Influence of Mutual Coupling on the Transmission Strategy Design for MIMO Correlated Rayleigh Fading Channels

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***Abstract*—The influence of mutual coupling among multiple input multiple output antenna (MIMO) systems is usually not negligible due to limited antenna spacing available in practice. A thorough understanding of such influence, however, is not available in the literature, although it is important to the design of a transmission strategy. In this paper, by formulating the equivalent channel matrix in terms of S-parameters that characterize mutual coupling, we obtain an explicit expression for the channel capacity. The maximization of the channel capacity so obtained leads to the optimal pre-coding strategy for the transmitter. Numerical examples are presented to illustrate the theory.**

***Index Terms*—Mutual coupling, multiple input multiple output (MIMO) channels, covariance matrix, channel capacity.**

I. Introduction

MIMO (Multiple Inputs Multiple Outputs) [1]–[15] is the improved technology in wireless communication system which has been introduced after making the prevailing and widely used technology (SISO) better. In SISO (Single Input Single Output) technology single antenna is used for both transmission and for the reception but now it has been improved, instead of single antenna now multiple or more than one antenna is been used for both transmission and reception. With the use of MIMO technology wireless communication system is now faster and the reception range and connection capacity has been improved. There are different types of technologies which uses smart antenna. Other than MIMO we have SIMO (Single Input Multiple Output) or MISO (Multiple Input Single Output) but in this paper we will focus on the MIMO technology and its wide acceptance in different wireless communication system. MIMO becomes more important technology because in recent years there has been a rapid growth in the use of wireless communication system. Now MIMO is even introduced in computer laptops.

It is known that the finite antenna spacing in array antennas introduces spatial correlation. This finite spacing is also responsible for mutual coupling which adversely affects signal transmission and reception due to the resulting antenna impedance mismatch. Mutual coupling in an antenna array refers to the phenomenon in which the current induced by a received voltage at one antenna can radiate an electromagnetic field affecting others. The mutual coupling effect is especially pronounced in tightly spaced arrays. Because of a considerable demand for compact size mobile station (MS) terminals, the effect of mutual coupling cannot be neglected and thus has to be taken into account while assessing the MIMO link performance. The problem of mutual coupling in MIMO systems for the case of peer-to-peer communication has been addressed via simulations and measurements in [16]–[21].

However, in most articles discussing MIMO systems, mutual coupling is not accounted for and studied because it is normally considered as a factor in antenna design or microwave field, rather than a concern in signal processing and communications. Thus, the antenna elements are usually assumed to be well isolated from each other so that no mutual coupling exists. But in practice, such an assumption is often unrealistic, particularly for compact MIMO systems, where the separation between adjacent antennas is small. In this case, the effect of mutual coupling should not be ignored and its impact on the channel capacity must be considered in the MIMO system design.

Previously published studies tackle the coupling issue in MIMO systems either from the viewpoint of signal processing and communications or from the perspective of antenna design and microwave circuits. In the former treatment, mutual coupling is normally characterized by a coupling matrix before and after the channel matrix, without going to the physical details. The consequence is failure to explicitly link the mutual coupling parameters to the MIMO system performance except for some numerical and simulation results [22]–[25]. The latter treatment, on the other hand, focuses on the concern of the microwave field, ending up with several good models to account for mutual coupling in antenna arrays [26]–[28]. The inclusion of these models in the overall communication system to form a solid framework for MIMO system design optimization remains an unsolved issue.

The issue of optimal transmission with perfect or statistic channel information feedback has been addressed in the literature by assuming that mutual coupling is negligible at both transmit and receiver antenna arrays [29], [32]. The corresponding strategy for the practical cases with mutual coupling remains unknown, and is therefore the focus of this

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paper.

The rest of this paper is organized as follows. In Section II, we present our system model and the equivalent MIMO channel matrix. In Section III, we present our transmission strategy. Section IV presents some numerical results based on our proposed scheme. Finally, Section V contains some concluding remarks.

II. EQUIVALENT MIMO CHANNEL MATRIX

Consider a wireless MIMO system with m transmit antennas and n receive antennas. Let $\mathbf{H}$ denote the $n \times m$ channel matrix for the ideal situation without mutual coupling at both transmit and receive arrays.

To obtain an equivalent channel matrix to account for mutual coupling, we invoke the Path-Based Channel Model described in [26], which can be briefly illustrated in Fig.1. The MIMO channel shown there is characterized by the scattering parameter S (or simply called S parameter) with $\mathbf{a}_T$ and $\mathbf{b}_R$ denoting, respectively, the inward and outward propagation wave vectors. The matrix $\mathbf{S}_{TT}$ describes the excitation port of the transmit array, containing the scattering parameters for self coupling and mutual coupling [31]. The matrix $\mathbf{S}_{RR}$ is for the receive array defined in a similar manner. By the same token, we can define $\mathbf{S}_{TR}$ and $\mathbf{S}_{RT}$. The S matrices are expressible in terms of their corresponding impedance matrices. For example, $\mathbf{S}_{RR}$ is related to the impedance matrix of receive-antenna array, $\mathbf{Z}_{RR}$, by [34]

$$\mathbf{Z}_{RR} = (\mathbf{I} + \mathbf{S}_{RR})(\mathbf{I} - \mathbf{S}_{RR})^{-1} Z_0. \tag{1}$$

where Z_0 is a chosen reference impedance for computing the S parameters. By using circuit-network theory, and appropriately setting boundary conditions for far field assumption, it turns out that [17]

$$\mathbf{b}_R = (\mathbf{I} + \frac{\mathbf{Z}_{RR}}{Z_0})^{-1} \frac{\mathbf{Z}_{RT}}{Z_0} (\mathbf{I} - \mathbf{S}_{TT}) \mathbf{a}_T. \tag{2}$$

