The Links Only for Protection with the Accurate Methods of the Finite Differences

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***Abstract*- In this paper, we considered the wavelength division multiplexing (WDM) optical networks that transmit large quantities of data. These networks need to be maintained fault-free at all times of their operation so the protection and restoration strategies are critical to provide optical network survivability from a variety of failure scenarios that can occur in this environment. Any failure may cause the failure of several optical channels and led to large data losses. This study examines the finite differences methods (linear and polynomial accurate) which are used to study the problem of the links only for protection and the restoration is executed dynamically.**

Key words: Planning, designing, WDM, mesh, protection , dynamic restoration

I. INTRODUCTION

When the WDM systems are used with the fibers the number of optical channels increases and the WDM optical networks are made. These optical fiber WDM mesh networks transmit large quantities of information due to multiple applications or because they serve large urban areas need to be maintained fault-free at all times of their operation [5]. Such protection from faults extends to both service providers and end-users [3], [4], [6], [7], [8] and [14]. To ensure service continuity, service providers plan and design suitable solutions to alleviate such disruptions. These solutions include protection systems and restoration paths. Planning depends on the demands or needs that the network satisfies as well as on the percentage of protection and fulfillment of the network systems. The protection is provided on optical fiber layer or on wavelength layer or combining both of them .In this paper the finite differences methods [1] are used to study the problem of the links that are dedicated for protection.

The term “protection” is usually defined as the method by which backup capacity on the link or path is statically reserved during connection setup. Protection is usually the first mechanism to deal with a failure. It needs to be fast and protection routes are preplanned so that traffic can switch immediately from the failed working routes to the protection routes. Protection schemes use 100 percent excess bandwidth in the network or more. The term “restoration” is usually defined as the method by which backup or spare capacity for the connection is dynamically discovered after connection has failed due to a link or node failure. Restoration is not the primary mechanism to deal with a failure. Instead, it is used to provide either more efficient routes and additional resilience against further failures before the first failure is fixed. As a result, it provides a slower alternative to protection. Complex algorithms can be used to reduce the excess bandwidth required.

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Protection is limited to simple topologies and relies on the hardware; it is preplanned and faster than restoration. Protection schemes operate either on the multiplex optical WDM signal layer or on the individual optical path layer 9],[10], [12],[13] and [15]. The mesh network has the best and fastest protection technique for every lightpath because there are more than one back-up paths for each main one. There are also the p-cycles that are an efficient way to obtain mesh like efficiency with ring like speed [17].

There are also links which are dedicated for protection using the availability of back-up optical fibers without working telecommunications traffic, new or old, to increase the restoration procedure performance and quite often it contributes to a better network restoration scheme. Such back-up fibers may be optical fibers of older optical cables that have been idle because of their increased, age or non-linearity weaknesses or other causes. The said weaknesses produce a downgraded transmission signal. Back-up optical fibers may be new fibers that have been installed to this link, i.e. in order to provide protection for some very important optical links or links susceptible to undergo cuts. Back-up optical fibers through such routes may be used in designing the network’s survivability, depending on the desired levels of survivability and costs which is wished to maintain. These links may be used for future network expansions, development of new services, management/handling of increased traffic etc.

The network topology and other parameters are known as WDM and optical fiber capacity, one optical fiber per link with an extension to a 1+1 fiber protection system. So this network is characterized by one working fiber per link, edges of two links, links of two optical fibers, one for working and one for protection. In this paper we are usually referred at the working optical fiber. The connections are lightpaths originating in the source nodes and terminating at the destination nodes proceeding from preplanned optical working paths. The connections of the same node pair by same preplanned optical paths form a connection group along the network. The nodes have wavelength conversion capability .The problem solution is to calculate the final available capacity of the network for a given traffic table first without any failure , second after a single failure of a cut link without using links only for protection and third after a single failure of a cut link with using links only for protection. The traffic table contains the number of the node pairs, the node pairs and the number of the connections of each node pair when their working paths are preplanned. The role of the Difference Calculus is in the study of the Numerical Methods. Computer solves these Numerical Methods. The subject of the Difference Equations is in the treatment of discontinuous processes. The network final available capacity is revealed as a difference equation because the final available capacity of the individual working optical fibers is also a difference equation. The reduction of the available capacity of each working optical fiber is a discontinuous process when connection groups of several sizes pass through it. Two methods, the linear function method and the accurate polynomial function method reveal each difference equation. These methods include accurate and arithmetical methods to study the optical networks and their problems.

This paper is broken down in the following sections: Section II shows how the finite differences are used for each optical fiber and illustrates the optical fiber residual available capacity; Section III describes the problem and provides a solution, synoptic description of method, an example, the attributes and a discussion; Section IV draws conclusions and finally ends with the references.

II. THE OPTICAL FIBER AND THE FINITE DIFFERENCES

Before studying finite differences and their use in optical WDM mesh networks survivability, it is necessary to provide a short comprehensive presentation of the finite differences computation. Let's assume that $y_1, y_2, \ldots, y_n$ is a sequence of numbers in which the order is determined by the index n. The number n is an integer and the y_n can be regarded as a function of n, an independent variable with function domain the natural numbers and it is discontinuous. Such a sequence shows the available capacity reduction of a telecommunication fibre network link between two nodes when the telecommunication traffic of 1, 2, …,n source-destination node pairs pass through. It is assumed that the telecommunication traffic unit is the optical channel that is one wavelength (1λ). The telecommunications traffic includes optical connections with their protections. The total connections of a node pair form its connection group. The first order finite differences represent symbolically the connection group of each node pair that passes through a fiber. This connection group occupies the corresponding number of optical channels and it is the bandwidth that is consumed by connections of a node pair through this fiber. The first order finite differences are used to represent the connection groups in optical channels of the node pairs that pass through an optical fiber .An equation of the first order finite differences gives the available capacity of an optical fiber network link when a connection group passes through it. This available capacity is provided for the connection groups of the other node pairs that their connections will pass through this optical fiber. When the first connection group of Δy_1 connections passes through an optical fiber network link with installed capacity of y_1 optical channels the first order finite difference equation gives the available capacity $y_{2\ (y_1+1)}$ which is written as following

