Performance Evaluation of Stochastic Algorithms for Linear Antenna Arrays Synthesis under Constrained Conditions

Armando Arce, David H. Covarrubias and Marco A. Panduro

***Abstract*—Particle Swarm Optimization (PSO) is a high performance optimization technique recently introduced to solve antenna array synthesis problems to handle multiple degrees of freedom. An important problem facing the user of PSO is its parameters selection, as well as an efficient scheme to improve the optimization process. This paper proposes the use of a global asynchronous PSO update scheme in the synthesis of linear antenna arrays to solve the complex design restrictions imposed by a constrained mask. It is shown that PSO is more efficient than the well-known method of Genetic Algorithms (GA) even under constrained conditions, in terms of simplicity and computational burden. To illustrate the effectiveness of this PSO scheme applied to linear array synthesis under these constrained conditions, modeling and simulation examples including both GA and PSO algorithms are shown and analyzed in different scenarios.**

***Index Terms*—Phased array, GA, PSO, antenna array synthesis.**

I. Introduction

DUE to the fact that analytical methods can not work with multiple degrees of freedom at the same time, analytical methods for array synthesis are not applicable. For this reason stochastic population-based optimizers are employed with the advantage to discover near to optimal solutions for NP-complete problems in polynomial search time; and with the benefit to handle multimodal, nonlinear, nonconvex and multidimensional optimization problems.

For this reason, bioinspired optimizers like Particle Swarm Optimization (PSO) have recently been used, due their multiple attributes, including the fact that the basic algorithm is very easy to understand and implement.

PSO originated in studies of synchronous bird flocking and fish schooling, when the researchers realized that their simulation algorithms possessed an optimizing characteristic [1].

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Consider an optimization problem that requires the simultaneous optimization of N variables. A collection or swarm of particles is defined, where each particle is assigned a random position in the N-dimensional problem space so that each particle’s position corresponds to a candidate solution to the optimization problem [2].

This optimization technique is promising, and researchers are still exploring its capabilities for solving electromagnetic problems.

Emerging like an effective alternative to the older and well-known method of Genetic Algorithms (GA) [3], [4], PSO has been applied in the electromagnetic field [5], [6] including antenna design [7], [8]. PSO is a bioinspired algorithm similar in some ways to evolutionary algorithms, such as GA and is commonly compared with them [9], [10]. Good performance can generally be obtained with both methods.

The evaluation of the cost function tends to dominate the overall computation budget for electromagnetic optimization, but the computational overhead requirements of both optimization algorithms are not always negligible [11].

Because antenna array synthesis often has a significant computational burden, finding ways to reduce the number of iterations and function evaluations required for stochastic algorithms represents an open line of research in the antenna field.

For the specific case of linear antenna arrays optimized by PSO we can found different approaches that are used to design a desired radiation pattern, some recent research are found in [11]-[13].

In this paper, an approach based on PSO for the synthesis of linear antenna arrays is presented. The objective of this paper is to present a comparative analysis between GA and PSO for the problem of linear array synthesis, in order to study the array factor through a constrained mask (lower and upper masks).

In particular, the study of the application of GA and PSO for this useful design problem is evaluated in terms of simplicity and computational burden.

The paper is organized as follows: Section II describes all the design formulation including the array pattern synthesis and fitness function formulation, in which it states the array geometry and excitations of the antenna elements. A short description of the stochastic algorithms used is included in Section III. Following this description the simulation results and comparisons are presented in Section IV. Finally, conclusions and references of this work are presented.

II. Theoretical Study

A. Array Pattern Synthesis Formulation

Consider a uniform linear array (ULA) of omnidirectional antenna elements in which all the elements are considered identical. Therefore the array factor can be obtained by considering the antenna elements as point sources with the first element of the array in the origin, as shown in Fig. 1.

The array factor for the linear array shown in Fig. 1 is given by [14]:

$$AF = \sum_{n=1}^{N} a_n e^{j(n-1)(kd\cos(\theta)+\beta_n)} \tag{1}$$

where N is the number of antenna elements or array radiating elements, a_n is the current distribution module of each antenna element, $k=2\pi/\lambda$, d is the spacing between elements, θ is the angle in relation with the array axis and β_n represents the phase current distribution for each antenna element.

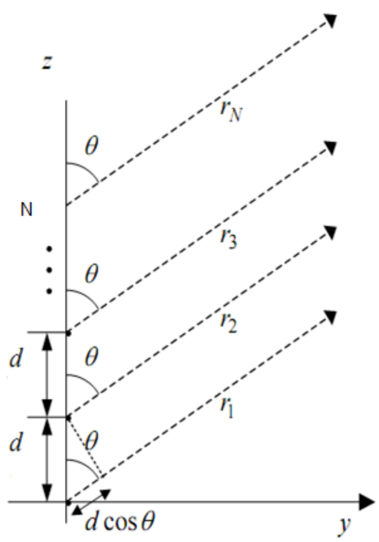

Fig. 1. Array geometry for an N element uniform linear array with inter-element spacing d.