Notice that $\mathbf{Z}_{RT}$ represents the trans-impedance matrix from the receive-antenna voltages to the transmit antenna currents. In the conventional usage in MIMO systems, $\mathbf{a}_T$ corresponds to the $m \times 1$ channel input vector $\mathbf{x}$, and $\mathbf{b}_R$ corresponds to the channel output vector. Let $\mathbf{y}$ denote the $n \times 1$ noisy output of the MIMO channel. We can rewrite (2) as

$$\mathbf{y} = \widehat{\mathbf{H}}\mathbf{x} + \mathbf{n}. \tag{3}$$

where $\mathbf{n}$ is zero-mean additive Gaussian noise vector with distribution $n \sim CN(0, \sigma^2 I)$ and $\hat{H}$ signifies the equivalent channel matrix given by

$$\widehat{\mathbf{H}} = (\mathbf{I} + \frac{\mathbf{Z}_{RR}}{Z_0})^{-1} \frac{\mathbf{Z}_{RT}}{Z_0} (\mathbf{I} - \mathbf{S}_{TT}). \tag{4}$$

For the ideal case without mutual coupling, we can set $\mathbf{S}_{TT}$=0 and $\mathbf{S}_{RR}$ = 0 whereby the above expression reduces to the conventional channel matrix, $\mathbf{H}$. Then we get

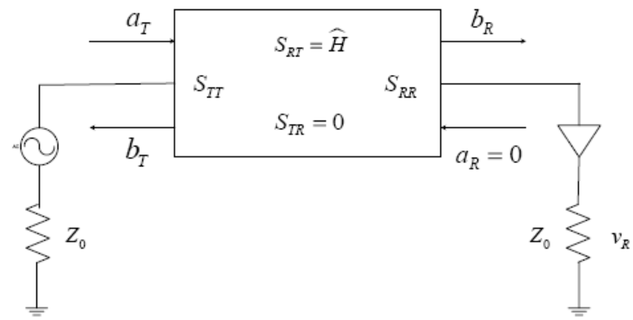

Fig. 1. Network model for entire MIMO communication system

$$\widehat{\mathbf{H}} = 2(\mathbf{I} + \frac{\mathbf{Z}_{RR}}{Z_0})^{-1} \mathbf{H} (\mathbf{I} - \mathbf{S}_{TT}). \tag{5}$$

The ergodic channel capacity can be easily obtained from [22], as shown by

$$C = E_H \left[\log\det \left(\mathbf{I} + \frac{\widehat{\mathbf{H}}^{\dagger} \widehat{\mathbf{Q}} \widehat{\mathbf{H}}}{\sigma^2} \right) \right]. \tag{6}$$

where $\hat{Q}$ represents the covariance matrix of the transmitted symbol vector $\mathbf{x}$, and $E_H[\cdot]$ denotes the expectation taken over random matrix $\mathbf{H}$. Given the constraint on the total transmitted power P, i.e., the problem is to optimize matrix $\hat{Q}$, such that

$$\widehat{\mathbf{Q}} = \arg \max_{\widehat{\mathbf{Q}}:\mathrm{trace}(\widehat{\mathbf{Q}})=P} E_H \left[\log\det \left(\mathbf{I} + \frac{\widehat{\mathbf{H}}^{\dagger} \widehat{\mathbf{Q}} \widehat{\mathbf{H}}}{\sigma^2} \right) \right]. \tag{8}$$

Throughout the paper, we assume that the transmitter knows all the impendence and S-parameter matrices of both the transmit and receive antennas through feedback.

III. OPTIMAL TRANSMIT STRATEGY

Similar to [29], we assume that the elements of $\mathbf{H}$ are all zero mean complex Gaussians and $\mathbf{H}$ has i.i.d. rows and correlated columns. The distribution of the i th row of $\mathbf{H}$ is given by

$$\mathbf{H}_i \sim CN(0, \mathbf{R}), \forall i \in 1, 2, ..., n. \tag{9}$$

So $\mathbf{H}$ could be written as

$$\mathbf{H} = \mathbf{W}\mathbf{R}^{\frac{1}{2}}. \tag{10}$$

where $\mathbf{W}$ has i.i.d. complex Gaussian distributed entries w_{ij} with zero mean and variance one. Now we consider the situation that mutual coupling exists among both the transmit and receive antenna elements. In this case, the equivalent channel matrix is given by (5).

