A Comparative Study of Amplify-and-Forward and Zero-Forcing Relaying with Different Power Constraints

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Abstract — *Cooperating nodes in wireless networks need to be power-efficient because once they deployed, as in sensor networks, they might not be recharged or replaced. The choice of relaying technique, therefore, becomes a crucial design parameter. Our work provides a detailed comparative performance analysis of amplify-and-forward and zero-forcing relaying. Our results give valuable insights into the robustness of these two common relaying techniques under different assumptions on power constraints and channel conditions and can be used as a practical guideline in the choice of relaying techniques.*

Index Terms — Distributed space-time block coding, fading channels, amplify-and-forward relaying, zero-forcing relaying.

I. INTRODUCTION

Multiple Input Multiple Output (MIMO) systems have demonstrated that the deployment of multiple antennas at the transmitter and/or receiver results in a great improvement in spectrum efficiency and reliability of a point-to-point wireless link [1]. However, in some scenarios, the use of multiple antennas at the receiver is not feasible because of additional hardware complexity and the market acceptability of MIMO systems. Therefore, researchers have begun looking into new communication paradigms to overcome these limitations. A particularly interesting proposal has been the development of *cooperative diversity* [2]-[5], which extends the benefits of MIMO systems to access points with only a single antenna.

Cooperative diversity systems create virtual antenna arrays by taking advantage of the broadcast nature of wireless transmission, i.e., the cost-free possibility of the transmitted signals being received by other than destination nodes, and thus a source node can get help from other nodes by relaying the information message to the destination nodes. Based on the transmission strategy at relays, cooperative schemes can be classified into two categories:

- Regenerative cooperative schemes where the relay decodes, re-encodes and re-transmits the signal. Decode-and-forward (DF) relaying [3] is typically used in such schemes.
- Non-regenerative cooperative schemes where the relay transmits a scaled version of its received noisy signal. The most common relaying technique in such cooperative schemes is amplify-and-forward relaying (AF) [4].

A relatively less known technique is zero-forcing (ZF) relaying [6], [7], where the relay terminal, similar to AF relaying, needs to scale its received signal before retransmission to satisfy an output power constraint.

In this paper, we focus on non-regenerative cooperative schemes and compare the performance of AF and ZF relaying under two different power constraints. Specifically, we consider a single-relay-assisted distributed space-time block coded (D-STBC) scheme similar to [5] noting that extension to more relays is straightforward. We derive the pairwise error probability (PEP) expressions for D-STBC in both AF and ZF relaying and discuss the achievable diversity order. For each relaying technique, we assume two different power constraints [4], i.e.,

$$\beta_1^2 = 1 \Big/ \mathop{\mathbf{E}}_{n, h_{SR}} [| r_R |^2] , \tag{1}$$

$$\beta_2^2 = 1 \Big/ \mathop{\mathbf{E}}_{n} [| r_R |^2] . \tag{2}$$

In the first constraint, the expectation is with respect to both n (which models the additive noise term) and h_{SR} (which models the fading coefficient in the source-to-relay link). This ensures that an average output power is maintained, but allows for the instantaneous output power to be much larger than the average. In the second constraint, the expectation is carried over only n while each realization of h_{SR} needs to be estimated and utilized in the computation of scaling term. This ensures that the same output power is maintained for each realization. Following [8], we refer the first and second constraints as *average power scaling* (APS) and *instantaneous power scaling* (IPS) constraints, respectively.

Our contributions in this paper are summarized as follows: We derive PEP expressions for D-STBC in four different scenarios: a) AF relaying with APS constraint, b) AF relaying

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with IPS constraint, c) ZF relaying with APS constraint and d) ZF relaying with IPS constraint.

1. For each scheme under consideration, we quantify the achievable diversity order assuming fading/non-fading relay-to-destination links under various assumptions imposed on signal-to-noise ratios (SNRs) in underlying links.
2. We present an extensive Monte Carlo simulation study to corroborate the analytical results and to provide detailed performance comparisons among the competing schemes.

