On Pilot-Symbol-Assisted Cooperative Systems with Cascaded Rayleigh and Rayleigh Fading Channels with Imperfect CSI

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***Abstract*—In this paper, we analyze the impact of imperfect channel estimation on the performance of pilot symbol assisted modulation (PSAM) scheme used in a cooperative communication system with distributed space time block code (STBC) operating with amplify-and-forward (AaF) relaying protocol. The fading channel is modeled as both Rayleigh fading and *cascaded Rayleigh fading*, also known as double Rayleigh fading. We derive the correlation coefficient r between the channel gain and its erroneous estimate, due to the imperfect CSI at the receiver terminal; when the $R \rightarrow D$ link is either non-fading or fading. We present an expression for r in terms of Doppler frequency, number of pilot symbols and SNR. Our performance analysis demonstrates that the presence of fading in the $R \rightarrow D$ link manifests itself by introducing additional Doppler frequency terms in r. It also reveals that there are additional Doppler frequency terms in case of cascaded Rayleigh channel compared to the conventional Rayleigh fading channel. Furthermore, we derive a tight lower bound for the bit error rate (BER) of both channel models with channel estimation errors, in terms of cross-correlation coefficients. Simulation results are also presented to corroborate our analytical studies.**

***Index Terms*— Amplify-and-forward relaying, Cascaded Rayleigh fading, Cooperative communications, Imperfect CSI, PSAM scheme.**

I. Introduction

THERE is an increasing demand for wireless multimedia and interactive internet services that require much higher data transmission speed and reliability compared to the current wireless communications systems. Spatial diversity is a widely used technique that promises significant improvement in link reliability and spectral efficiency through the use of multiple antennas at the transmitter and/or receiver side [1]-[3]. One of the most effective tools to exploit distributed spatial diversity in the wireless networks is cooperative diversity, also known as user cooperation [4] that can bring about spatial diversity via creating a virtual antenna array at the receiver terminal by applying space-time coding techniques [5]-[6].

Most of the current works on cooperative diversity have assumed that perfect knowledge of the channel fading gains is available at the receiver side, e.g. [7]-[8]. In practical scenarios, these coefficients must be estimated and then used in the detection process [9]. The effect of channel estimation errors for AaF relaying protocol have been extensively studied in the literature for conventional Rayleigh fading channels, see e.g. [10]-[14] and references therein.

A. Related works

Although most researchers assume that the channel is Rayleigh fading in analyzing the performance of cooperative communication systems, it has been shown that *cascaded Rayleigh* distribution, also known as *double Rayleigh* distribution, provides a more accurate model for mobile-to-mobile communications [15], especially in such applications as inter-vehicular communications (IVC) systems and ad-hoc networks that both source and destination terminals are in motion. To the best of our knowledge there are only few analyses on the performance of STBC-assisted systems over cascaded Rayleigh fading channels. In [16], Uysal has derived an expression for pair-wise error probability for space-time trellis codes over cascaded Rayleigh fading channels under the assumption of perfect channel state information (CSI) at the receiver terminal. The same author investigates the error rate performance of coherent M-ary phase shift keying (M-PSK) modulation over cascaded Rayleigh fading with receive antenna diversity where he also assumes that perfect CSI is available at all terminals [17]. However, cooperative transmission is considered in neither [16] nor [17]. Amin *et al.* investigate the performance of AaF relaying with two different pilot-symbol-assisted channel estimation methods in [18] where they compare the performance of two different estimation methods.

For fading channels, pilot symbol assisted modulation (PSAM) scheme is used for coherent detection by applying pilot symbols to estimate the channel on minimum-mean-squared-error (MMSE) basis [19]. In [20], the symbol error rate of a cooperative communication system operating in the AaF mode for a PSAM scheme is derived in the presence of channel estimation errors considering the effect of Doppler frequency. The asymptotic BER bound for the high SNR regime in a multi-relay network is provided in [21]. However, the bound is loose for practical SNR's. The impact of

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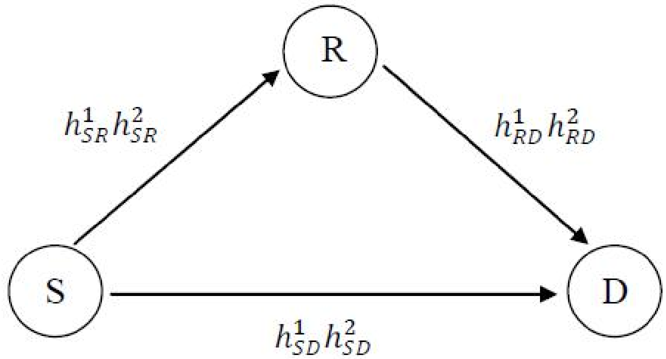

Fig. 1. Cascaded relay-assisted fading channel.

imperfect channel estimation on the error performance of distributed space time codes is also analyzed in [22] in terms of the diversity order but without considering PSAM.

B. Contributions

Our main contributions in this paper are summarized as follow:

We analyze the impact of imperfect channel estimation on the performance of PSAM scheme for a distributed STBC system operating in AaF relaying protocol. We derive the correlation coefficients of channel gains and their estimates in terms of Doppler frequency, number of pilot symbols and SNR. Having known this relation, one can optimally choose the number of pilot symbols in order to compensate for the estimation error when the fading and Doppler effects are severe. It is demonstrated that the presence of fading in the $R \to D$ link manifests itself by introducing additional Doppler frequency terms.