This way the new channel matrix with mutual coupling could be written as

$$\begin{aligned} \widehat{\mathbf{H}} &= 2(\mathbf{I} + \frac{\mathbf{Z}_{RR}}{Z_0})^{-1} \mathbf{W}\mathbf{R}^{\frac{1}{2}} (\mathbf{I} - \mathbf{S}_{TT}) \\ &= \mathbf{B}\mathbf{W}\mathbf{A}. \end{aligned} \tag{11}$$

where

$$\mathbf{A} = \mathbf{R}^{\frac{1}{2}}(\mathbf{I} - \mathbf{S}_{TT}). \quad (12)$$

$$\mathbf{B} = 2(\mathbf{I} + \frac{\mathbf{Z}_{RR}}{Z_0})^{-1}. \quad (13)$$

We assume the coupling matrix $\mathbf{S}_{RR}$ and $\mathbf{S}_{TT}$ are hermitian, resulting that **A** and **B** are also hermitian.

From (6), the ergodic capacity could be written as

$$\begin{aligned} C &= E[\log\det(\mathbf{I} + \frac{\widehat{\mathbf{H}}^{\dagger}\widehat{\mathbf{H}}\widehat{\mathbf{Q}}}{\sigma^2})] \\ &= E[\log\det(\mathbf{I} + \frac{1}{\sigma^2}(\mathbf{BWA})^{\dagger}\mathbf{BWA}\widehat{\mathbf{Q}})] \\ &= E[\log\det(\mathbf{I} + \frac{1}{\sigma^2}\mathbf{W}^{\dagger}\mathbf{B}^{\dagger}\mathbf{BWA}\widehat{\mathbf{Q}}\mathbf{A}^{\dagger})]. \quad (14) \end{aligned}$$

Let the eigenvalue decomposition of **A** and $\widehat{Q}$ be

$$\begin{aligned} \mathbf{A} &= \mathbf{R}^{\frac{1}{2}}(\mathbf{I} - \mathbf{S}_{TT}) \\ &= \mathbf{U}_A^{\dagger}\mathbf{\Lambda_A}\mathbf{U}_A \\ &= \mathbf{U}_A^{\dagger}\mathrm{diag}(\lambda_1^A, \ldots, \lambda_m^A)\mathbf{U}_A. \quad (15) \end{aligned}$$

$$\begin{aligned} \widehat{\mathbf{Q}} &= \mathbf{U}_{\widehat{Q}}^{\dagger}\mathbf{\Lambda}_{\widehat{\mathbf{Q}}}\mathbf{U}_{\widehat{Q}} \\ &= \mathbf{U}_{\widehat{Q}}^{\dagger}\mathrm{diag}(\lambda_1^{\widehat{Q}}, \ldots, \lambda_m^{\widehat{Q}})\mathbf{U}_{\widehat{Q}}. \quad (16) \end{aligned}$$

Continue with (14), we obtain

$$\begin{aligned} C &= E[\log\det(\mathbf{I} + \frac{1}{\sigma^2}\mathbf{W}^{\dagger}\mathbf{B}^{\dagger}\mathbf{BWA}\widehat{\mathbf{Q}}\mathbf{A}^{\dagger})] \\ &= E[\log\det(\mathbf{I} + \frac{1}{\sigma^2}\mathbf{W}^{\dagger}\mathbf{B}^{\dagger}\mathbf{BWU}_A^{\dagger}\mathbf{\Lambda_A}\mathbf{U}_A\widehat{\mathbf{Q}}\mathbf{U}_A^{\dagger}\mathbf{\Lambda_A}^{\dagger}\mathbf{U}_A)] \\ &= E[\log\det(\mathbf{I} + \frac{1}{\sigma^2}\mathbf{U}_A\mathbf{W}^{\dagger}\mathbf{B}^{\dagger}\mathbf{BWU}_A^{\dagger}\mathbf{\Lambda_A}\mathbf{U}_A\widehat{\mathbf{Q}}\mathbf{U}_A^{\dagger}\mathbf{\Lambda_A}^{\dagger})]. \quad (17) \end{aligned}$$

In order to proceed, we need to derive the distribution of $W_i^{'}$, the i th row of matrix $W' = BW$.

We know that for any matrix **X**,

$$\mathbf{X}^{-1} = \frac{1}{\det(\mathbf{X})}\mathrm{adj}\ \mathbf{X}. \quad (18)$$

where adj X is the adjugate(or adjoint) matrix of X.