The rest of the paper is organized as follows: In Section II, the relay-assisted transmission model is introduced. In Section III, we present the PEP derivations and discuss the achievable diversity for each scenario. Numerical results are presented in Section IV and the paper is concluded in Section V.

II. TRANSMISSION MODEL

A wireless communication system scenario where the source terminal S transmits information to the destination terminal D with the assistance of a single relay terminal R is considered (See Fig.1). All terminals are equipped with single transmit and receive antennas. We assume a quasi-static frequency-flat Rayleigh fading channel and adopt the user cooperation protocol proposed in [5]: In the so-called Protocol I, the source terminal communicates with the relay and destination terminals during the first signaling interval. In the second time slot, both the relay and source terminals communicate with the destination terminal. Let the signals transmitted by the source terminal during the first and second time slots denoted as x_1 and x_2. In the first time slot, the signal received at the relay terminal is given as

$$r_{R,1} = \sqrt{E_{SR}} h_{SR} x_1 + n_{R,1} \tag{3}$$

The signal received at the destination terminal in the first time slot is given by

$$r_{D,1} = \sqrt{E_{SD}} h_{SD} x_1 + n_{D,1}. \tag{4}$$

In (3) and (4), E_{SD} and E_{SR} represent the average energies available at the destination and relay terminals, respectively, taking into account for possibly different path loss and shadowing effects between source-to-destination $(S \to D)$ and source-to-relay $(S \to R)$ links. h_{SD} and h_{SR} denote the complex fading coefficients over $S \to D$ and $S \to R$ links. Both of them are modeled as complex Gaussian random variables with variance 0.5 per dimension. $n_{R,1}$ and $n_{D,1}$ are complex Gaussian random variables with zero-mean and variance $N_0/2$ per dimension, which model the additive noise terms.

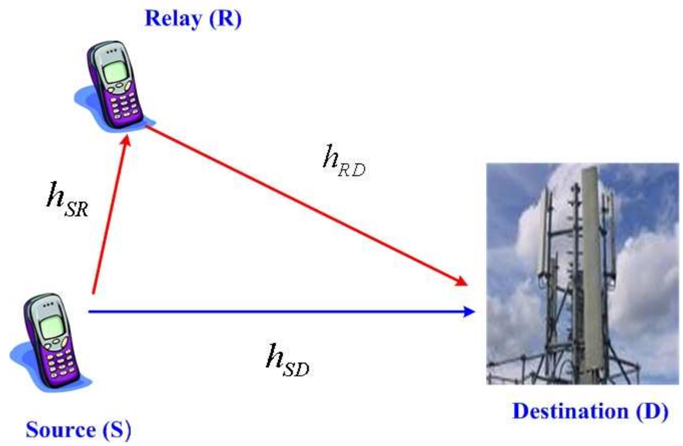

Fig. 1. Schematic representation of relay-assisted transmission

At the relay terminal, we will either assume AF or ZF relaying. In both relaying techniques, the relay terminal needs to scale its received signal before retransmission. In the following, we present our received signal models under APS and IPS constraints.

A. *Amplify-and-Forward (AF) Relaying*

Assuming AF relaying, the relay terminal transmits a scaled version of its received signal, $r_{D,1}$. The received signal at the destination terminal in the second time slot is given by

$$r_{D,2} = \sqrt{E_{SD}} h_{SD} x_2 + \sqrt{E_{RD}} h_{RD} \beta_i r_{R,1} + n_{D,2}, \tag{5}$$

where β_i, $i = 1,2$, is

$$\beta_i^2 = \begin{cases} 1/\left(E_{SR} + N_0\right), & \text{for APS} \\ 1/\left(E_{SR} \left|h_{SR}\right|^2 + N_0\right), & \text{for IPS} \end{cases}$$