We also present a tight lower bound for the BER of a cooperative communication system with BPSK modulation in the presence of channel estimation errors in terms of the cross-correlation coefficient for the practical SNR regime.

We further expand the work of [23] and consider the cascaded Rayleigh fading channel as well as the conventional Rayleigh fading channel. We compare the performance of the two models which reveals that there is an additional Doppler frequency term in r when the cascaded distribution is adopted.

The rest of the paper is organized as follows. In Section II, the considered system model is introduced. In Section III, the PSAM scenario is presented and the expressions for the cross-correlation coefficients of the channel gains and their estimates are derived. Performance analysis and BER derivation are provided in Section IV. In Section V, simulation results are presented to confirm the analytical results, and conclusions are drawn in Section VI.

II. System Model

A wireless communications system where the source terminal S transmits information to the destination terminal D with the assistance of a relay terminal R is considered as shown in Fig. 1. We assume a time-varying frequency-flat cascaded Rayleigh fading channel and adopt the user cooperation protocol III proposed in [24]: The source terminal communicates with the relay terminal during the first signaling interval. There is no transmission from source to destination within this period. In the second signaling interval, both the relay and the source terminals communicate with the destination terminal. For relay to destination link, the AaF mode is used.

Let two consecutive signals transmitted by the source terminal, using BPSK modulation, be denoted as x_1 and x_2. The received signal at the destination terminal after the second time interval is as follows:

$$r = a\, h_{SR} h_{RD} x_1 + b\, h_{SD} x_2 + n \tag{1}$$

where h_{SD}, h_{SR} and h_{RD} are the channel gains over $S \to D$, $S \to R$ and $R \to D$ links, respectively.

If the channel is modeled as conventional Rayleigh fading, each channel gain is a zero-mean complex Gaussian random variable, denoted by $CN \bullet (0, s^2)$ and their magnitude $|h_{SD}|$, $|h_{SR}|$, and $|h_{RD}|$ follow a Rayleigh distribution given by [25]

$$f(x) = \frac{x}{s^2} \exp\left(\frac{-x^2}{2s^2} \right) \tag{2}$$

On the other hand, in cascaded Rayleigh fading channels, each channel gain is a product of two independent complex Gaussian random variables, i.e., $h_{SD} = h_{SD}^1 h_{SD}^2$, $h_{SR} = h_{SR}^1 h_{SR}^2$ and $h_{RD} = h_{RD}^1 h_{RD}^2$; each of which has zero mean and variance $s_{SD}^2/2$, $s_{SR}^2/2$ and $s_{RD}^2/2$ per dimension, respectively[1]. Therefore, their magnitudes follow a cascaded Rayleigh distribution given by [25]

$$f(x) = 2\sqrt{x}\,\mathrm{K}_0\left(2\sqrt{x}\right) \tag{3}$$

where $\mathrm{K}_0(.)$ is the zero-order modified Bessel function of the second kind.

The noise term n in (1) is assumed to be a zero-mean complex Gaussian random variable with variance $N_0/2$ per dimension. a and b are normalization coefficients due to the AaF mode and are as follow [26]

$$a = \sqrt{\frac{(E_{SR}/N_0)E_{RD}}{1 + E_{SR}/N_0 + |h_{RD}|^2\, E_{RD}/N_0}} \tag{4}$$

$$b = \sqrt{\frac{(1 + E_{SR}/N_0)E_{SD}}{1 + E_{SR}/N_0 + |h_{RD}|^2\, E_{RD}/N_0}} \tag{5}$$

[1] For the sake of simplicity, most of the time it is assumed that $s_{SD}^2 = s_{SR}^2 = s_{RD}^2 = 1$.

where E_{SR} is the average energy available at the relay terminal, and E_{RD} and E_{SD} represent the average energies available at the destination terminal considering different path loss and possible shadowing effects in the $S \rightarrow R$, $R \rightarrow D$ and $S \rightarrow D$ links, respectively.

We now employ STBC to exploit its inherent orthogonality as an essential feature for channel estimation and data detection. For the case of single relay terminal, we need to use STBC designed for two transmit antennas i.e., Alamouti's scheme [3]. For this purpose, the two data signals x_1 and x_2 are simultaneously sent during four consecutive signaling time slots as shown in Table I, where "NT" stands for "no transmission".