We assume matrix **D** is the adjugate matrix of $I + \frac{Z_{RR}}{Z_0}$ and $\mathbf{D}_i$ is the i th row of **D**, for receive coupling matrix **B**, we have

$$\begin{aligned} \mathbf{B} &= 2(\mathbf{I} + \frac{\mathbf{Z}_{RR}}{Z_0})^{-1} \\ &= \frac{2}{\det(\mathbf{I} + \frac{\mathbf{Z}_{RR}}{Z_0})}\mathrm{adj}(\mathbf{I} + \frac{\mathbf{Z}_{RR}}{Z_0}) \\ &= \frac{2}{\det(\mathbf{I} + \frac{\mathbf{Z}_{RR}}{Z_0})}\mathbf{D}. \quad (19) \end{aligned}$$

We let B_i denote the i th row of matrix B and W_j denote the jth column of matrix W. This way,

$$\mathbf{W}_i^{'} = (\mathbf{BW})_i = (\mathbf{B}_i\mathbf{W}_1, \ldots, \mathbf{B}_i\mathbf{W}_m). \quad (20)$$

and

$$\begin{aligned} E[\mathbf{W}_i^{'\dagger}\mathbf{W}_i^{'}] &= E[(\mathbf{BW})_i^{\dagger}(\mathbf{BW})_i] \\ &= E[\begin{pmatrix} \overline{\mathbf{B}_i\mathbf{W}_1} \\ \vdots \\ \overline{\mathbf{B}_i\mathbf{W}_m} \end{pmatrix}(\mathbf{B}_i\mathbf{W}_1, \ldots, \mathbf{B}_i\mathbf{W}_m)] \\ &= E[\sum_{j=1}^{n}\sum_{k=1}^{n}\begin{pmatrix} \overline{b_{ij}w_{j1}}b_{ik}w_{k1} & \cdots & \overline{b_{ij}w_{j1}}b_{ik}w_{km} \\ \vdots & \ddots & \vdots \\ \overline{b_{ij}w_{jm}}b_{ik}w_{k1} & \cdots & \overline{b_{ij}w_{jm}}b_{ik}w_{km} \end{pmatrix}] \\ &= \begin{pmatrix} E[||\mathbf{B}_i||^2||\mathbf{W}_1||^2] & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & E[||\mathbf{B}_i||^2||\mathbf{W}_m||^2] \end{pmatrix} \\ &= \frac{4}{(\det(\mathbf{I} + \frac{\mathbf{Z}_{RR}}{Z_0}))^2}||\mathbf{D}_i||^2\mathbf{I}. \quad (21) \end{aligned}$$

where $\| x \|$ means the norm of vector **x**, i.e. $(\sum_{i=1}^{m} x_i^2)^{1/2}$. This way, we derive

$$\mathbf{W}_i^{'} = (\mathbf{BW})_i \sim CN(0, \frac{4}{(\det(\mathbf{I} + \frac{\mathbf{Z}_{RR}}{Z_0}))^2}||\mathbf{D}_i||^2\mathbf{I}). \quad (22)$$

Here, we also note that if we do not consider transmit mutual coupling, then the distribution of the i th row of the channel matrix $H' = BH$ only with receive mutual coupling is

$$\begin{aligned} E[(\mathbf{H}_i^{'})^{\dagger}\mathbf{H}_i^{'}] &= E[(\mathbf{H}_i^{'})^{\dagger}\mathbf{H}_i^{'}] \\ &= E[(\mathbf{BWR}^{\frac{1}{2}})_i^{\dagger}(\mathbf{BWR}^{\frac{1}{2}})_i] \\ &= \frac{4}{(\det(\mathbf{I} + \frac{\mathbf{Z}_{RR}}{Z_0}))^2}||\mathbf{D}_i||^2\mathbf{R}. \quad (23) \end{aligned}$$

which leads to

$$\mathbf{H}_i^{'} \sim CN(0, \frac{4}{(\det(\mathbf{I} + \frac{\mathbf{Z}_{RR}}{Z_0}))^2}||\mathbf{D}_i||^2\mathbf{R}). \quad (24)$$

Compared with (9), we can see that if each row of the channel matrix H has independent and identical distribution, after receive mutual coupling is included, each row of the new channel matrix H_ is not independent yet, and the distribution of each row of H_ is also different. But the difference among the covariance matrix of each row only lies in an coefficient $\frac{4}{(\det(\mathbf{I}+\frac{\mathbf{Z}_{RR}}{Z_0}))^2}\mathbf{D}_i$, which is determined by the mutual coupling of the receive antenna.

Now come back to the ergodic capacity (17), since $\mathbf{U}_A$ is unitary, it is easy to see that $\mathbf{W}_i^{'}\mathbf{U}_A^{\dagger} \sim \mathbf{W}_i^{'}$, i.e. $\mathbf{W}_i^{'}\mathbf{U}_A^{\dagger}$ and $\mathbf{W}_i^{'}$ are identically distributed. We derive

$$C = E[\log\det(\mathbf{I} + \frac{1}{\sigma^2}\mathbf{W}'^{\dagger}\mathbf{W}'\mathbf{\Lambda_A}\mathbf{U}_A\widehat{\mathbf{Q}}\mathbf{U}_A^{\dagger}\mathbf{\Lambda_A}^{\dagger})]. \quad (25)$$

with power constraint $trace(\widehat{Q}) = P$.