B. Fitness Function

As it is well known, the nexus between the methods of optimization and the linear antenna array synthesis problem is the fitness function. For this reason we must suitably select a fitness function that presents a low computation burden.

For the problem of linear array synthesis we have selected and analyzed the performance of 2 fitness functions in order to analyze and compare the radiation pattern optimization through a mask.

Equation (2) can be found in [15] and (3) is a modification introduced to try to get better results. These fitness functions are the following and are referred to in the next section:

$$\begin{aligned} F_1 = {} & \sum_{p=1}^{P} \max\left(\left|AF_p\right| - UM_p, 0\right)^2 + \ldots \\ & \ldots + \sum_{p=1}^{P} \max\left(LM_p - \left|AF_p\right|, 0\right)^2 . \end{aligned} \tag{2}$$

$$\begin{aligned} F_2 = {} & \sum_{p=1}^{P} \min\left(\left|AF_p(dB)\right| - \left|UM_p(dB)\right|, 0\right)^2 + \ldots \\ & \ldots + \sum_{p=1}^{P} \min\left(\left|LM_p(dB)\right| - \left|AF_p(dB)\right|, 0\right)^2 . \end{aligned} \tag{3}$$

In (2) and (3), LM and UM represent the lower and upper masks to which the array factor should be fitted, P is the set of points used to specify the array factor, and AF_p is the array factor value in each angular position.

III. Stochastic Optimization Algorithms

As mentioned earlier, the objective of this paper is to present a comparative evaluation of GA and PSO for optimizing the array factor of linear antenna arrays through a constrained mask with a specific design.

The design that involves a mask includes certain side lobe level (SLL) and can include a null in some angular direction. We should mention that this type of mask is very useful in cellular mobile communications.

The algorithms and their main characteristics are described in the next subsections.

A. Genetic Algorithms

Genetic algorithms are an extremely popular method of optimization used by the research electromagnetic community to tackle a vast variety of problems [16].

GA are based on Darwin’s theories of evolution and the concept of “survival of the fittest”, genetic algorithms use processes that emulate the genetic recombination and mutation to evolve a population that best satisfies a predefined goal.

In general in order to apply GA to antenna array synthesis, a summary of steps to be followed is shown in Fig. 2.

GA with real codification version with added elitism was utilized for the design problems, tournament selection and uniform crossover is applied to the population, where random mutation with certain percentage is used with offspring from the crossover process [17].

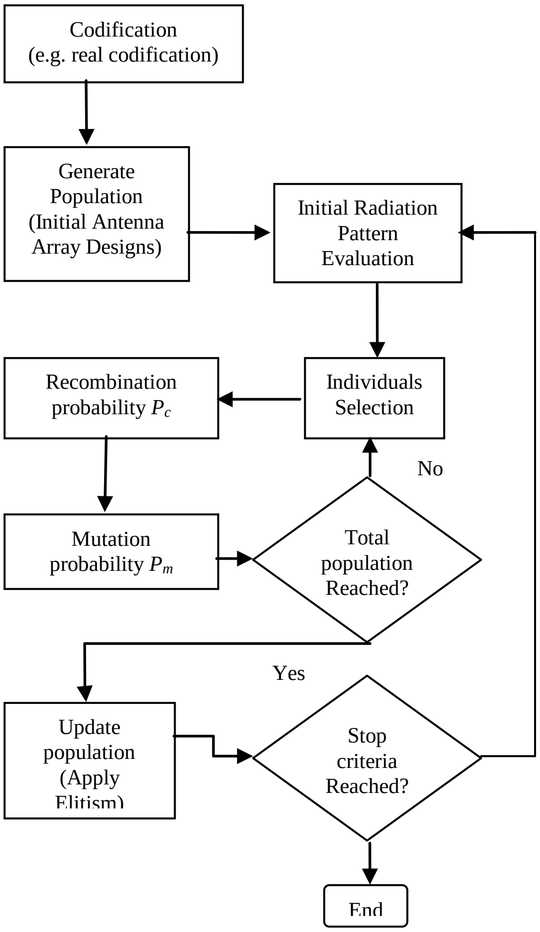

Fig. 2. Genetic Algorithm flowchart applied to antenna array synthesis.

B. *Particle Swarm Optimization*

The PSO algorithm is based on a population of individuals (swarm), where each individual, called agent or particle represents a possible solution within the multidimensional solution space.