The corresponding detected signals at the destination terminal can then be written as

$$\hat{x}_1 = a\,\hat{h}_{SR}^{*}\hat{h}_{RD}^{*}r_1 + b\,\hat{h}_{SD}r_2^{*} \quad (6)$$

$$\hat{x}_2 = b\,\hat{h}_{SD}^{*}r_1 - a\,\hat{h}_{SR}\hat{h}_{RD}r_2^{*} \quad (7)$$

where $\hat{h}_{SD}$, $\hat{h}_{SR}$ and $\hat{h}_{RD}$ are the estimates of h_{SD}, h_{SR} and h_{RD}, respectively. r_1 and r_2 are received signals at the destination terminal after the second and the fourth signal intervals given by (1) as

$$r_1 = a\,h_{SR}h_{RD}x_1 + b\,h_{SD}x_2 + n_1 \quad (8)$$

$$r_2 = -a\,h_{SR}h_{RD}x_2^{*} + b\,h_{SD}x_1^{*} + n_2 \quad (9)$$

III. PSAM FOR DISTRIBUTED STBC

In the considered PSAM scenario, each frame consists of 2 pilot symbols, P_1 and P_2, and $M-2$ data symbols as shown in Fig. 2. We assume that each frame consists of $M/2$ sub blocks each of which comprising two symbols. Under the assumption of non-fading $R \rightarrow D$ link, i.e., $h_{RD}=1$, and that the channel gains remain constant over four symbol intervals, the received signals at the k^{th} ($0 \le k < M/2$) sub block in the j^{th} frame can be obtained from (8) and (9) as

$$r_1^{k,j} = a\,h_{SR}^{k,j}x_1^{k,j} + b\,h_{SD}^{k,j}x_2^{k,j} + n_1^{k,j} \quad (10)$$

$$r_2^{k,j} = -a\,h_{SR}^{k,j}x_2^{k,j} + b\,h_{SD}^{k,j}x_1^{k,j} + n_2^{k,j} \quad (11)$$

Assuming, without loss of generality, that pilot symbols (P_1=1, P_2=1) are positioned at the beginning of each frame, i.e., sub-block $k=0$, we can write the received pilot signals at the destination terminal as

$$r_1^{0,j} = a\,h_{SR}^{0,j}x_1^{0,j} + b\,h_{SD}^{0,j}x_2^{0,j} + n_1^{0,j} \quad (12)$$

$$r_2^{0,j} = -a\,h_{SR}^{0,j}x_2^{0,j} + b\,h_{SD}^{0,j}x_1^{0,j} + n_2^{0,j} \quad (13)$$

Based on the received signals corresponding to pilot symbol

TABLE I
THE ENCODING AND TRANSMISSION SEQUENCE FOR A SINGLE RELAY STBC SYSTEM

Time Slot \ Transmission Link	$S \rightarrow R$	$R \rightarrow D$	$S \rightarrow D$
1	x_1	NT	NT
2	x_2	x_1	x_2
3	$-x_2^{*}$	NT	NT
4	NT	$-x_2^{*}$	x_1^{*}

transmissions, the destination terminal employs a Wiener filter to estimate the fading coefficients. As depicted in Fig. 2, we assume that $\lfloor L/2 \rfloor$ pilot symbols from the following frames and $\lfloor (L-1)/2 \rfloor$ pilot symbols from the previous frames and 1 current frame are employed in this estimation.

A. *Conventional Rayleigh fading*

In conventional Rayleigh fading model, the channel estimates for both $S \rightarrow R \rightarrow D$ and $S \rightarrow D$ links at the k^{th} sub block in the j^{th} frame are obtained as

$$\hat{h}_{SR}^{k,j} = \sum_{j=-\lfloor (L-1)/2 \rfloor}^{\lfloor L/2 \rfloor} w_j^k \left(a\,h_{SR}^{0,j} + \frac{n_1^{0,j} - n_2^{0,j}}{2} \right) \quad (14)$$

$$\hat{h}_{SD}^{k,j} = \sum_{j=-\lfloor (L-1)/2 \rfloor}^{\lfloor L/2 \rfloor} w_j^k \left(b\,h_{SD}^{0,j} + \frac{n_1^{0,j} - n_2^{0,j}}{2} \right) \quad (15)$$

where w_j^k's are the interpolation coefficients in the j^{th} frame. It is worth mentioning that the value of the channel fading gain in (14) is the same for the same sub block in different frames whereas it is different for different sub blocks in a single frame. The same also holds true for (15).

The variance of $\hat{h}_{SR}^{k,j}$ is given as

$$\sigma_{\hat{h}_{SR}^{k,j}}^{2} = \frac{a^2}{2} \sum_{i=-\lfloor (L-1)/2 \rfloor}^{\lfloor L/2 \rfloor} \sum_{j=-\lfloor (L-1)/2 \rfloor}^{\lfloor L/2 \rfloor} w_i^k \left(w_j^k\right)^{*} \mathrm{E}\left[h_{SR}^{0,i} \left(h_{SR}^{0,j}\right)^{*} \right] + \frac{N_0}{4} \sum_{j=-\lfloor (L-1)/2 \rfloor}^{\lfloor L/2 \rfloor} |w_j^k|^2 \quad (16)$$

where we assume Clarke's Bessel-type auto-correlation function

$$\mathrm{E}\left[h_{SR}^{0,i} \left(h_{SR}^{0,j}\right)^{*} \right] = 2\sigma_{SR}^{2} J_0\left(2\pi\, fT_{SR}\,|i-j|\,M\right) \quad (17)$$

where $J_0(.)$ is the zero-order Bessel function of the first kind and fT_{SR} is the normalized Doppler frequency for $S \rightarrow R$ link.