If we let

$$\mathbf{Q}' = \mathbf{\Lambda_A}\mathbf{U}_A\widehat{\mathbf{Q}}\mathbf{U}_A^{\dagger}\mathbf{\Lambda_A}^{\dagger} = \mathbf{U}'^{\dagger}\mathbf{\Lambda}'\mathbf{U}' \quad (26)$$

where U' and Λ' are the spectral decomposition of Q', then we have

$$C = E[\log\det(\mathbf{I}_n + \mathbf{W}'^{\dagger}\mathbf{W}'\mathbf{Q}')] \quad (27)$$

Since U' is unitary and does not change the distribution of W',

$$C = E[\log\det(\mathbf{I}_n + \mathbf{W}'^{\dagger}\mathbf{W}'\mathbf{\Lambda}')] \quad (28)$$

with power constraint

$$\begin{aligned} \mathrm{tr}(\widehat{\mathbf{Q}}) &= \mathrm{tr}(\mathbf{U}_A^{-1}\mathbf{\Lambda}_{\mathbf{A}}^{-2}\mathbf{Q}'(\mathbf{U}_A^{\dagger})^{-1}) \\ &= \mathrm{tr}(\mathbf{\Lambda}_{\mathbf{A}}^{-2}\mathbf{U}'^{\dagger}\mathbf{\Lambda}'\mathbf{U}') \\ &= P \quad (29) \end{aligned}$$

So the capacity maximization problem could be expressed as

$$C = \max_{\mathrm{tr}(\mathbf{\Lambda}_{\mathbf{A}}^{-2}\mathbf{U}'^{\dagger}\mathbf{\Lambda}'\mathbf{U}')=P} E[\log\det(\mathbf{I}_n + \mathbf{W}'^{\dagger}\mathbf{W}'\mathbf{\Lambda}')] \quad (30)$$

where the maximization is over all unitary U' and diagonal Λ'.

By lemma 1 in [19], $tr(\Lambda_A^{-2}U'^{+}\Lambda' U')$ is simultaneously minimized for all Λ' by $U' = I_m$. Thus, $U' = I_m$ will maximize (30). Inserting $U' = I_m$ into (26), we get

$$\widehat{\mathbf{Q}} = \mathbf{U}_A^{\dagger}\mathbf{\Lambda}_{\mathbf{A}}^{\dagger}\mathbf{\Lambda}'|\mathbf{\Lambda}_{\mathbf{A}}\mathbf{U}_A \quad (31)$$

By (16), we have

$$\mathbf{U}_{\widehat{Q}} = \mathbf{U}_A \quad (32)$$

and

$$\mathbf{\Lambda}_{\widehat{\mathbf{Q}}} = \mathbf{\Lambda}_{\mathbf{A}}^{\dagger}\mathbf{\Lambda}'\mathbf{\Lambda}_{\mathbf{A}} \quad (33)$$

Inserting (32) and (33) into (30), we have

$$C = \max_{\mathrm{tr}(\mathbf{\Lambda}_{\widehat{\mathbf{Q}}})=P} E[\log\det(\mathbf{I}_n + \mathbf{W}'^{\dagger}\mathbf{W}'\mathbf{\Lambda}_{\mathbf{A}}\mathbf{\Lambda}_{\widehat{\mathbf{Q}}}\mathbf{\Lambda}_{\mathbf{A}}^{\dagger})] \quad (34)$$

Therefore, the optimal input covariance matrix $\widehat{Q}$ that maximizes the ergodic capacity in (6) has the same eigenvectors as the matrix $A = R^{1/2}(I - S_{TT})$. That is, the optimal transmit strategy is to employ independent complex circular Gaussian inputs along the eigenvectors of **A**. For comparison, when no mutual coupling exists, the optimal input covariance matrix $\widehat{Q}$ that maximizes the ergodic capacity in (6) has the same eigenvectors as the matrix $R^{1/2}$. Here we note that the receive mutual coupling does not have an effect on the optimal transmit direction. Only the transmit mutual coupling influences the optimal transmit direction by multiplying an coupling matrix $\mathbf{I} - \mathbf{S}_{TT}$ with the channel matrix $R^{1/2}$. From (30), the optimal power allocation could be given by

$$\begin{aligned} \mathbf{\Lambda}_{\widehat{\mathbf{Q}}} &= \arg\max_{\mathbf{\Lambda}_{\widehat{\mathbf{Q}}}:\mathrm{trace}(\mathbf{\Lambda}_{\widehat{\mathbf{Q}}})=P} E[\log\det(\mathbf{I} + \\ &\qquad \frac{1}{\sigma^2}\mathbf{W}'^{\dagger}\mathbf{W}'\mathbf{\Lambda}_{\mathbf{A}}\mathbf{\Lambda}_{\widehat{\mathbf{Q}}}\mathbf{\Lambda}_{\mathbf{A}}^{\dagger})] \\ &= \arg\max_{\lambda_i^{\widehat{Q}}:\sum_{i=1}^m \lambda^{\widehat{Q}}=P} E[\log\det(\mathbf{I} + \\ &\qquad \sum_{i=1}^m \frac{1}{\sigma^2}\lambda_i^{\widehat{Q}}\lambda_i^A\mathbf{W}_i'^{\dagger}\mathbf{W}_i')]. \quad (35) \end{aligned}$$

where

$$\mathbf{W}_i' = \begin{pmatrix} \frac{2}{\det(\mathbf{I}+\frac{Z_{RR}}{Z_0})}||\mathbf{D}_1||w_{1i} \\ \frac{2}{\det(\mathbf{I}+\frac{Z_{RR}}{Z_0})}||\mathbf{D}_2||w_{2i} \\ \vdots \\ \frac{2}{\det(\mathbf{I}+\frac{Z_{RR}}{Z_0})}||\mathbf{D}_n||w_{ni} \end{pmatrix}^T. \quad (36)$$