$$y_{1+1} = y_1 - \Delta y_1 \qquad (1)$$

The sequence Δy_1, Δy_2, Δy_3,…, Δy_n represents the connection groups that pass through this optical fiber network link .When Δy_1 subtracted from y_1, creates y_2, when Δy_2 subtracted from y_2, creates y_3,..., when Δy_n subtracted from y_n creates y_{n+1} which is the total unused available capacity of this optical fiber. Thus the total unused available capacity of each network optical fiber is calculated after n connections groups pass through it. This could be written with two methods, the linear function method and the polynomial function method. So these methods could be used to check each other. Table 1 gives a short comprehensive presentation of the computation of the finite differences for a given link that can be arranged quite simply.

The optical fiber available capacity for connections (linear function method) of the link (i) is given by the following equation and it is written

$$y_{i,\,n(i)+1} = y_{i,1} - \sum_{l=1}^{n(i)} a_{i,l} * x_l - dy_i \qquad (2)$$

whereby $a_{i,l}$ is a coefficient that takes the value one (1) if the node pair (l) passes all its working connections from this fiber (i) and zero (0) if no passes.
x_l is the total of the connections of the node pair (l) that is the connections groups and it is called connections group size.
n the total number of the node pairs and n>= n(i) .
dy_i is the restoration optical channels (wavelengths) of the link (*i*).

TABLE 1
FINITE DIFFERENCE SYNOPTIC TABLE

Node Pair	Node pair index	Available capacity connection group	Number of the optical connection group	Variation of the optical differences number	Higher order differences number
(S_1,D_1)	1	y_1			
			Δy_1		
(S_2,D_2)	2	y_2		$\Delta^2 y_1$	
			Δy_2		$\Delta^3 y_1$
(S_3,D_3)	3	y_3		$\Delta^2 y_2$	
			Δy_3		
(S_4,D_4)	4	y_4			
.....					
(S_{n-1},D_{n-1})	n-1	y_{n-1}			
			$\Delta y_{,n-1}$		
(S_n,D_n)	n	y_n		$\Delta^2 y_{n-1}$	
			Δy_n		
(S_{n+1},D_{n+1})	n+1	y_{n+1}			

The optical fiber available capacity for connections (polynomial function method) of the link (i) is given by general form of a polynomial function after the serving n(i) working connection groups and it written is as follows

n(i)

$$y_{i,\ n(i)\ +1} = \sum_{r=0} \alpha_r * (\, n(i) + 1)^{\,r} \qquad (3)$$

The assessment of the polynomial function coefficients is done with the values that the polynomial function represents for 1,2,…, n, n +1. The values of the function y_{n+1} for each n must be integral because each value represents optical channels.

Some notes on the polynomial function are presented.

-When no connection group passes through an optical fiber, the polynomial function is constant and independent of n.

-When only one connection group passes through an optical fiber the polynomial function is of the first degree.

-When only a two-connection group passes through an optical fiber the polynomial function is of the second degree, etc.

-The degree of the polynomial function of an optical fiber depends on the connection group number that passes through it.

-The polynomial function of an optical fiber is different when there is full or partial servicing of the connection groups that pass through it.

-Two polynomial functions with the same available capacity have different coefficients when the order and the size of the same number connection groups are different.

-If the equation (3) is written analytically as follows

$$y_{0+1} = \alpha_0 * (\, 0+1)^0$$
$$y_{1+1} = \alpha_0 * (\, 1+1)^0 + \alpha_1 * (\, 1+1)^1$$
$$y_{2+1} = \alpha_0 * (\, 2+1)^0 + \alpha_1 * (\, 2+1)^1 + \alpha_2 * (\, 2+1)^2$$
$$y_{3+1} = \alpha_0 * (\, 3+1)^0 + \alpha_1 * (\, 3+1)^1 + \alpha_2 * (\, 3+1)^2 + \alpha_3 * (\, 3+1)^3$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$y_{n+1} = \alpha_0 * (\, n+1)^0 + \alpha_1 * (\, n+1)^1 + \alpha_2 * (\, n+1)^2 + \ldots + \alpha_n * (\, n+1)^n$$

The value of the function has high accuracy of 15 decimal digits. This method is an *accurate* one. They are systems of n+1equations with n+1 unknown coefficients. The values of the coefficients depend of the number of the connection groups and the connections of each connection group. It is ought to the Table I column 4. This method is more accurate because only one factor is added to new equation when the polynomial degree increases.

The polynomials that calculate the available capacity of each optical link for the accurate method for all possible values up to four and for the following cases are represented. The number, the order and the size of $\underline{\Delta y_{i,j}}$ are critical. The WDM system capacity is 30 λ. So for the accurate method it is written.

-If no one-connection group passes through an optical link the polynomial function is constant, etc.

$\underline{y_{j,0+1} = \alpha_0 * (\, 0+1)^0}$

$y_{j,0+1} = 30.$

-If only one-connection group passes through an optical link the polynomial function is of the first degree.

$\Delta y_{i,1}$,	$y_{i,1+1} = \alpha_0 * (\, 1+1)^0 + \alpha_1 * (\, 1+1)^1$
1	$y_{i,1+1} = 30 * (\, 1+1)^0 - (1/2) * (\, 1+1)^1 = 29$
2	$y_{i,1+1} = 30 * (\, 1+1)^0 - (2/2) * (\, 1+1)^1 = 28$
3	$y_{i,1+1} = 30 * (\, 1+1)^0 - (3/2) * (\, 1+1)^1 = 27$
4	$y_{i,1+1} = 30 * (\, 1+1)^0 - (4/2) * (\, 1+1)^1 = 26$

-If only two-connection groups pass through an optical link the polynomial function is of the second degree.