Furthermore, the cross-correlation coefficient of the channel gain and its estimate can be calculated as

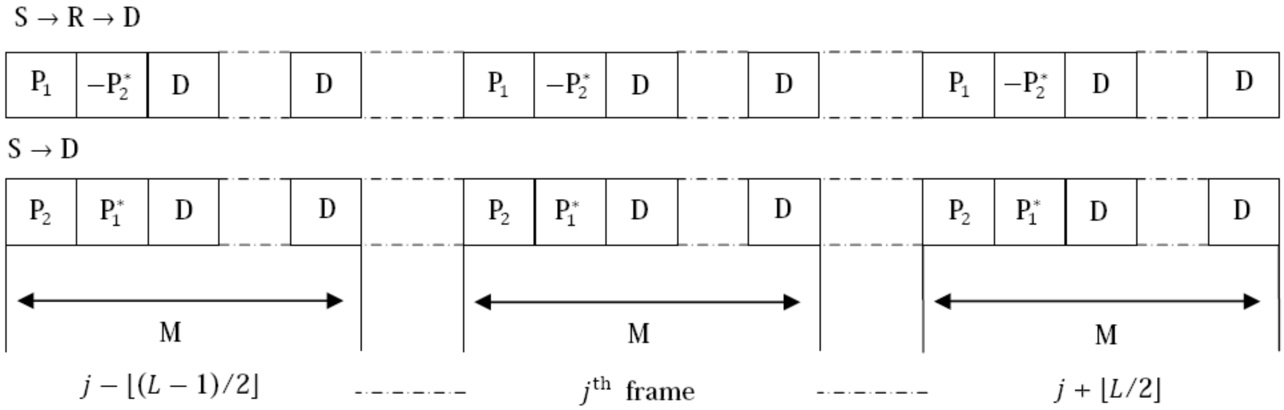

Fig. 2. Frame structure for pilot-symbol-assisted channel estimation. P : Pilot symbol, D : Data, M : Frame length.

$$\begin{aligned} \rho^{k,j}_{h_{SR}\hat{h}_{SR}} &= \frac{1}{2}\left[h^{k,j}_{SR}\left(\hat{h}^{k,j}_{SR}\right)^{*}\right] \\ &= \alpha \sum_{j=-\lfloor (L-1)/2 \rfloor}^{\lfloor L/2 \rfloor} \left(w^{k}_{j}\right)^{*} \sigma^{2}_{SR} J_{0}\left(2\pi\ fT_{SR}\,|\,2k-j\,|\,M\right) \end{aligned} \tag{18}$$

Therefore, following [27], the correlation coefficient of the squared amplitude of the channel estimates for $S \to R \to D$ link with underlying non-fading $R \to D$ link[2], $\rho^{s}_{S\to R\to D}$, can be written as in (19). For the asymptotic case of $E_{SD}/N_o = E_{RD}/N_0 \cdot 1$ with perfect power control and sufficiently large $E_{SR}/N_0 > E_{SD}/N_0$ values, the normalization coefficients in (4) and (5) reduce to $\alpha = \beta = \sqrt{E_{SD}}$. Then, the correlation coefficient in (19) is a function of Doppler frequency, the SNR= $2\sigma^{2}_{SR}E_{SD}/N_0$, and the number of interpolation coefficients.

Similarly, it can be shown that the correlation coefficient for $S \to D$ link, $\rho^{s}_{S\to D}$, is given by (20) where fT_{SD} is the normalized Doppler frequency for $S \to D$ link.

When the underlying $R \to D$ link is subject to fading, we can write the j^{th} fading channel estimate for $S \to R \to D$ link as

$$\hat{h}^{k,j}_{SR}\hat{h}^{k,j}_{RD} = \sum_{j=-\lfloor (L-1)/2 \rfloor}^{\lfloor L/2 \rfloor} w^{k}_{j}\left(\alpha\, h^{0,j}_{SR}h^{0,j}_{RD} + \frac{n^{0,j}_{1} - n^{0,j}_{2}}{2}\right) \tag{21}$$

Following similar steps used in the derivation of (19), we can find the correlation coefficient, $\rho^{f}_{S\to R\to D}$, as in (22). Here, fT_{RD} is the normalized Doppler frequency for the fading $R \to D$ link. Comparing (19) and (22), it can be observed that the presence of fading in the $R \to D$ link manifests itself with the introduction of additional Doppler frequency terms. In other words, the time-varying nature of $R \to D$ link will increase the effective Doppler speed observed by the destination terminal. It should also be noted that (19) and (22) are the same when h_{RD} is non-fading, i.e., fT_{RD}=0 [3]. It is worth mentioning that due to the embedded orthogonality, $\rho^{f}_{S\to D}$ under the effect of fading $R \to D$ link is still given by (20).