depending on the choice of power constraint. In (5), E_{RD} represents the average energy available at the destination terminal considering the path loss and shadowing effects in relay-to-destination ($R \to D$) link and h_{RD} denotes the complex fading coefficient over the same link and is modeled as a complex Gaussian random variable with variance 0.5 per dimension, i.e. $E[|h_{SR}^2|] = 1$. Under the assumption that APS is used, we can rewrite (5) as

$$r_{D,2} = \sqrt{E_{SD}} h_{SD} x_2 + \sqrt{\frac{E_{SR} E_{RD}}{E_{SR} + N_0}} h_{SR} h_{RD} x_1 + \tilde{n}, \tag{6}$$

where the effective noise term $\tilde{n}$ is given as

$$\tilde{n} = \sqrt{\frac{E_{RD}}{E_{SR} + N_0}} h_{RD} n_{R,1} + n_{D,2}. \tag{7}$$

Here, $n_{D,2}$ are the independent samples of a zero-mean complex Gaussian random variable with variance $N_0/2$ per dimension, which models the additive noise term. Therefore, $\tilde{n}$ (conditioned on h_{RD}) is zero-mean complex Gaussian with variance of

$$E\left[|\tilde{n}|^2 \middle| |h_{RD}|\right] = N_0\left(1+\frac{E_{RD}|h_{RD}|^2}{E_{SR}+N_0}\right). \quad (8)$$

Following [5], [9], we can write the received signal after normalizing (6) with (8) as

$$r_{D,2} = \sqrt{\gamma_1}\sqrt{E_{RD}}h_{RD}h_{SR}x_1 + \sqrt{\gamma_2}\sqrt{E_{SD}}h_{SD}x_2 + n, \quad (9)$$

where n turns out to be zero mean complex Gaussian random variable with variance $N_0/2$ per dimension. This does not affect the SNR, but simplifies the ensuing presentation [5]. In (9), γ_1 and γ_2 are defined respectively, as

$$\gamma_1 = \gamma_{1,APS}^{AF} = \frac{E_{SR}/N_0}{1+E_{SR}/N_0+|h_{RD}|^2 E_{RD}/N_0}, \quad (10)$$

$$\gamma_2 = \gamma_{2,APS}^{AF} = \frac{1+E_{SR}/N_0}{1+E_{SR}/N_0+|h_{RD}|^2 E_{RD}/N_0}. \quad (11).$$

Under IPS constraint, the received signal model preserves a similar form as in (9) where γ_1 and γ_2 are now defined respectively, as

$$\gamma_1 = \gamma_{1,IPS}^{AF} = \frac{E_{SR}/N_0}{1+|h_{SR}|^2 E_{SR}/N_0+|h_{RD}|^2 E_{RD}/N_0}, \quad (12)$$

$$\gamma_2 = \gamma_{2,IPS}^{AF} = \frac{1+|h_{SR}|^2 (E_{SR}/N_0)}{1+|h_{SR}|^2 E_{SR}/N_0+|h_{RD}|^2 E_{RD}/N_0}. \quad (13)$$

We now introduce space-time coding across the transmitted signals, i.e. x_1 and x_2. Although different classes of space-time coding proposed originally for co-located antennas can be applied to cooperative diversity schemes in a *distributed* fashion, we assume here STBCs with their attractive orthogonality feature [9], [10]. Assuming that the destination terminal makes an observation for a duration length of 4 symbol periods, the received signals at the destination terminal can be written as

$$r_1 = \sqrt{E_{SD}}h_{SD}x_1 + n_1, \quad (14)$$

$$r_2 = \sqrt{\gamma_1}\sqrt{E_{RD}}h_{RD}h_{SR}x_1 + \sqrt{\gamma_2}\sqrt{E_{SD}}h_{SD}x_2 + n_2, \quad (15)$$

$$r_3 = -\sqrt{E_{SD}}h_{SD}x_2^* + n_3, \quad (16)$$

$$r_4 = -\sqrt{\gamma_1}\sqrt{E_{RD}}h_{RD}h_{SR}x_2^* + \sqrt{\gamma_2}\sqrt{E_{SD}}h_{SD}x_1^* + n_4. \quad (17)$$

In (14)-(17), n_j, $j=1,2,3,4$, are zero mean, complex Gaussian random variables with variance $N_0/2$ per dimension.