$\Delta y_{i,1}$,	$\Delta y_{i,2}$,	$y_{i,1+1} = \alpha_0 * (\, 2+1)^0 + \alpha_1 * (\, 2+1)^1 + \alpha_2 * (\, 2+1)^2$
1	1	$y_{i,2+1} = 30 * (\, 2+1)^0 - 0.5 * (\, 2+1)^1 - 5.55555555555429E\text{-}2 * (\, 2+1)^2 = 28$
2	1	$y_{i,2+1} = 30 * (\, 2+1)^0 - 1.0 * (\, 2+1)^1 - 0.00000000000000E{+}0 * (\, 2+1)^2 = 27$
1	2	$y_{i,2+1} = 30 * (\, 2+1)^0 - 0.5 * (\, 2+1)^1 - 1.66666666667420E\text{-}1 * (\, 2+1)^2 = 27$
3	1	$y_{i,2+1} = 30 * (\, 2+1)^0 - 1.5 * (\, 2+1)^1 + 5.55555555555429E\text{-}2 * (\, 2+1)^2 = 26$
2	2	$y_{i,2+1} = 30 * (\, 2+1)^0 - 1.0 * (\, 2+1)^1 - 1.11111111111086E\text{-}1 * (\, 2+1)^2 = 26$
1	3	$y_{i,2+1} = 30 * (\, 2+1)^0 - 0.5 * (\, 2+1)^1 - 2.77777777777828E\text{-}1 * (\, 2+1)^2 = 26$
4	1	$y_{i,2+1} = 30 * (\, 2+1)^0 - 2.0 * (\, 2+1)^1 + 1.11111111111086E\text{-}1 * (\, 2+1)^2 = 25$
3	2	$y_{i,2+1} = 30 * (\, 2+1)^0 - 1.5 * (\, 2+1)^1 - 5.55555555555429E\text{-}2 * (2+1)^2 = 25$
2	3	$y_{i,2+1} = 30 * (\, 2+1)^0 - 1.0 * (\, 2+1)^1 - 2.22222222221720E\text{-}1 * (\, 2+1)^2 = 25$
1	4	$y_{i,2+1} = 30 * (\, 2+1)^0 - 0.5 * (\, 2+1)^1 - 3.88888888886870E\text{-}1 * (\, 2+1)^2 = 25$
4	2	$y_{i,2+1} = 30 * (\, 2+1)^0 - 2.0 * (\, 2+1)^1 + 0.00000000000000E{+}0 * (\, 2+1)^2 = 24$
3	3	$y_{i,2+1} = 30 * (\, 2+1)^0 - 1.5 * (\, 2+1)^1 - 1.66666666667420E\text{-}1 * (2+1)^2 = 24$
2	4	$y_{i,2+1} = 30 * (\, 2+1)^0 - 1.0 * (\, 2+1)^1 - 3.33333333333485E\text{-}1 * (\, 2+1)^2 = 24$

e.t.c

The checking and studying of the optical fiber available capacity are also presented. The equations, (2) and (3) must be greater or equal to zero, for full servicing all connection groups that pass through an optical fiber. So the number of connections on each link is bounded.

TABLE 2

THE SYMBOLS OF THIS PAPER

SN	Symbol	Comments
1	q	**The node number**
2	p	**The edge number**
3	G(V,E)	The network graph
4	V(G)	**The network node set**
5	E(G)	**The network edge set**
6	2p	The number of working and backup fiber for 1+1 line protection
7	n	**The number of source – destination nodes pairs of the network**
8	Xn	Column matrix with dimension (nx1) and elements the connection group sizes of the corresponding source-destination nodes pairs
9	n(i)	**The number of the connection groups that passes through the fiber (*i*) and means that each fiber has different number of connection groups pass through it**
10	k	**The number of the wavelengths channels on each fiber that is the WDM system capacity**
11	Y_1	**Column matrix (2px1) with elements the installed capacity of fiber network links of the linear function method**
12	Y_2	Column matrix (2px1) with elements the unused available capacity of each fiber network link of the linear function method
13	A	**Matrix (2p x n) which shows the network active links that corresponding to working fibers**
14	$a_{i,j}$	Element of the matrix A and takes the value one if the node pair (j) passes all its working connections from the fiber (i) and zero (0) if no passes
15	$\Delta y_{i,j}$	First order of finite difference that corresponds to a group of optical connections that pass through the optical fiber *i* with serial number *j* and valid 1<=i<=2p and 1<=j<=n(i)
16	$y_{i,j}$	Unused available capacity of the optical fiber i that it is offered for optical connections group with serial number j and valid 1<=i<=2p and 1<=j<=n(i)
17	Dy	**Column matrix (2px1) and elements the restoration capacity of each optical fiber that is used**
18	dy_i	Element of the Dy column matrix and shows restoration capacity of the optical fiber (*i)*
19	d_j	**Demand of each node pairs**
20	$a_{i,r}$	Real number coefficient for the polynomial function method for the link (*i*).
21	Cav	The total network available capacity

22	Cb	The total network busy capacity
23	Cinst	The total network installed capacity

III. THE PROBLEM AND ITS SOLUTION

A. The problem

The network topology and other parameters are known as WDM and optical fiber capacity, one optical fiber per link with an extension to a 1+1 fiber protection system. So this network is characterized by one working fiber per link, edges of two links, links of two optical fibers, one for working and one for protection. The connections are lightpaths originating in the source nodes and terminating at the destination nodes proceeding from preplanned optical working paths. The nodes have wavelength conversion capability. The problem solution is to calculate the final available capacity of the network for a given traffic table first without any failure, second after a single failure of a cut link without using any links only for protection and third after a single failure of a cut link with using links only for protection. The traffic table contains the number of the node pairs, the node pairs and the number of the connections of each node pair when their working paths are preplanned.