B. *Cascaded Rayleigh fading*

In this model, each channel gain is a product of two independent complex Gaussian random variables. Therefore, it is easy to confirm that each channel gain could be expressed in the same fashion as

$$\begin{aligned} \hat{h}^{k,j}_{SR} &= \hat{h}^{1,\{k,j\}}_{SR} \times \hat{h}^{2,\{k,j\}}_{SR} \\ &= \sum_{j=-\lfloor (L-1)/2 \rfloor}^{\lfloor L/2 \rfloor} w^{k}_{j}\left(\alpha\, h^{1,\{0,j\}}_{SR}h^{2,\{0,j\}}_{SR} + \frac{n^{0,j}_{1} - n^{0,j}_{2}}{2}\right) \end{aligned} \tag{23}$$

Similarly, we can write h_{SD} and h_{RD} in terms of interpolation coefficients w^{k}_{j}'s. The auto-correlation function is therefore

$$\begin{aligned} \mathrm{E}\left[h^{0,i}_{SR}\left(h^{0,j}_{SR}\right)^{*}\right] &= \mathrm{E}\left[h^{1,\{0,i\}}_{SR}\left(h^{1,\{0,j\}}_{SR}\right)^{*}\right] \\ &\quad\times \mathrm{E}\left[h^{2,\{0,i\}}_{SR}\left(h^{2,\{0,j\}}_{SR}\right)^{*}\right] \\ &= 4\sigma^{4}_{SR}J^{2}_{0}\left(2\pi\ fT_{SR}\,|\,i-j\,|\,M\right) \end{aligned} \tag{24}$$

Considering (23) and (24) and using the same approach as in the derivation of cross-correlation $\rho^{k,j}_{h_{SR}\hat{h}_{SR}}$ in (16) and the variance $\sigma^{2}_{\hat{h}^{k,j}_{SR}}$ in (18), we finally find the correlation coefficient over $S \to R \to D$ link with underlying fading $R \to D$ link as in (25). It is evident that the cascaded channel incurs a severer Doppler effect pertaining to the additional

[2] The "s" in ρ^{s} stands for "static". In contrary, "f" stands for "fading".

[3] Note that $\alpha^{2} = E_{SD}$ and $\sigma^{2}_{RD} = 1$.

$$\rho^{s}_{S\to R\to D}=\frac{\left(r^{k,j}_{h_{SR}\hat{h}_{SR}}\right)^2}{\sigma^2_{h^{k,j}_{SR}}\sigma^2_{\hat{h}^{k,j}_{SR}}}=\frac{\left(\sum_j\left(w^k_j\right)^* J_0\left(2\pi\, fT_{SR}\,|\,2k-j\,|\,M\right)\right)^2}{\sum_i\sum_j w^k_i\left(w^k_j\right)^* J_0\left(2\pi\, fT_{SR}\,|\,i-j\,|\,M\right)+\frac{N_0}{4\sigma^2_{SR}\alpha^2}\sum_j|\,w^k_j\,|^2} \tag{19}$$

$$\rho^{s}_{S\to D}=\frac{\left(\sum_j\left(w^k_j\right)^* J_0\left(2\pi\, fT_{SD}\,|\,2k-j\,|\,M\right)\right)^2}{\sum_i\sum_j w^k_i\left(w^k_j\right)^* J_0\left(2\pi\, fT_{SD}\,|\,i-j\,|\,M\right)+\frac{1}{2\mathrm{SNR}}\sum_j|\,w^k_j\,|^2} \tag{20}$$

Doppler frequency terms. The correlation coefficients $\rho^{s}_{S\to D}$, $\rho^{f}_{S\to D}$ and $\rho^{s}_{S\to R\to D}$ are straightforward to derive.

IV. Bit Error Rate Analysis

In this section we present detailed derivation of the BER expression for the aforementioned system model. We adopt BPSK modulation where $x_1 = x_2$ or $x_1 = -x_2$, each with probability 1/2. According to the BPSK decision rule, if $\mathrm{Re}\{\hat{x}_i\} > 0$ (i=1,2), then $\hat{x}_i$ is demodulated to 1, otherwise $\hat{x}_i = -1$ is chosen. Without loss of generality, we consider the detection of $\hat{x}_1$, noting that the same steps can be followed in the detection of the symbol $\hat{x}_2$.

A. Conventional fading channel

Since h_{SR} and $\hat{h}_{SR}$ are jointly Gaussian, conditioned on $\hat{h}_{SR}$, the channel gain h_{SR} can be written as [14]

$$h_{SR} = \rho_{SR}\hat{h}_{SR} + d_{SR} \tag{26}$$

Where $\sqrt{\rho_{SR}} = \mathrm{E}[h_{SR}\hat{h}^*_{SR}]/2\sigma_{h_{SR}}\sigma_{\hat{h}_{SR}}$ and d_{SR} is a complex Gaussian random variable with zero mean and variance $\sigma^2_{d_{SR}} = (1-\rho_{SR})\sigma^2_{h_{SR}}$ per dimension. We can also write $h_{SD} = \rho_{SD}\hat{h}_{SD} + d_{SD}$ and $h_{RD} = \rho_{RD}\hat{h}_{RD} + d_{RD}$. Assuming that all links experience identical statistics, we have $\rho_{SD} = \rho_{SR} = \rho_{RD} = \rho$ and $\sigma^2_{d_{SD}} = \sigma^2_{d_{SR}} = \sigma^2_{d_{RD}} = \sigma^2_d$.