B. Zero Forcing (ZF) Relaying

The relay terminal first applies zero-forcing to its received signal as

$$\tilde{r}_{R,1} = h_{SR}^{-1}\left(\sqrt{E_{SR}}h_{SR}x_1 + n_{R,1}\right), \quad (18)$$

then scales the resulting signal, i.e. (18), by $\overline{\beta}_i$, $i=1,2$,

$$\overline{\beta}_i^2 = \begin{cases} 1/(E_{SR}+N_0), & \text{for APS} \\ 1/\left(E_{SR}+N_0/|h_{SR}|^2\right), & \text{for IPS} \end{cases}$$

and retransmits the signal during the second time slot. Therefore, the received signal at the destination terminal in the second time slot is given by (5) where β_i and $r_{R,1}$ are now replaced by $\tilde{\beta}_i$ and $\tilde{r}_{R,1}$. Following similar steps in the previous section, we obtain a normalized version of the received signal as given by (9) where γ_1 and γ_2 are now defined as

$$\gamma_1 = \gamma_{1,APS}^{ZF} = \frac{E_{SR}/N_0}{1+E_{SR}/N_0+\left(|h_{RD}|^2/|h_{SR}|^2\right)E_{RD}/N_0}, \quad (19)$$

$$\gamma_2 = \gamma_{2,APS}^{ZF} = \frac{1+E_{SR}/N_0}{1+E_{SR}/N_0+\left(|h_{RD}|^2/|h_{SR}|^2\right)E_{RD}/N_0}, \quad (20)$$

under APS constraint and

$$\gamma_1 = \gamma_{1,IPS}^{ZF} = \frac{E_{SR}/N_0|h_{SR}|^2}{1+|h_{SR}|^2 E_{SR}/N_0+|h_{RD}|^2 E_{RD}/N_0}, \quad (21)$$

$$\gamma_2 = \gamma_{2,IPS}^{ZF} = \frac{1+|h_{SR}|^2 E_{SR}/N_0}{1+|h_{SR}|^2 E_{SR}/N_0+|h_{RD}|^2 E_{RD}/N_0}, \quad (22)$$

under IPS constraint.

III. Diversity Order Analysis

In this section, we investigate the achievable diversity gains assuming AF and ZF relaying under the APS and IPS power constraints. Defining the transmitted codeword matrix and the erroneously-decoded codeword matrix as $\mathbf{X}$ and $\hat{\mathbf{X}}$, respectively, a Chernoff bound on conditional PEP is given as [11]

$$P(\mathbf{X},\hat{\mathbf{X}}\,|\,\mathbf{h}) \le \exp\left(-\frac{d^2(\mathbf{X},\hat{\mathbf{X}})}{4N_0}\right), \quad (23)$$

assuming maximum likelihood (ML) decoder with perfect knowledge of the channel state information (CSI) at the receiver side. Here, $\mathbf{h} = [h_{SD} \quad h_{SR}h_{RD}]$ and

$$\mathbf{X} = \begin{bmatrix} \sqrt{E_{SD}}x_1 & \sqrt{\gamma_2}\sqrt{E_{SD}}x_2 & -\sqrt{E_{SD}}x_2^* & \sqrt{\gamma_2}\sqrt{E_{SD}}x_1^* \\ 0 & \sqrt{\gamma_1}\sqrt{E_{RD}}x_1 & 0 & -\sqrt{\gamma_1}\sqrt{E_{RD}}x_2^* \end{bmatrix}. \quad (24)$$