B. The formulation

A difference table (table 1) is calculated for each optical fiber and the problem is solved using two methods, the first is the linear function and the second the accurate polynomial function.

The formulation of the *linear function* method is presented below. The available capacity of all optical fibers is written as a column matrix. The equation of the linear function method is written as

$$Y_2=Y_1-A*Xn-Dy \qquad (4)$$

A is a matrix that shows the active network links Y_2,Y_1, Xn, *Dy* are column matrices consisting of elements related to the available capacity of each fiber, capacity that offered for connections groups that pass through each, connection group size, restoration optical channels in wavelengths of each fiber respectively. When all connections have been done then each element of Y_2 must be greater than or equal to zero. In other cases some connections are not set up.

The equation of the polynomial function method is similar to that of the equation (3) but for all network fibers there are two column matrices, the left one that is equals with the right one. When all connections have been set up then each element of the column matrix must be greater or equal to zero. In other cases some connections are not possible. The total final available capacity of the network for the linear function method is the following

$$\sum_{i=1}^{2p} y_{i,\,n(i)+1} = \sum_{i=1}^{2p} y_{1,i} - \sum_{i=1}^{2p}\sum_{j=1}^{n(i)} a_{i,j}*x_j - \sum_{i=1}^{2p} dy_i \qquad (5)$$

$$y_{i,\,n(i)+1} >=0 \;,\; y_{1,i} >=0,\; a_{i,j} >0 \;,\; dy_i >0 \;,\; x_j>0$$

The total final available capacity of the network for the polynomial function method is the following

$$\sum_{i=1}^{2p} y_{i,\,n(i)+1} = \sum_{i=1}^{2p}\sum_{r=0}^{n(i)} \alpha_r * (n(i)+1)^r \qquad (6)$$

$$y_{i,\,n(i)+1} >=0 \;,\; n(i)>=0$$

C. Synoptic description of the method

The method describes the operation of the WDM optical fiber mesh network with 1+1 optical fiber protection, the working connections passed through preplanned optical paths but the restoration is done dynamically by shortest path algorithm. It has two parts, the first part or the planning and designing phase and the second part or network with failure phase. So when a cut occurs, the network has failure and the restoration method is activated. The network has links with a finite, nonzero capacity and the link capacity is not exceeded. This method is driven by suitable data and then simulates the actual dynamic behavior of the network. Simulation language is critical to the economic feasibility of this entire investigation. TURBO PASCAL is used to program the model [2], [11] and [16]. The complexity of this method for the node number q depends on the square of the node number and the total number of the requests for connection (s) so it is written as O $(s*q^2)$. The time complexity of that algorithm is 'order q^2, $O(s*q^2)$. Thus, on a 133MHz computer, q=6 and s=12, the time is 4 hundredths of second. It means that worst time consuming depends by the network size for the same computer. The consuming time of each method is about equal and their time differences are negligent. The symbols with tone mean modifications. The shortest path algorithm is an algorithm that finds the shortest path between two given vertices in an undirected graph G= (V, E). The addition links are used during the network failure phase. The synoptic description of the method is as follows.

TABLE 2
THE SYNOPTIC PRESENTATION OF THE METHODS

FIRST PART(*Planning and designing Phase*)
First step, network parameters reading (q, p, V(G), E(G), G(V,E), 2,2p, k)
Second step, connection selections (n, (S_n,D_n), Xn, Preplanned working lightpaths)
Third step, wavelength allocation (Routing and wavelength assignment method)
Forth step, Finite difference presentation (Cavl versus Cavp)
Fifth step, results (Cavl,Cbl,Cinst,Cavp,Cbp)
SECOND PART(*Network with failure Phase*)
Sixth step. Network parameter modifications (cut link, q, p, V(G'), E'(G'), G'(V,E'), 2,2p-1 or 2p+1, k)
Seventh step. New Wavelength allocation (Restoration method)
Eighth step. New Results (C'avl,C'bl,Ci'nst,C'avp,C'bp)

TABLE 3
THE NETWORK PARAMETERS

Network parameters	Amount
Node number	6
Edge number without addition links	8
Edge number with addition links	9
Working fiber per edge	2
Working fiber per link	1
Network working fiber	16
Protection fiber per edge	2
Protection fiber per link	1
Protection fiber without addition links	16
Protection fiber with addition links	18
WDM system capacity	30

D. Example

The network here below is studied for the best presentation of the results. It is because for larger networks it is difficult to present the results as well as the number, the size of tables are larger, the dimensions of the matrices are also larger as well as the degree of the polynomial higher. It is assumed that the topology of the network is presented by the graph G (V, E). This mesh topology is used because it is a simple, palpable and an analytical example of the finite differences and it is easy to expand to any mesh topology. The vertex set has q=6 elements which are V={v_1, v_2, v_3, v_4, v_5, v_6} and the edge set has p=8 elements which are E={e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8}. Each edge is an optical link of two directions with one working fiber for each direction. Thus there are 2*p=2*8=16 optical fibers. When the links only for protection are added the number of fibers increase to 2*p'=2*9=18 and the edge set elements change and total used optical fibers. The symbols with tone mean modifications. The capacity of WDM system is 30 OCh. Table 3 presents the network parameters.

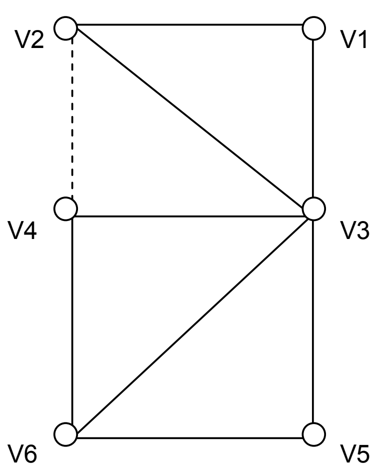

Fig. 1.The mesh topology of the network.