$(\cdot)^T$ denotes transpose. (32), (33) represent the optimal power allocation with both transmit and receive mutual coupling. And the corresponding capacity could be given by

$$C = E[\log\det(\mathbf{I} + \frac{1}{\sigma^2}\mathbf{W}'^{\dagger}\mathbf{W}'\mathbf{\Lambda}_{\mathbf{A}}\mathbf{\Lambda}_{\widehat{\mathbf{Q}}}\mathbf{\Lambda}_{\mathbf{A}}^{\dagger})]. \quad (37)$$

where $\Lambda_A, W', \Lambda_{\widehat{Q}}$ are given by (15), (22) and (32) respectively.

Specifically, if mutual coupling only exists among the transmit antenna arrays, we let

$$\widehat{\mathbf{R}} = E[\widehat{\mathbf{H}}_i^{\dagger}\widehat{\mathbf{H}}_i] = (\mathbf{I} - \mathbf{S}_{TT})^{\dagger}\mathbf{R}(\mathbf{I} - \mathbf{S}_{TT}) = \mathbf{U}^{\dagger}\mathbf{\Lambda}\mathbf{U}. \quad (38)$$

where U is unitary, $\Lambda = \mathrm{diag}(\lambda_1, ..., \lambda_m)$, and $\lambda_1 \geqslant \lambda_2 \geqslant ... \geqslant \lambda_m$.

The optimal covariance matrix _**Q** could be given by

$$\widehat{\mathbf{Q}} = \mathbf{U}^{\dagger}\mathbf{\Omega}\mathbf{U} = \mathbf{U}^{\dagger}\mathrm{diag}(\gamma_1, ..., \gamma_m)\mathbf{U}. \quad (39)$$

The corresponding eigenvalues or power allocation γ_i can be determined by

$$\mathbf{\Omega} = \arg\max_{\mathbf{\Omega}:\mathrm{trace}(\mathbf{\Omega})=P} E[\log\det(\mathbf{I} + \sum_{i=1}^m \frac{1}{\sigma^2}\lambda_i\lambda_i^A\mathbf{W}_i^{\dagger}\mathbf{W}_i)]. \quad (40)$$

where Wi is the i th row of matrix W with i.i.d. complex Gaussian distributed entries with zero mean and variance one

And the corresponding capacity is given by

$$\begin{aligned} C &= E[\log\det(\mathbf{I} + \frac{\widehat{\mathbf{H}}^{\dagger}\widehat{\mathbf{H}}\widehat{\mathbf{Q}}}{\sigma^2})] \\ &= E[\log\det(\mathbf{I} + \frac{1}{\sigma^2}(\mathbf{I} - \mathbf{S}_{TT})^{\dagger}\mathbf{H}^{\dagger}\mathbf{H}(\mathbf{I} - \mathbf{S}_{TT})\widehat{\mathbf{Q}})]. \quad (41) \end{aligned}$$

If no mutual coupling exists among both transmit and receive antenna arrays and we let $R = U^{\dagger}\Lambda U$, where **U** is unitary, $\mathbf{\Lambda} = \mathrm{diag}(\lambda_1, ..., \lambda_m)$, and $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_m$. The optimal covariance matrix **Q** is given by

$$\widehat{\mathbf{Q}} = \mathbf{U}^{\dagger}\mathbf{\Omega}\mathbf{U} = \mathbf{U}^{\dagger}\mathrm{diag}(\gamma_1, ..., \gamma_m)\mathbf{U}. \quad (42)$$

where

$$\mathbf{\Omega} = \arg\max_{\mathbf{\Omega}:\mathrm{trace}(\mathbf{\Omega})=P} E[\log\det(\mathbf{I} + \sum_{i=1}^m \frac{1}{\sigma^2}\lambda_i\lambda_i^R\mathbf{W}_i^{\dagger}\mathbf{W}_i)]. \quad (43)$$

and

$$C = E[\log\det(\mathbf{I} + \frac{\mathbf{H}^{\dagger}\mathbf{H}\mathbf{Q}}{\sigma^2})]. \quad (44)$$

which turns to the traditional case [29].