In (23), $d^2(\mathbf{X},\hat{\mathbf{X}})$ denotes the Euclidean distance between $\mathbf{X}$ and $\hat{\mathbf{X}}$ and is given by

$$d^2(\mathbf{X},\hat{\mathbf{X}}) = \mathbf{h}\Delta\,\mathbf{h}^{\mathrm{H}} \quad (25)$$

where $\boldsymbol{\Delta} = \left(\mathbf{X}-\hat{\mathbf{X}}\right)\left(\mathbf{X}-\hat{\mathbf{X}}\right)^{\mathrm{H}}$ is

$$\boldsymbol{\Delta} = \begin{bmatrix} E_{SD}(1+\gamma_2)\left(|x_1-\hat{x}_1|^2+|x_2-\hat{x}_2|^2\right) & 0 \\ 0 & E_{RD}\gamma_1\left(|x_1-\hat{x}_1|^2+|x_2-\hat{x}_2|^2\right) \end{bmatrix}. \quad (26)$$

Since $\boldsymbol{\Delta}$ turns out to be a diagonal matrix, the eigenvalues of $\boldsymbol{\Delta}$ can simply be defined as

$$\lambda_1 = E_{SD}(1+\gamma_2)\chi, \tag{27}$$

$$\lambda_2 = E_{RD}\gamma_1\chi, \tag{28}$$

where $\chi = |x_1-\hat{x}_1|^2 + |x_2-\hat{x}_2|^2$.

A. PEP Analysis under APS Constraint

Non-fading $R\to D$ link: In our PEP analysis, we first assume that $S\to R$ and $S\to D$ links experience fading while $R\to D$ link is non-fading, i.e. $h_{RD}=1$. Physically, this assumption corresponds to the case where the destination and relay terminals have a very strong line-of-sight (LOS) connection [5]. In this case, the Euclidean distance $d^2(\mathbf{X},\hat{\mathbf{X}})$ in (25) reduces to

$$d^2(\mathbf{X},\hat{\mathbf{X}}) = |h_{SD}|^2 E_{SD}(1+\gamma_2)\chi + |h_{SR}|^2 E_{RD}\gamma_1\chi, \tag{29}$$

$$d^2(\mathbf{X},\hat{\mathbf{X}}) = |h_{SD}|^2 E_{SD}(1+\gamma_2)\chi + E_{RD}\gamma_1\chi, \tag{30}$$

for AF and ZF, respectively. Substituting (29)-(30) in (23) and averaging the resulting expressions with respect to $|h_{SD}|$ and $|h_{SR}|$ which are Rayleigh distributed, we obtain the final PEP expressions as

$$P_{AF}(\mathbf{X},\hat{\mathbf{X}}) \le \left(1+\frac{E_{SD}(1+\gamma_2)\chi}{4N_0}\right)^{-1}\left(1+\frac{E_{RD}\gamma_1\chi}{4N_0}\right)^{-1}, \tag{31}$$

$$P_{ZF}(\mathbf{X},\hat{\mathbf{X}}) \le \left(1+\frac{E_{SD}(1+\gamma_2)\chi}{4N_0}\right)^{-1}\exp\left(-\frac{E_{RD}\gamma_1\chi}{4N_0}\right), \tag{32}$$

for AF and ZF, respectively.

Assume perfect power control where $S\to D$ and $R\to D$ links are balanced and high SNRs for the underlying links, i.e. $E_{SD}/N_0 = E_{RD}/N_0 >> 1$. Further let SNR in $S\to R$ be sufficiently large, i.e. $E_{SR}/N_0 > E_{SD}/N_0$. Under these assumptions, (31) reduces to

$$P_{AF}(\mathbf{X},\hat{\mathbf{X}}) \le \left(\frac{E_{SD}}{2\sqrt{2}N_0}\right)^{-2}\chi^{-2}. \tag{33}$$