For the case of $x_1 = x_2$, conditioned on h_{RD}, i.e., the receiver estimates h_{RD} correctly, and substituting (8) and (9) into (6) and (7), we obtain

$$\begin{aligned}\hat{x}_1 &= h\, x_1\Big[\alpha^2\,|\,h_{RD}\,|^2|\,\hat{h}_{SR}\,|^2+\beta^2\,|\,\hat{h}_{SD}\,|^2\\ &\quad+\alpha\beta\,\hat{h}_{SD}\Big(\left(\hat{h}_{SR}h_{RD}\right)^*-\hat{h}_{SR}h_{RD}\Big)\Big]\\ &\quad+x_1\Big[\alpha^2\,|\,h_{RD}\,|^2\,\hat{h}^*_{SR}+\beta^2\hat{h}_{SD}d^*_{SD}\\ &\quad+\alpha\beta\Big(\left(\hat{h}_{SR}h_{RD}\right)^* d_{SD}-\hat{h}_{SD}h_{RD}d_{SR}\Big)\Big]\\ &\quad+\alpha\,\hat{h}^*_{SR}h^*_{RD}n_1+\beta\,\hat{h}_{SD}n^*_2\end{aligned} \tag{27}$$

It is worth mentioning that the assumption that h_{RD} is correctly estimated, simplifies the BER derivation at the cost of 1 dB difference in higher SNR's compared to the same actual BER. However, the final expression for the lower bound is quite tight for small SNR's.

Now, conditioned on $\hat{h}_{SR}$ and $\hat{h}_{SD}$, $\mathrm{Re}\{\hat{x}_1\}$ is

$$\mathrm{Re}\left[\hat{x}_1\right] = Cx_1 + y \tag{28}$$

where y is a zero-mean Gaussian random variable with variance

$$\begin{aligned}\sigma^2_y &= \sigma^2_d\left(\beta^2+\alpha^2\,|\,h_{RD}\,|^2+\frac{N_0}{2}\right)\\ &\quad\times\left(\alpha^2\,|\,h_{RD}\,|^2|\,\hat{h}_{SR}\,|^2+\beta^2\,|\,\hat{h}_{SD}\,|^2\right)\end{aligned} \tag{29}$$

and

$$C = \rho\left(\alpha^2\,|\,h_{RD}\,|^2|\,\hat{h}_{SR}\,|^2+\beta^2\,|\,\hat{h}_{SD}\,|^2\right) \tag{30}$$

Therefore, it can be shown that

$$P^{h_{RD},\hat{h}_{SD},\hat{h}_{SR}}_{e|x_1=+x_2} = Q\left(\sqrt{\frac{\rho^2\left(\alpha^2\,|\,h_{RD}\,|^2|\,\hat{h}_{SR}\,|^2+\beta^2\,|\,\hat{h}_{SD}\,|^2\right)}{\sigma^2_d\left(\beta^2+\alpha^2\,|\,h_{RD}\,|^2+\frac{N_0}{2}\right)}}\right) \tag{31}$$

We also have $P^{h_{RD},\hat{h}_{SD},\hat{h}_{SR}}_{e|x_1=+x_2} = P^{h_{RD},\hat{h}_{SD},\hat{h}_{SR}}_{e|x_1=-x_2}$. Therefore the BER, conditioned on h_{RD}, $\hat{h}_{SD}$ and $\hat{h}_{SR}$, is given by (31).

Subsequently, assuming realistically that $E_{SD}/N_0 = E_{RD}/N_0 \gg 1$ and for sufficiently large $E_{SR}/N_0 > E_{SD}/N_0$, the normalization factors α and β reduce to $\sqrt{E_{SD}}$. Thus, we can rewrite (31) in terms of the channel correlation coefficient $\rho^{f}_{S\to R\to D}$ and the end-to-end instantaneous SNR, γ, as follows

$$r^{f}_{S\rightarrow R\rightarrow D} = \frac{\left(\sum_j \left(w_j^k\right)^* J_0\left(2p\ fT_{SR}\ |2k-j|\ M\right) J_0\left(2p\ fT_{RD}\ |2k-j|\ M\right)\right)^2}{\sum_i \sum_j w_i^k \left(w_j^k\right)^* J_0\left(2p\ fT_{SR}\ |i-j|\ M\right) J_0\left(2p\ fT_{RD}\ |i-j|\ M\right) + \frac{N_0}{4s^2_{SR}s^2_{RD}E_{SD}} \sum_j |w_j^k|^2} \tag{22}$$

$$r^{f,\text{Cascaded}}_{S\rightarrow R\rightarrow D} = \frac{\left(\sum_j \left(w_j^k\right)^* J_0\left(2p\ fT_{SR}\ |2k-j|\ M\right) J_0\left(2p\ fT_{RD}\ |2k-j|\ M\right)\right)^4}{\sum_i \sum_j w_i^k \left(w_j^k\right)^* J_0^2\left(2p\ fT_{SR}\ |i-j|\ M\right) J_0^2\left(2p\ fT_{RD}\ |i-j|\ M\right) + \frac{N_0}{4s^2_{SR}s^2_{RD}E_{SD}} \sum_j |w_j^k|^2} \tag{25}$$

$$P_{e|h_{RD},\hat{h}_{SD},\hat{h}_{SR}} = Q\left(\sqrt{\frac{gr\left(|h_{RD}|^2|\hat{h}_{SR}|^2+|\hat{h}_{SD}|^2\right)}{\left(1+|h_{RD}|^2\right)\left(1+(1-r)g\right)}}\right) \tag{32}$$

where $r = r^{f}_{S\rightarrow R\rightarrow D}$, given by (22) when the underlying $R\rightarrow D$ link is subject to fading, and