In this example, the linear function method and the final result of the polynomial function are presented. The problem is solved for an instance with n=12 of 30 possible node pairs. These have their order, their working paths and sizes for as shown in table 4 and it is studied for three cases.

In the first case, the finite difference methods are investigated and presented when the network is operating under normal conditions, in the second case, when the network is operating under failure conditions and the restoration is executed dynamically by shortest path algorithm without adding new links and in the third case, when the network is operating under failure conditions and the restoration is executed dynamically by shortest path algorithm with adding new links only for protection.

TABLE 4
THE NODE PAIRS WITH PREPLANNED PATHS AND THE CONNECTION GROUP SIZE

Node Pair [Si,Di]	Node pair [v_i, v_j]	Working Path	Group size
[S_1, D_1]	[v_1, v_2]	v_1, v_2	4
[S_2, D_2]	[v_1, v_3]	v_1 ,v_3	2
[S_3, D_3]	[v_1, v_5]	v_1, v_3 ,v_5	8
[S_4, D_4]	[v_2, v_3]	v_2, v_3	1
[S_5, D_5]	[v_2, v_4]	v_2, v_3, v_4	9
[S_6, D_6]	[v_2, v_5]	v_2, v_3,v_5	2
[S_7, D_7]	[v_3, v_4]	v_3, v_4	1
[S_8, D_8]	[v_4, v_6]	v_4 ,v_6	1
[S_9, D_9]	[v_4, v_1]	v_4 ,v_3,v_1	5
[S_{10}, D_{10}]	[v_4, v_5]	v_4, v_6, v_5	1
[S_{11}, D_{11}]	[v_5, v_4]	v_5, v_3 ,v_4	1
[S_{12}, D_{12}]	[v_6, v_1]	v_6, v_3 ,v_2,v_1	5

TABLE 5
THE CONNECTION GROUPS OF EACH FIBER AND THE HIGH ORDER FINITE DIFFERENCES

		Without Failure No more Links		**With Failure No more Links**		**With Failure Yes more Links**	
Fiber i	**Optical fiber link**	**n(i)**	**m(i)**	**n(i)**	**m(i)**	**n(i)**	**m(i)**
1	<v_1, v_2>	1	1	1	1	1	1
2	<v_2, v_1>	1	1	4	4	3	3
3	<v_1, v_3>	2	2	5	5	4	4
4	<v_3, v_1>	1	1	1	1	1	1
5	<v_2, v_3>	3	3	0	0	0	0
6	<v_3, v_2>	1	1	1	1	1	1
7	<v_3, v_4>	3	3	3	3	2	2
8	<v_4, v_3>	1	1	1	1	1	1
9	<v_3, v_5>	2	2	2	2	2	2
10	<v_5, v_3 >	1	1	1	1	1	1
11	<v_3, v_6>	1	1	1	1	1	1
12	<v_6, v_3>	1	1	1	1	1	1
13	<v_4, v_6>	1	1	1	1	1	1
14	<v_6, v_4>	0	0	0	0	0	0
15	<v_5, v_6>	0	0	0	0	0	0
16	<v_6, v_5 >	1	1	1	1	1	1
17	<v_2, v_4>	-	-	-	-	1	1
18	<v_4, v_2>	-	-	-	-	0	0

In the *first case*, the table 1 (finite difference table) of each fiber is not presented because the number of these tables is fourteen (14). The number of connection groups that passes through and the high order finite differences of each optical fiber are showed in the following table 5. (Fiber, i) shows the optical fiber numbering. Optical fiber link means the corresponded link of this fiber. “Without failure No more Links” means the network phase without failure and no extra links used. “With failure No more Links” means the corresponded network phase but no extra links used for restoration. “With failure yes more Links” means the corresponded network phase and extra links used for restoration. The n(i) shows the number of the connection groups that pass through each optical fiber and it is here corresponded to the m(i)=1, 2, 3, 4, 5 of the fiber (i) which is the order finite differences with $\Delta^{m(i)}y_i$ the value of this order which now is not represented. The intermediate order finite differences are not showed. Fibers 5 and 7 has 3rd order finite difference. It is obvious that optical fibers five (5) and seven (7) have the larger difference tables.

For the linear function (5), Y_2, Y_1 and Dy have dimension of (16x1), A has a dimension of (16x12), X_n has a dimension of (12x1). Matrix A is a known matrix (16x12) which is always constant because it depends on the optical paths that are constant for all examples. So the linear function provides

$$\begin{vmatrix} 26 \\ 25 \\ 20 \\ 25 \\ 18 \\ 25 \\ 19 \\ 25 \\ 20 \\ 29 \\ 29 \\ 25 \\ 29 \\ 30 \\ 30 \\ 29 \end{vmatrix} = \begin{vmatrix} 30 \\ 30 \\ 30 \\ 30 \\ 30 \\ 30 \\ 30 \\ 30 \\ 30 \\ 30 \\ 30 \\ 30 \\ 30 \\ 30 \\ 30 \\ 30 \end{vmatrix} - \begin{vmatrix} 4 \\ 5 \\ 10 \\ 5 \\ 12 \\ 5 \\ 11 \\ 5 \\ 10 \\ 1 \\ 1 \\ 5 \\ 1 \\ 0 \\ 0 \\ 1 \end{vmatrix} - \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix}$$

When no connection group goes through the optical fiber, then the degree of the polynomial function is 0, when one connection group goes through, then the degree of the polynomial function is 1, when two connection groups go through, then the degree of the polynomial function is 2, etc. The following matrix provides the final available capacity of all optical fibers and its dimension is (16x1).

