber because the number of tables is thirteen (13). In table 5, with failure phase columns, there is the number of connection groups that pass through each link after the link < v_3, v_4> cut that corresponds to the fiber 9. When the restoration is executed by shortest path algorithm, the follows are written.

TABLE 6
THE AVAILABLE, BUSY AND INSTALLED CAPACITIES

	Capacity	Linear	Approximate Polynomial	Absolute Error
Without Failure	Cav	481	481,026	0.026
	Cb	59	58,974	0.026
	Cinst	540	540	0
With Failure	Cav	452	452,0180	0.018
	Cb	58	57,982	0.018
	Cinst	510	510	0

For the linear function (3) the dimensions remain as previously but the elements of the effected links change. So the linear function provides

28		30		2		0
30		30		0		0
22		30		8		0
20		30		10		0
25		30		5		0
29		30		0		1
27		30		2		1
30	=	30	-	0	-	0
0		0		0		0
25		30		5		0
21		30		7		2
25		30		5		0
29		30		1		0
28		30		0		2
30		30		0		0
30		30		0		0
28		30		0		2
25		30		5		0

The final available capacity for each optical fiber is positive or zero so there is no problem with their connections. The results are written in table 6, lines "With Failure".

As previously, the order of finite differences of each fiber is dependent on the number of node pairs that passes their connection groups through. The following matrix provides the final available capacity of all optical fibers and its dimension is (18x1) but some elements change. The comments for this case are the same with the previous ones but the coefficients, the base and degree of polynomials of the links that effected change. When the network operates under these conditions, the network available capacity given by the equations (4) and (5) and the results are given in the table 6, lines "With Failure". The restoration is full because there is sufficient available capacity.

$y_{1,2}$		32 - 2*2		28
$y_{2,1}$		30		30
$y_{3,3}$		31 –1*9		22
$y_{4,3}$		35 -5*3		20
$y_{5,3}$		34 -4.5*3 +0.5*9		25
$y_{6,2}$		31 - 1*2		29
$y_{7,3}$		33 -3.5*3 +0.5*9		27
$y_{8,1}$		30		30
$y_{9,0}$	=	0 (cut)	=	0
$y_{10,2}$		35 - 5*2		25
$y_{11,4}$		41 - 14,9907*4+4.5*16-0.5003*64		21.0180
$y_{12,2}$		35 - 5*2		25
$y_{13,2}$		31 - 1*2		29
$y_{14,2}$		32 -2*2		28
$y_{15,1}$		30		30
$y_{16,1}$		30		30
$y_{17,2}$		32 -2*2		28
$y_{18,2}$		35 - 5*2		25

I change the traffic table 4, column *group size* by putting the same value at all group sizes starting from 1 and ending to 10. Larger values are not put because the routing by normal conditions is not possible. It is ought to the preplanned lightpaths which pass from link 9, 3*11= 33>30 that is the link capacity.

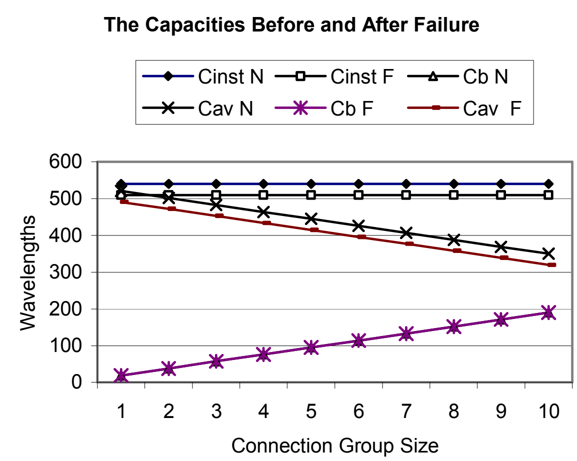

Fig 2.The installed, busy and available capacities before (letter N) and after the failure (letter F).

Some notes on the figure 2 are presented below.

- -The figure was produced by linear function method.
- -The installed and the available capacities are reduced after the failure occurs.
- -The busy capacity remains the same before and after the failure because the lengths of the cut and restoration lightpaths are the same.
- -The failure is the cut link <3, 4>.

In the figure 3, the errors of polynomial method versus the linear one for the available and busy capacities are showed. Each compensates the other. They are same before and after the failure (link <3, 4> cut) because the criteria of degrees, orders and group sizes are also same.

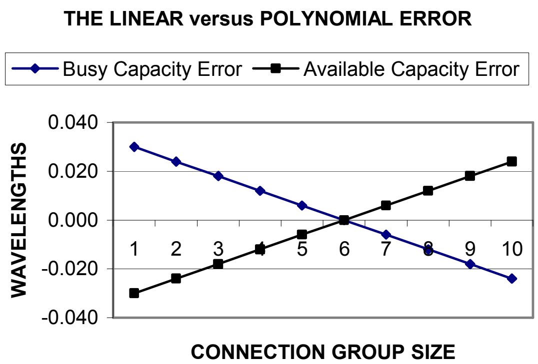

Fig. 3.The error of the approximate polynomial method versus linear one for busy and available capacities.

E. Discussion

In communications, network survivability is defined as the capability of a communication network to resist any interruption or disturbance of service, particularly by warfare, fire, earthquake, harmful radiation or other physical or natural catastrophes.

The network of this study has physical protection on line layer and logical protection on optical path layer. The finite differences are used on optical path layer.

In the dynamical restoration, the dynamic routing method requires each OXC network controller to store only necessary local information and the rerouting decision is made according to the network status (e.g. configuration, available spare capacity and so on) at the time of network component failures .

The use of the finite differences is possible for the study of the problems of the protection and restoration of the optical connections. The solution of each problem with two different ways is used for the verification and validation of results. Differences between them must be in accepted tolerances. These methods solve problems with rather small networks because when the connections groups that pass through a link increases, then the number of the n+1equations with n+1 unknown of the system also increases and it is difficult to be solved. It is assumed that all of the values, polynomial coefficients and results of mathematical computations are presented with suitable precision. An optical mesh network designed under the infinite precision assumption will not perform up to design expectations if implemented with reduced precision. In many cases the degradations can be severe. In this problem, the failures that can restore are single wavelength crash, optical fiber cut and partial node failure.

IV. CONCLUSION

By applying WDM the optical networks are capable of carrying many independent channels, which are carried on different wavelengths, over a single optical fiber. This allows the network to transport huge amounts of data that are needed for many current and future communication services, which play a very important role in many of our daily social and economical activities. The main drawback is the failures that can lead to the loss of a large amount of data. Thus the suitable strategy must use to minimize all such failures effects.

In this paper, the link failure restoration is searched by approximate finite difference method, compare it with linear function one and show their use to solve these problems with an efficient way. When a failure occurs, new lightpaths can be set up or torn down to alleviate the failure repercussions. The strategy is done by traffic rerouting during failures. The calculation of the new lightpaths is done by shortest path algorithm using the network spare resources and the traffic rerouting is presented by finite difference methods, the linear and the approximate polynomial ones. The method of finite difference polynomial approximate method presents the results with a reduced precision way but the linear one with full precision one. This recovery technique is importance because uses the spare resources with a more efficient way and provides increasingly stringent levels of reliability that operator demand for their future networks.

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