Therefore, from (32), (33) (40), we note that, when both transmit and receive mutual coupling exist, the optimal power allocation $\lambda_i^{\widehat{Q}}$ is also determined by the power constraint P and eigenvalues $\lambda_i^{A^2}$ of the matrix A^2. But it is the eigenvalue

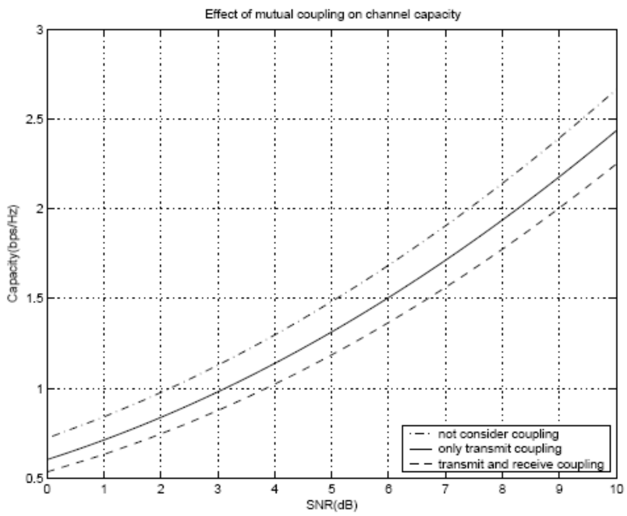

Fig. 2. Comparison of the effect of mutual coupling on channel capacity with different assumptions.

$\lambda_i^{A^2}$ of matrix $\mathbf{A}2 = (\mathbf{I} - \mathbf{S}_{TT})^{\dagger}\mathbf{R}(\mathbf{I} - \mathbf{S}_{TT})$ that is used, instead of the channel covariance $\mathbf{R}$. And the matrix $\mathbf{I} - \mathbf{S}_{TT}$ expresses the effect of transmit mutual coupling on optimal power allocation.

Also we note that the W_i' becomes the function of W_i, where the coefficient $\frac{4}{(\det(\mathbf{I}+\frac{Z_{RR}}{Z_0}))^2}||\mathbf{D}_i||^2$ expresses the effect of receive mutual coupling on optimal power allocation. So the solution also resembles water-filling in the sense that $\Lambda_{\hat{\mathbf{Q}}}$ and Λ_{A^2} are both arranged in descending order, i.e., larger $\lambda_i^{A^2}$ corresponds to stronger channel modes, which get allocated more power than weaker modes. But the value of the water level $\lambda_i^{A^2}$ will be influenced by receive mutual coupling, which leads to some change on the power value in each transmit direction. For example, when $\| D_i \|^2$ are all equal, (32) becomes

$$\Lambda_{\hat{\mathbf{Q}}} = \arg \max_{\lambda_i^{\hat{Q}}:\sum_{i=1}^{m}\lambda^{\hat{Q}}=P} E[\log\det(\mathbf{I} + \sum_{i=1}^{m}\frac{1}{\sigma^2}\frac{4}{(\det(\mathbf{I}+\frac{Z_{RR}}{Z_0}))^2}||\mathbf{D}_i||^2\lambda_i^{\hat{Q}}\lambda_i^{A^2}\mathbf{W}_i^{\dagger}\mathbf{W}_i)]. \quad (45)$$

It is obvious to see that the solution when mutual coupling exists is the same as (40), except that the water-filling level changes from λ_i^R to $\frac{4}{(\det(\mathbf{I}+\frac{Z_{RR}}{Z_0}))^2}||\mathbf{D}_i||^2\lambda_i^{A^2}$. And this illustrates clearly the effect of mutual coupling on the optimal power allocation.

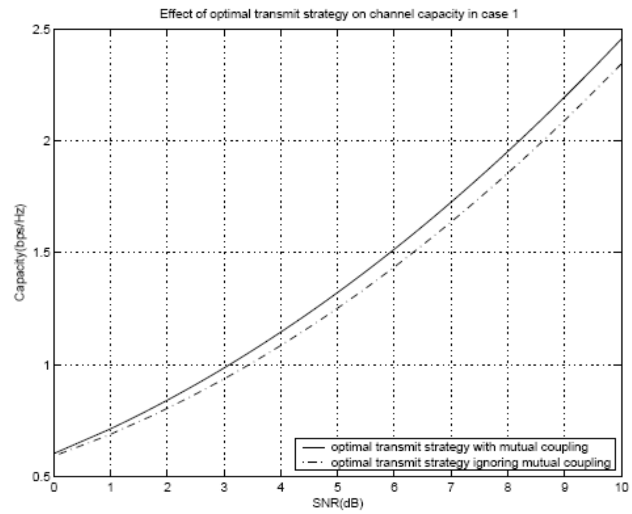

Fig. 3. Effect of optimal transmit strategy with or ignoring mutual coupling at the transmitter on channel capacity

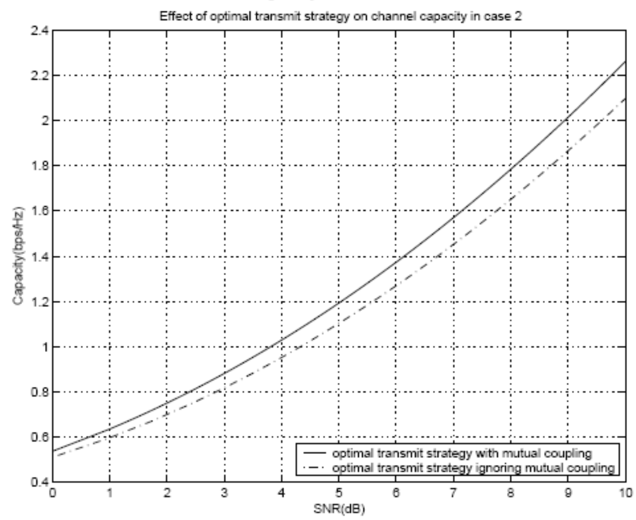

Fig. 4. Effect of optimal transmit strategy with or ignoring mutual coupling at both the transmitter and receiver on channel capacity