It is clear from (33) that the second order diversity is achieved for AF under APS constraint assuming non-fading $R\to D$ link. Under the similar assumptions, we obtain

$$P_{ZF}(\mathbf{X},\hat{\mathbf{X}}) \le \left(\frac{E_{SD}}{2N_0}\right)^{-1}\chi^{-1}\exp\left(-\frac{E_{SD}\chi}{4N_0}\right), \tag{34}$$

for ZF relaying. In (34), the exponential term becomes dominant and, therefore, the diversity order is much larger than two. Therefore, if a strong LOS is present in $R\to D$ link, ZF outperforms AF (under APS constraint) significantly.

Effect of Fading in the $R\to D$ link: Due to the presence of $|h_{RD}|^2$ terms in (10) and (11), the derivation of PEP becomes analytically intractable unless some assumptions are imposed on the SNR in the underlying links. To have some insight into the achievable diversity order, we consider the asymptotic case of $E_{SD}/N_0 = E_{RD}/N_0 >> 1$ with perfect power control and sufficiently large $E_{SR}/N_0 > E_{SD}/N_0$ values. Under these assumptions, the eigenvalues in (27) and (28) reduce to $\lambda_1 = 2\chi E_{SD}$ and $\lambda_2 = \chi E_{SD}$. Averaging the resulting expression with respect to $|h_{RD}|$, we obtain the PEP as

$$P_{AF}(\mathbf{X},\hat{\mathbf{X}}) \le \left(\frac{E_{SD}}{2\sqrt{2}N_0}\right)^{-2}\chi^{-2}\exp\left(\frac{4N_0}{\chi E_{SD}}\right)\Gamma\left(0,\frac{4N_0}{\chi E_{SD}}\right), \tag{35}$$

which has a similar form to the result reported earlier in [5] for so-called Protocol III. Here, $\Gamma(a,b)=\int_b^\infty q^{a-1}\exp(-q)\,dq$ [12] denotes the incomplete gamma function. It is observed from (35) that the second order diversity is achieved. Comparison of (33) and (35) reveals that the effect of fading in the $R\to D$ link incurs only a coding gain loss. Under similar assumptions, we obtain

$$P_{ZF}(\mathbf{X},\hat{\mathbf{X}}) \le \left(\frac{E_{SD}}{2\sqrt{2}N_0}\right)^{-2}\chi^{-2}, \tag{36}$$

for ZF relaying. Comparison of (35) and (36) points out that ZF will provide the same diversity order as AF and will outperform it in terms of coding gain (i.e. horizontal shift in the performance).

B. PEP Analysis under IPS Constraint

Non-fading $R\to D$ link: Due to the presence of $|h_{SR}|^2$ in (12) and (13), the analysis becomes more involved even for the case of non-fading $R\to D$ link. However, under certain assumptions imposed on the SNR of the underlying links, the derivation of PEP becomes analytically intractable. Assuming high SNR values for all underlying links, i.e. $E_{SD}/N_0 = E_{RD}/N_0 >> 1$ with perfect power control and sufficiently large $E_{SR}/N_0 > E_{SD}/N_0$ values, the eigenvalues in (27) and (28) reduce to $\lambda_1 = 2E_{SD}\chi$ and $\lambda_2 = \chi E_{SD}/|h_{SR}|^2$. Then, using (29), $d^2(\mathbf{X},\hat{\mathbf{X}})$ yields

$$d^2(\mathbf{X},\hat{\mathbf{X}}) = 2|h_{SD}|^2 E_{SD}\chi + E_{SD}\chi, \tag{37}$$

in AF case. Substituting (37) in (23) and averaging the resulting expression with respect to $|h_{SD}|$ which is Rayleigh distributed, we obtain the PEP expression as

$$P_{AF}(\mathbf{X},\hat{\mathbf{X}}) \le \left(1+\frac{E_{SD}\chi}{2N_0}\right)^{-1}\exp\left(-\frac{\chi E_{SD}}{4N_0}\right). \tag{38}$$

Under similar SNR assumptions and using (21)-(22) for ZF, the eigenvalues in (27) and (28) reduce to $\lambda_1 = 2E_{SD}\chi$ and $\lambda_2 = \chi E_{SD}$. Using (30), we obtain the final PEP expression for ZF which turns out to be identical to (38). It is observed from (38) that under IPS constraint and assuming non-fading $R\to D$ link, the diversity order for both AF and ZF is much higher than two due to the presence of the exponential term.