$$\begin{vmatrix} y_{1,2} \\ y_{2,2} \\ y_{3,3} \\ y_{4,2} \\ y_{5,4} \\ y_{6,2} \\ y_{7,4} \\ y_{8,2} \\ y_{9,3} \\ y_{10,2} \\ y_{11,2} \\ y_{12,2} \\ y_{13,2} \\ y_{14,1} \\ y_{15,1} \\ y_{16,2} \end{vmatrix} = \begin{vmatrix} 30 - 2*2 \\ 30 - 2.5*2 \\ 30 - 1*3 - 0.777*9 \\ 30 - 2.5*2 \\ 30 - 0.5*4 - 0.944*16 + 0.07986*64 \\ 30 - 2.5*2 \\ 30 - 4.5*4 + 0.388*16 + 0.012*64 \\ 30 - 2.5*2 \\ 30 - 4*3 + 0.222*9 \\ 30 - 0.5*2 \\ 30 - 0.5*2 \\ 30 - 2.5*2 \\ 30 - 0.5*2 \\ 30 \\ 30 \\ 30 - 0.5*2 \end{vmatrix} = \begin{vmatrix} 26 \\ 25 \\ 20 \\ 25 \\ 18 \\ 25 \\ 19 \\ 25 \\ 20 \\ 29 \\ 29 \\ 25 \\ 29 \\ 30 \\ 30 \\ 29 \end{vmatrix}$$

TABLE 6

THE AVAILABLE, BUSY AND INSTALLED CAPACITIES

		Capacity Symbol	Linear	Accurate Polynomial
1	Without Failure No more Links	Cav	404	404
		Cb	76	76
		Cinst	480	480
2	With Failure No more Links	Cav	362	362
		Cb	88	88
		Cinst	450	450
3	With Failure Yes more Links	Cav	440	440
		Cb	70	70
		Cinst	510	510

If the degree of polynomials increases then the writing of the polynomial numerical coefficients has error because they are difficult to be represented. This method is an accurate one but if all coefficients are not written completely with 15 digits there are errors in the results. These values are rounded to the closest integer to agree with the real ones. The available, busy and installed capacities for both cases are showed in the table 6. The values of the linear function method are the same with these of accurate polynomial method. The installed working network capacity is Cinst=16*30=480 wavelengths.

In the second case, the network is operating under failure conditions with the link < v_2, v_3> cut that corresponds to the fiber 5. The results are showed in the table 7, column 4 (With Failure No more Links).

TABLE 7

THE RESULTS OF THE CUT LINK < v_2, v_3>

		Without Failure No more Links	With Failure No more Links	With Failure Yes more Links
Node pairs	Group size	path	path	path
[v_2, v_3]	1	v_2, v_3	v_2, v_1, v_3	v_2, v_1, v_3
[v_2, v_4]	9	v_2, v_3, v_4	v_2, v_1, v_3, v_4	v_2, v_4
[v_2, v_5]	2	v_2, v_3, v_5	v_2, v_1, v_3, v_5	v_2, v_1, v_3, v_5
Total length		5	8	6
Total connection length		23	35	17

The restoration is executed dynamically by shortest path algorithm and connections are done by new optical paths. So the restoration connections lengths are longer than the working connections lengths.

As in the first case, I don’t present table 1 (finite difference table) of each fiber because the number of tables is thirteen (13). In table 5, “With failure phase No more links” columns, there is the number of connection groups that pass through each link after the link < v_2, v_3> cut that corresponds to the fiber 5. When the restoration is executed by shortest path algorithm, the follows are written. Table 5 here gives the connection groups number that pass through each fiber when the network is working under failure conditions with the link < v_2, v_3> cut that corresponds to the fiber 5 and the restoration is executed by shortest path algorithm. For the linear function (5), the dimensions of the matrixes remain as in the case one. So the linear function provides

$$\begin{vmatrix} 26\\13\\8\\25\\0\\25\\19\\25\\20\\29\\29\\25\\29\\30\\30\\29 \end{vmatrix} = \begin{vmatrix} 30\\30\\30\\30\\0\\30\\30\\30\\30\\30\\30\\30\\30\\30\\30\\30 \end{vmatrix} - \begin{vmatrix} 4\\5\\10\\5\\0\\5\\2\\5\\8\\1\\1\\5\\1\\0\\0\\1 \end{vmatrix} - \begin{vmatrix} 0\\12\\12\\0\\0\\0\\9\\0\\2\\0\\0\\0\\0\\0\\0\\0 \end{vmatrix}$$

As previously, the order of finite differences of each fiber is dependent on the number of node pairs that passes their connection groups through. The following matrix provides the final available capacity of all optical fibers and its dimension is (16x1) but some elements change. The comments for this case are the same with the previous ones but the coefficients, the base and degree of polynomials of the links that effected change. When the network works under these conditions, the network available capacity given by the equations (5) and (6) and the results are given in the table 6, lines "With Failure No more Links". The restoration is full because there is sufficient available capacity.

$$\begin{vmatrix} y_{1,2}\\ y_{2,5}\\ y_{3,6}\\ y_{4,2}\\ y_{5,0}\\ y_{6,2}\\ y_{7,4}\\ y_{8,2}\\ y_{9,3}\\ y_{10,2}\\ y_{11,2}\\ y_{12,2}\\ y_{13,2}\\ y_{14,1}\\ y_{15,1}\\ y_{16,2} \end{vmatrix} = \begin{vmatrix} 30 - 2*2\\ 30 - 2.5*5 + 0.17*25 - 0.12*125 + 0.01*625\\ 30 - 1*6 - 0.77*36 + 0.085*216 - 0.0099*1296 + 0.0008*7776\\ 30 - 2.5*2\\ 0\\ 30 - 2.5*2\\ 30 - 0.5*4 - 0.055*16 - 0.125*64\\ 30 - 2.5*2\\ 30 - 4*3 + 0.222*9\\ 30 - 0.5*2\\ 30 - 0.5*2\\ 30 - 2.5*2\\ 30 - 0.5*2\\ 30\\ 30\\ 30 - 0.5*2 \end{vmatrix} = \begin{vmatrix} 26\\13\\8\\25\\0\\25\\19\\25\\20\\29\\29\\25\\29\\30\\30\\29 \end{vmatrix}$$

In the *third case*, the network is operating under failure conditions with the link < v_2, v_3> cut that corresponds to the fiber 5 but the addition links only for protection are set up. The results are showed in the table 7, column 5 (With Failure Yes more Links).The restoration is executed dynamically by shortest path algorithm and connections are done by new optical paths.