$$g = \frac{2s^2_{SR}E_{SD}}{N_0}\left(1+|h_{RD}|^2\right) \tag{33}$$

Using the alternative definition of Q-function, $Q(x)=1/p\int_0^{p/2}\exp\left(-x^2/2\sin^2 q\right)dq$, and the MGF approach; after some mathematical manipulations, (32) is simplified to

$$P_e = \frac{1}{p}\int_o^{p/2}\left(\frac{\text{Sin}^2 q}{\text{Sin}^2 q+\bar{z}_1}\right)\left(\frac{\text{Sin}^2 q}{\text{Sin}^2 q+\bar{z}_2}\right)dq \tag{34}$$

where

$$\bar{z}_1 = \frac{gr\ |h_{RD}|^2}{\left(1+|h_{RD}|^2\right)\left(1+(1-r)g\right)} \tag{35}$$

$$\bar{z}_1 = \frac{gr}{\left(1+|h_{RD}|^2\right)\left(1+(1-r)g\right)} \tag{36}$$

Therefore, the BER is given by a single finite integral which is a function of the channel correlation coefficient expressed in (22) that is a function of Doppler frequency, SNR and the number of pilot symbols in a PSAM-assisted STBC network.

For the non-fading $R\rightarrow D$ link, i.e., $h_{RD}=1$, the BER expression is reduced to

$$P_e^{h_{RD}=1} = \frac{1}{4}\left(1-\sqrt{\frac{\bar{z}}{2+\bar{z}}}\right)^2\left(2+\sqrt{\frac{\bar{z}}{2+\bar{z}}}\right) \tag{37}$$

where $\bar{z}$ is calculated by substituting $h_{RD}=1$ in either (35) or (36).

B. Cascaded Rayleigh channels

Similar to the conventional Rayleigh channel model, since h^i_{SR} and $\hat{h}^i_{SR}$ for i=1,2 are jointly Gaussian, conditioned on $\hat{h}^i_{SR}$, the channel gain h^i_{SR} can be written as

$$h^i_{SR} = r_{SR,i}\hat{h}^i_{SR} + d_{SR,i}, \quad i=1,2 \tag{38}$$

which in turn results in

$$h_{SR} = r_{SR,1}r_{SR,2}\hat{h}_{SR} + d_{SR} \tag{39}$$

where, in this case, d_{SR} is no longer a simple complex Gaussian random variable. Instead, it comprises the products of different independent random variables. However, due to mathematical complexity, we assume that it is approximated by a complex Gaussian random variable with variance $s^2_{d_{SR}}$. We should mention that a simpler though more realistic model to express the channel gain h_{SR} in terms of $\hat{h}_{SR}$ is $h^1_{SR}h^2_{SR}=\hat{h}^1_{SR}\hat{h}^2_{SR}+e$ where e is a Gaussian error. However, it is no longer in terms of the correlation coefficient r.

Identical expressions for the channel gains over $S\rightarrow D$ and $R\rightarrow D$ are straightforward to derive. Therefore, conditioned on h_{RD}, and taking similar approach as in the conventional Rayleigh channel model, and also assuming that all links experience identical statistics, we obtain the conditional BER expression as given by (32). Now, in order to calculate P_e, we have

$$P_e = \frac{1}{p}\left[\underbrace{\int_0^\infty e^{-sX}f_X(x)dx}_{\Phi_X(-s)}\underbrace{\int_0^\infty e^{-sY}f_Y(y)dy}_{\Phi_Y(-s)}\right]dq \tag{40}$$

where $X=\bar{z}_1|\hat{h}_{SR}|^2$ and $Y=\bar{z}_2|\hat{h}_{SD}|^2$, $s=1/2\sin^2 q$, $f_X(.)$ and $f_Y(.)$ are the PDF of X and Y, respectively, given by (3). Finally, $\Phi_X(-s)$ and $\Phi_Y(-s)$ are the MGF of X and Y reported in Eq. (5) of [17]. Therefore, the BER P_e is found as

$$P_e = \frac{1}{p}\int_0^{p/2}\frac{1}{t^2\bar{z}_1\bar{z}_2} \times\exp\left[-\frac{1}{t}\left(\frac{1}{\bar{z}_1}+\frac{1}{\bar{z}_2}\right)\right]\Gamma(0,1/t\bar{z}_1)\Gamma(0,1/t\bar{z}_2)dq \tag{41}$$

where $t = 2\sin^2 q$. In this case too, the BER is given by a single finite integral which is a function of the channel correlation coefficient expressed in (25). Although it is a lower bound, due to its simplicity it can be used as a framework which can facilitate the investigation of diversity order in Multi-input-multi-output cooperative communications systems in the absence of perfect CSI at the receiver terminal. For the perfect CSI counterpart see [17].

V. Simulation Results

In this section, simulation results are presented to verify our analytical expressions. In our simulation study, we consider BPSK modulation and assume $E_{SD} = E_{RD}$, i.e., $S \to D$ and $R \to D$ links are balanced, which can be achieved through power control. As for the $S \to R$ link, we set $E_{SR} / N_0 = 30$ dB. First, we assume a non-fading $R \to D$ link, i.e., $h_{RD} = 1$. Fig. 3 shows BER versus SNR with respect to various values for correlation coefficient r . As it can be seen, there is a perfect match between our theoretical results and simulations for non-fading $R \to D$ link. For the fading $R \to D$ link, our theoretical result is a tight lower bound for BER.