IV. Numerical Results

As illustration, we consider an MIMO wireless system with correlated Rayleigh fading channels with channel covariance feedback. We assume each row of the channel matrix $\mathbf{H}$ has independent and identical distribution. The transmitter only knows the covariance matrix of each row of $\mathbf{H}$ and the coupling matrix of both transmit antenna elements and receive antenna elements if any. Here we give an example and assume the covariance matrix $\mathbf{R} = [1, 0.3; 0.3, 1]$. In engineering, scattering parameter matrix $\mathbf{S}_{TT}$ and $\mathbf{S}_{RR}$ are very small, thus we assume they both equal to -10 dB level. Since we aim to explore the effect of the mutual coupling, we assume that s_{11} and s_{12} equals to 0 and 0.3, respectively. We also assume that the transmit and receive antenna have the same S parameter matrix. The characteristic impendence Z_0 is 50 Ω. And in each simulation, the capacity for each SNR point is obtained through 10,000 independent

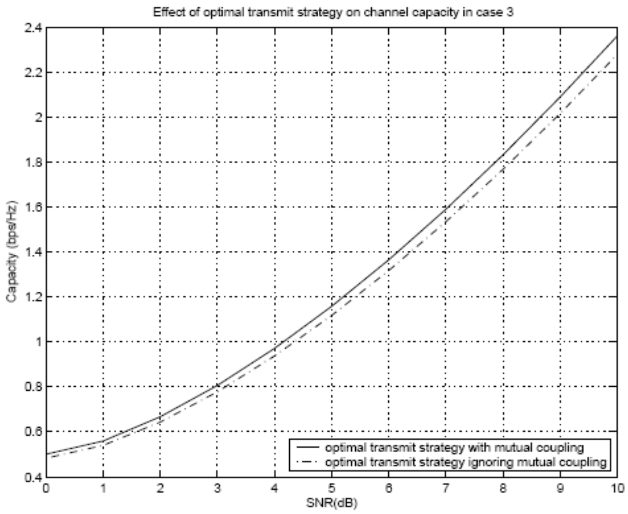

Fig. 5. Effect of optimal transmit strategy with or ignoring mutual coupling at the receiver on channel capacity

computer runs.

We consider the mutual coupling only exists among transmit antenna elements and mutual coupling exists both among transmit antenna elements and receive antenna elements first.

Figure 2 illustrates the effect of mutual coupling on the channel capacity corresponding to the proposed optimal transmit strategy in these two cases. You can see that the mutual coupling among either the transmit or receive antenna has a significant influence when mutual coupling is medium. And besides the transmit mutual coupling, the effect of receive mutual coupling on channel capacity can not be ignored, although we have proved that the receive mutual coupling does not change the optimal transmit direction and only influence the optimal power allocation. More numerical results show that when the mutual coupling is larger, the reduction of the channel capacity is even larger.

Figure 3 and 4 illustrate the effect of the optimal transmit strategy on the channel capacity these two cases. You can see that, when mutual coupling exists among either the transmit or receive antenna, if ignore the effect of mutual coupling and adopt the original optimal transmit strategy without mutual coupling, the resulted channel capacity has a drop compared with that corresponding to the optimal transmit strategy using the information of mutual coupling. And this drop becomes larger with the increase of SNR. More numerical results show that the larger the mutual coupling, the larger the influence of the optimal transmit strategy is, which illustrates the necessity to consider mutual coupling when it is large, for example, compact MIMO system. Also we have provided a simulation result that illustrates the influence of the mutual coupling at the receiver. It is easy to see that the influence of mutual coupling at the receiver is not very much.

Finally, from this work, we also know that if the feedback information is limited, the cost of considering mutual coupling is high, or the demand for capacity is not strict, we can simply ignore the mutual coupling, with the cost of some capacity loss. This applies to many situations except for compact MIMO system.

V. CONCLUSIONS

In this paper, we discuss the effect of mutual coupling on optimal transmit strategy with statistic channel information feedback. First, we derive a channel model using S-parameter from the Path-Based Channel Model. Then by analyzing the characteristic of the coupling matrix, we give the expression for the optimal input covariance matrix and the corresponding channel capacity when the transmitter only knows the channel statistic information. Numerical results clearly illustrate the mutual coupling has an effect on optimal transmit strategy and corresponding capacity, especially when mutual coupling or SNR is large, which illustrates the necessity to consider mutual coupling in compact MIMO system. But in other situations, if the feedback information is limited, the cost of considering mutual coupling is high, or the demand for capacity is not strict, we can simply ignore the mutual coupling, with the cost of some capacity loss.

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