As in the previous case, I don't present table 1 (finite difference table) of each fiber because the number of tables is fourteen (14). In table 5, "With failure phase Yes more links" columns, there is the number of connection groups that pass through each link after the link < v_2, v_3> cut that corresponds to the fiber 5. When the restoration is executed by shortest path algorithm, the follows are written. Table 5 here gives the connection groups number that pass through each fiber when the network is working under failure conditions with the link < v_2, v_3> cut that corresponds to the fiber 5, the addition links are used and the restoration is executed by shortest path algorithm. The matrixes have their dimensions to be increased so for the linear function (4), Y_2, Y_1 and Dy have dimension of (18x1), A has a dimension of (18x12), X_n has a dimension of (12x1). Matrix A is a known matrix (18x12) which is modified by cuts and the setting new protection links. It depends on the optical paths that change. So the linear function gives

$$\begin{vmatrix} 26\\22\\17\\25\\0\\25\\28\\25\\20\\29\\29\\25\\29\\30\\30\\29\\21\\30 \end{vmatrix} = \begin{vmatrix} 30\\30\\30\\30\\0\\30\\30\\30\\30\\30\\30\\30\\30\\30\\30\\30\\30\\30 \end{vmatrix} - \begin{vmatrix} 4\\5\\10\\5\\0\\5\\2\\5\\8\\1\\1\\5\\1\\0\\0\\1\\0\\0 \end{vmatrix} - \begin{vmatrix} 0\\3\\3\\0\\0\\0\\0\\0\\2\\0\\0\\0\\0\\0\\0\\0\\9\\0 \end{vmatrix}$$

As previously, the order of finite differences of each fiber is dependent on the number of node pairs that passes their connection groups through. The following matrix provides the final available capacity of all optical fibers and its dimension is (18x1) but some elements change. The coefficients, the base and degree of polynomials of the links that effected, change. When the network operates under these conditions, the network available capacity given by the equations (5) and (6) and the results are given in the table 6, lines "With Failure Yes more Links". The restoration is full because there is sufficient available capacity. The final available capacity for each optical fiber is positive or zero so there is no problem with their connections. The working network installed capacity after link cut is Cinst =17*30= =510 wavelengths.

$$\begin{vmatrix} y_{1,2}\\ y_{2,4}\\ y_{3,5}\\ y_{4,2}\\ y_{5,0}\\ y_{6,2}\\ y_{7,3}\\ y_{8,2}\\ y_{9,3}\\ y_{10,2}\\ y_{11,2}\\ y_{12,2}\\ y_{13,2}\\ y_{14,1}\\ y_{15,1}\\ y_{16,2}\\ y_{17,2}\\ \mathbf{y_{18,1}} \end{vmatrix} = \begin{vmatrix} 30 - 2*2\\ 30 - 2.5*4 + 0.167*16 - 0.01*64\\ 30 - 1*5 - 0.78*25 + 0.085*125 + 0.0014*625\\ 30 - 2.5*2\\ 0\\ 30 - 2.5*2\\ 30 - 0.5*3 - 0.055*9\\ 30 - 2.5*2\\ 30 - 4*3 + 0.222*9\\ 30 - 0.5*2\\ 30 - 0.5*2\\ 30 - 2.5*2\\ 30 - 0.5*2\\ 30\\ 30\\ 30 - 0.5*2\\ 30 - 4.5*2\\ \mathbf{30} \end{vmatrix} = \begin{vmatrix} 26\\22\\17\\25\\0\\25\\28\\25\\20\\29\\29\\25\\29\\30\\30\\29\\21\\ \mathbf{30} \end{vmatrix}$$

The fig.2 presents the comparison of network busy capacity when the network does not use links only for protection (16 links) and when it uses these (18 links) for corresponded link cuts.

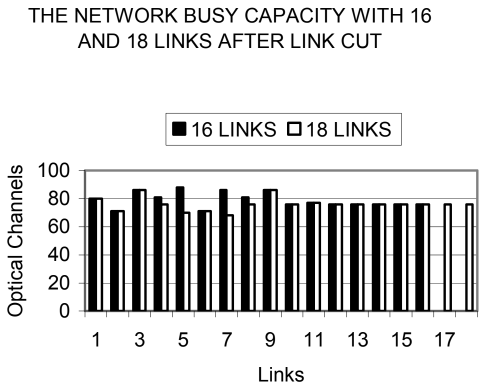

Fig. 2.The network busy capacity without and with links only for protection after the corresponded link cuts

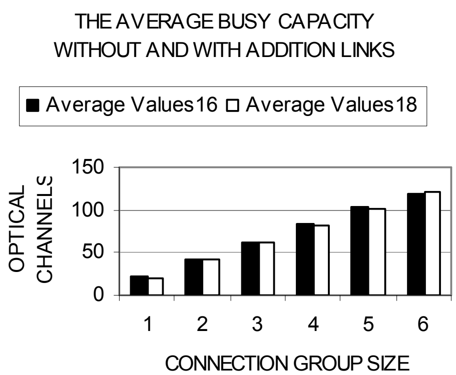

Fig. 3.The average busy capacity without and with addition links

In the figure 3 the average values of busy capacities are showed when no more (case 2) and more extra (case 3) links are used. These average values are obtained setting all connection group size to the 1, 2, 3, 4, 5 and 6. For each value and cutting one by one the network links a busy capacity is produced. All these busy capacities are summed and take the average value. After them the averages are compared and it is showed in figure 3. The difference between them is small and the added links when reduce the busy capacity, increase the available capacity. Larger values than 6 are not used because the link capacity is not adequate. The average is an integer that is closest to the obtained decimal value.