Effect of Fading in the $R \rightarrow D$ ***link****:* For the general case where all underlying links experience fading and under similar assumptions on SNRs elaborated in the previous section, we can write $d^2(\mathbf{X},\hat{\mathbf{X}})$ as

$$d^2(\mathbf{X},\hat{\mathbf{X}}) = 2|h_{SD}|^2 E_{SD}\chi + |h_{RD}|^2 E_{SD}\chi, \tag{39}$$

which turn to be identical for both AF and ZF. Substituting (39) in (23) and averaging the resulting expression with respect to $|h_{SD}|$ and $|h_{RD}|$ which are Rayleigh distributed, we obtain the PEP expression as

$$P_{AF}(\mathbf{X},\hat{\mathbf{X}}) = P_{ZF}(\mathbf{X},\hat{\mathbf{X}}) \leq \left(\frac{E_{SD}}{2\sqrt{2}N_0}\right)^{-2} \chi^{-2}. \tag{40}$$

It is interesting to see here that (40) is identical to (36) indicating that ZF achieves identical performance under both APS and IPS constraints. On the other hand, comparison of (40) and (35) reveals that AF yields a better performance if used in conjunction with IPS constraint instead of APS constraint.

IV. NUMERICAL RESULTS

In this section, we present Monte-Carlo simulation results for D-STBC in AF and ZF relaying assuming a quasi-static Rayleigh fading channel and QPSK modulation. We assume perfect power control, i.e. $S \rightarrow D$ and $R \rightarrow D$ links are balanced.

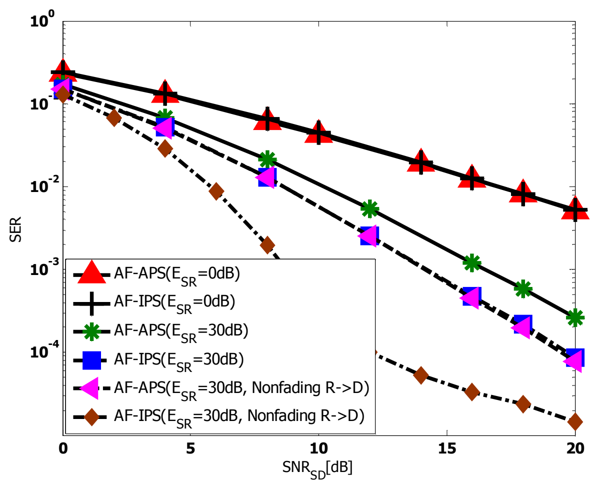

Fig. 2. SER performance of D-STBC with AF-APS and AF-IPS.