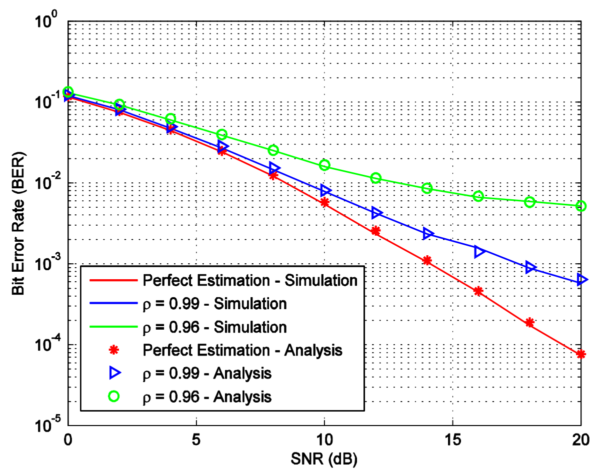

Fig. 3. BER versus SNR of Rayleigh fading relay-assisted transmission with non-fading $R \to D$ link for various correlation coefficients.

As shown in Fig. 4, it also perfectly matches the Monte Carlo simulation at lower SNR's and gradually diverges from it at higher SNR's due to the approximations made during the derivation of analytical results.

For the sake of performance comparison, we have plotted the BER versus SNR for both cascaded Rayleigh and Rayleigh distributions. We observe performance degradation in cascaded Rayleigh fading case as predicted by our derived expressions in (25) and (41). Since the cascaded channel incurs a severer Doppler effect pertaining to the additional Doppler frequency terms, the correlation coefficients between the actual channel gains and their estimates are smaller compared to that of conventional Rayleigh fading channels

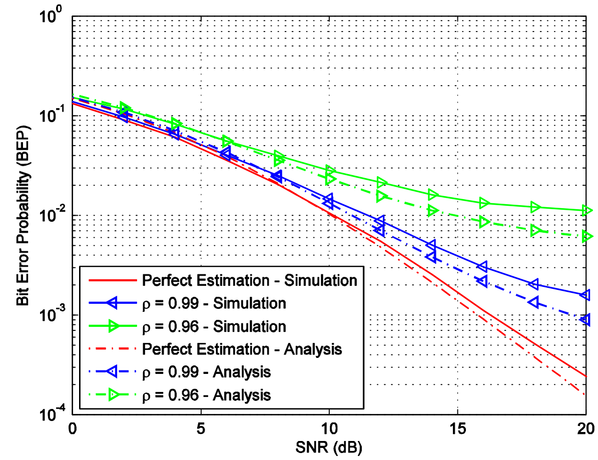

Fig. 4. BER versus SNR of Rayleigh fading relay-assisted transmission with fading $R \to D$ link for various correlation coefficients.

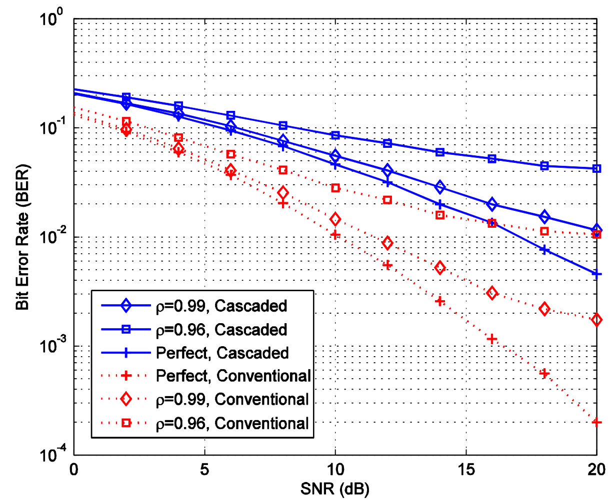

Fig. 5. BER of cascaded Rayleigh and conventional Rayleigh fading relay-assisted transmission with fading $R \to D$ link for various correlation coefficients.

which leads to a higher error probability.

VI. Conclusion

We analyzed the impact of imperfect channel estimation on the performance of PSAM for a distributed STBC system with AaF relaying. Through the derivation of the correlation coefficient of a channel coefficient and its estimate, we demonstrated that the presence of fading in the $R \to D$ link manifests itself with introducing additional Doppler frequency terms. In other words, the time varying nature of $R \to D$ link will increase the effective Doppler speed observed by the destination terminal. This would bring about more errors in the estimation. However, with the relation of the correlation coefficients and the number of pilot symbols derived in this paper, one can optimally choose the number of pilots in order to compensate for the estimation error when the fading and Doppler effects are severe. In addition, tight lower bounds for the BER of both cascaded Rayleigh and conventional Rayleigh cooperative communications systems with BPSK modulation in the presence of channel estimation errors were also presented in terms of the cross correlation coefficient. These bounds are single finite integrals that can be used as frameworks which can facilitate the investigation of diversity order in Multi-input-multi-output cooperative communications systems.

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