E. Attributes

A survivable network with extra links only for protection has the following attributes. First, *spare capacity,* a network with extra back up links only for protection uses more spare capacity than one without such links. In our example, the network without such links uses 16 optical fibers for protection and with such links uses 18 optical fibers for protection. Second, *restoration time*, a network with extra back up links only for protection obtains faster full restoration than another without such links. The restoration time depends primarily of the connections quantity and secondary of the restoration rerouting hops and the fiber-cut position. In this study, the same link cuts, at the same position with the same quantity of connections (same connection group number passing through), but the restoration rerouting hops contribute to the different restoration time. In the example, the link < v_2, v_3> cuts and the connections groups of the node pair [v_2, v_3], [v_2, v_4] and [v_2, v_5] are also cut. For full restoration when the extra back up links only for protection are used the total connections hops are six (6) and without such links the total connections hops are eight (8). The difference is ought to the restoration rerouting hops of the connection groups of the node pair [v_2, v_4] that uses such link < v_2, v_4>. So this case obtains less restoration time. The fiber length contributes very small to the restoration time and it is negligent. Third, *restoration complexity*, a network with extra back up links only for protection obtains restoration with simpler way than another without such links when these protection links are used. It is because the network has more connectivity, more available spare capacity and less restoration hops because such links also provide most direct restoration lightpaths so they use fewer network facilities and generally can provide better transmission quality. In the example, the link <v_2, v_3> cuts and the connections groups of the node pair [v_2, v_3], [v_2, v_4] and [v_2, v_5] are also cut. The lengths of the restoration lightpaths of the node pairs [v_2, v_3] and [v_2, v_5] do not change but the length of the restoration lightpath of the node pair [v_2, v_4] changes from three to one .So the restoration lightpath of the last node pair is more direct. Fourth, *network connectivity*, a network with extra back up links only for protection has more connectivity than a network without such links. In our example, the degree of the vertexes v_2 and v_4 when the extra protection back up link is used only for protection is three and without such links two. Fifth, *network reliability*, a network with extra back up links only for protection has more reliability than a network without such links because there are more restoration paths. Generally, the extra links only for protection are used when they improve the network.

F. Discussion

In the dynamical restoration , the dynamic routing method requires each OXC network controller to store only necessary local information and the rerouting decision is made according to the network status (e.g. configuration, available spare capacity and so on) at the time of network component failures .

As it is discussed previously, the use of such links improves the network attributes. However, the great disadvantage of these links is the cost. If their use reduces the network topological cost, they are always used. Some cases in which they are used are a) when the back-up optical fibers are old and their use is limited in time, in order to cover an extraordinary situation despite the whatever loss of quality in restoring traffic that may occur due to the age of fibers, b) when we use new optical fibers through existing ducts or special cable transmission channels (lower placement cost), c) the existing restoring routes require the use of optical amplifiers (EDFAs) that increase the cost much more than this method ,d)when they are going to use for network expansions or other future uses.

The use of the finite differences is possible for the study of the problems of the protection and restoration of the optical connections. These methods are accurate arithmetically for the study of the optical networks and problems associated thereto. The solution of each problem with two different ways is used for the verification and validation of results. These methods solve problems with small networks because when the number of the links and the number of the connections groups that pass through a link increases, then the number of the n+1equations with n+1 unknown of the system also increases and it is difficult to be solved. All of the optical network designs presented up until now have been based on suitable mathematics. That is, I assumed that all of the values, polynomial coefficients and results of mathematical computations are presented with one way, this of high, infinite precision. In Turbo Pascal for the PC, the floating-point formats are used and dictate the precision in representations of the polynomial coefficients and computation results. An optical mesh network designed under the infinite precision assumption will probably present severe degradations. The agreement of methods in this example is represented with accuracy of 15 decimal digits but some differences that appeared are ought to the difficulty to write all decimal digits for each polynomial coefficient of the polynomial method. In this problem, the failures that can restore are single wavelength crash, optical fiber cut and partial node failure.

IV. CONCLUSION

By using WDM the optical networks are capable of carrying many independent channels, which are carried on different wavelengths, over a single optical fiber. This allows the network to transport huge amounts of data that are needed for many current and future communication services, which play a very important role in many of our daily activities. The main drawback is the failures that can lead to the loss of a large amount of data. Thus the suitable strategy must use to minimize all such failures effects.

In this present paper I study the links which are used only for protection and show the advantages. When a failure occurs, new lightpaths can be set up or torn down to alleviate the failure repercussions. The strategy is done by traffic rerouting during failures by using extra links only for protections. The calculation of the new lightpaths is done by shortest path algorithm and the traffic rerouting is presented by finite difference methods, the linear and the accurate polynomial ones. These extra links use new or old optical fibers. The new one will increase the operation costs of the network overall, however it will eliminate most quality-related problems; in light of the latter, restoring capabilities may constitute a specification of quality in service-level agreements (SLAs). A service-level agreement is a part of a service contract where the level of service is formally defined. In practice, the term SLA is sometimes used to refer to the contracted delivery time (of the service) or performance. So the extra links only for protection together with the back-up optical fibers can cover our protection related needs in a fuller and far easier way than any other method.

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