Fig. 2 demonstrates the SER (symbol error rate) performance assuming AF relaying considering both APS and IPS constraints, labeled as AF-APS and AF-IPS, respectively. We consider scenarios with fading/non-fading $R \rightarrow D$ links and assume $E_{SR}/N_0 = 30$ dB (unless otherwise noted). For fading $R \rightarrow D$ link, we observe that AF-IPS outperforms AF-APS by ≈ 3 dB (at SER=10^{-3}) although both of them are able to achieve the same diversity order. This confirms our observations from the derived PEP expressions in (35), (40). For non-fading $R \rightarrow D$ link, the performance characteristics of AF-APS and AF-IPS differ from each other significantly. AF-IPS provides a diversity order much larger than two in most of the considered $E_{SD}/N_0 \left(<< E_{SR}/N_0 = 30\text{dB} \right)$ range confirming our observation in (38), while it later converges to its asymptotical diversity order. On the other hand, AF-APS consistently provides the same diversity order of two. It is actually interesting to note that the performance of AF-APS under non-fading $R \rightarrow D$ link is identical to the performance of AF-IPS under fading $R \rightarrow D$ link. In Fig. 2, we also study the performance of AF for $E_{SR}/N_0 = 0$ dB. The performance difference earlier observed between AF-APS and AF-IPS vanishes for lower E_{SR}/N_0 values. Furthermore, AF relaying under both constraints suffer from a diversity order loss and the performance is now dominated by $S \rightarrow D$ direct transmission. It should be further noted that the performance curves for fading and non-fading $R \rightarrow D$ links overlap for this case.

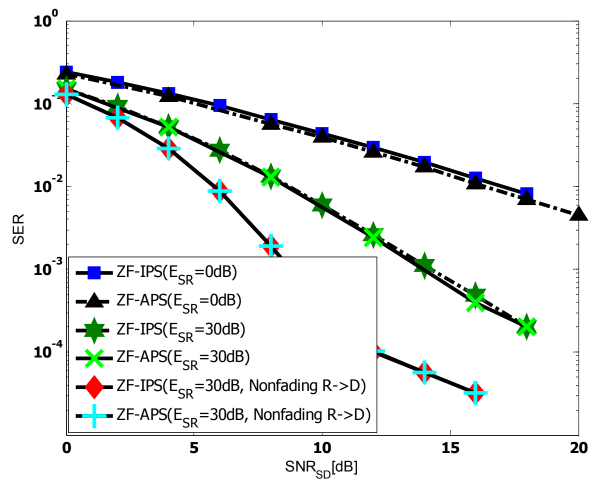

Fig. 3. SER performance of D-STBC with ZF-APS and ZF-IPS.

Fig. 3 demonstrate the SER performance assuming ZF relaying considering both APS and IPS constraints labeled as ZF-APS and ZF-IPS, respectively. For fading $R \rightarrow D$ link, regardless of E_{SD}/N_0 values, performances of ZF-APS and ZF-IPS are similar to each other. For $E_{SR}/N_0 = 30$dB, we observe a diversity order of two confirming our observations in (36) and (40) while a diversity order of one is observed for $E_{SR}/N_0 = 0$dB. Under non-fading $R \rightarrow D$ link, we observe that performances of ZF-APS and ZF-IPS are the same. Both of them are able to provide a diversity order much larger than two (as confirmed by the presence of exponential terms in (32) and (38)).

Comparison of Fig.2 and Fig.3 further points out that

1. For fading $R \rightarrow D$ link and sufficiently large E_{SR}/N_0 values, ZF outperforms AF under APS constraint although their achievable diversity orders are still the same. Under similar assumptions, AF-IPS and ZF-IPS provide identical performance. For lower E_{SR}/N_0 values, ZF and AF have the same performance regardless of the power constraint

choice.

2. For non-fading $R \rightarrow D$ link, ZF outperforms AF significantly under APS constraint while providing a similar performance under IPS constraint.

V. Conclusion

In this paper, we have studied the error rate performance of a distributed STBC through the derivation of PEP expressions under different relaying/power constraint assumptions. Specifically, we have provided an extensive comparison among four scenarios: a) AF relaying with APS constraint, b) AF relaying with IPS constraint, c) ZF relaying with APS constraint and d) ZF relaying with IPS constraint. Our results provide valuable insight into the choice of AF and ZF relaying and power constraint. In particular, ZF outperforms AF under APS constraint while AF-IPS and ZF-IPS provide identical performance. For the special case of non-fading $R \rightarrow D$ link which can be justified with a strong LOS component, ZF becomes the obvious choice as the performance improvement over AF even becomes larger under APS constraint.

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