The swarm movement within the solution search space is given by the velocity of adaptation and position equations ((4) and (5)) for each particle, considering the inertia weight model [18]:

$$v_{ij}(t+1) = \omega v_{ij}(t) + c_1 r_{1j}(t)[y_{ij}(t) - x_{ij}(t)] + \ldots \\ \ldots + c_2 r_{2j}(t)[\hat{y}_{ij}(t) - x_{ij}(t)]. \quad (4)$$

$$x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1) \quad (5)$$

where v_{ij} represents the particle velocity i in dimension j, ω is the inertia weight that regulates the impact of the previous velocities in the new particle velocity, c_1 is the cognitive parameter that indicates the maximum influence of the personal best experience of the particle and c_2 is the social parameter that indicates the maximum influence of the social information. The terms r_{1j} and r_{2j} are two random numbers uniformly distributed between 0 and 1, i.e., $U[0,1]$. The personal best and global best are represented by y_{ij} and $\hat{y}_{ij}$, respectively. Finally, x_{ij} represents particle position.

The way to establish how the vicinity of a particle is defined as well as the form in which other individuals influence a specific particle have a great impact on the algorithm's performance.

Therefore, the relevance to use a scheme adapted in the algorithm, according to the problem to treat, in this specific case the antenna array synthesis under a constrained condition.

In the definition of a particle's vicinity, two main topologies can be discerned: global and local topologies [19].

In a global topology, all the particles are interrelated and have immediate access to the findings of their fellows. In a local topology each particle finds its trajectory influenced by its adjacent neighbors only, remaining isolated from distant particles of the swarm.

In regards to the form in which a particle is influenced by other individuals, two types of updates schemes can be distinguished in PSO: synchronous and asynchronous [20]. The type of update scheme depends on the step of the iterative process in which each particle's memory is updated, as well as the group knowledge.

In this work, the use of a global asynchronous PSO scheme is proposed to be used, since it has been shown that the basic PSO algorithm is not always effective for solving complex electromagnetic problems, modifications in its parameters as well as in the general scheme have been suggested in literature [8],[18],[20],[21].

In Fig. 3 a flowchart of the proposed PSO applying a global asynchronous scheme to antenna array synthesis is shown.

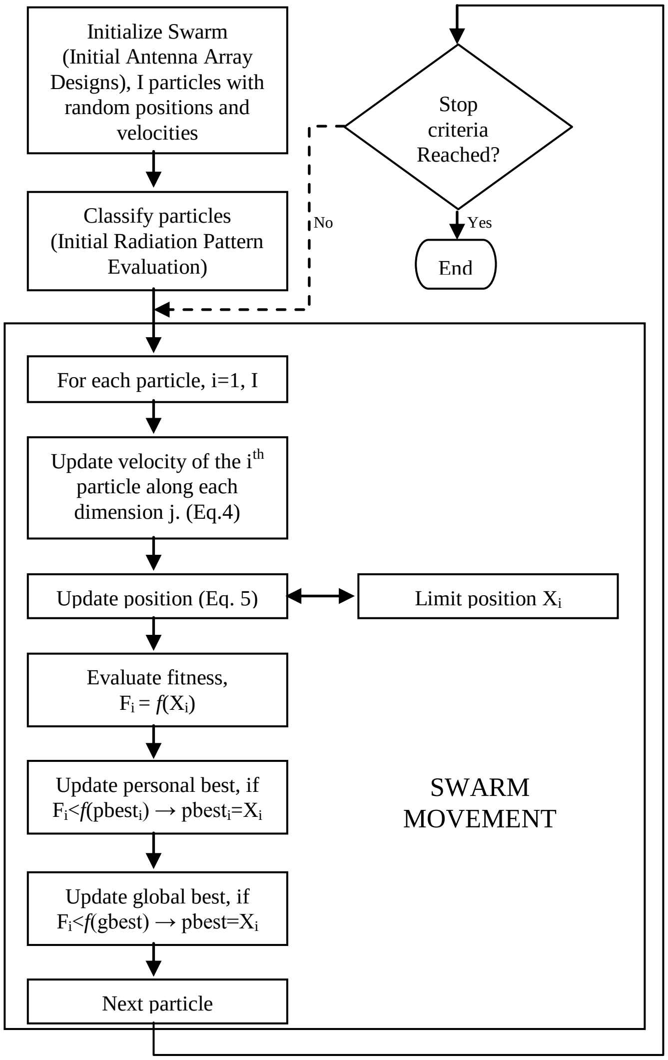

Fig. 3. Particle swarm flowchart applied to antenna array synthesis with global asynchronous update scheme.

IV. Simulation Results and Comparison Analysis

The GA and PSO methods were implemented to study the behavior of the array factor for linear antenna arrays. Published literature and simulation results carried were followed to set the parameters of each algorithm in an attempt to make a fair comparison between them.

In the case of genetic algorithms, we carried out simulations and we set some simulation parameters like in [17], as follows: crossover probability p_c = 1.0, mutation probability p_m = 0.1, maximum number of generations $r_{\max}$ = 500, elitism as added method and a population size to match the double number of parameters to optimize a complex excitation. These settings have shown good results applied to linear antenna arrays.

For PSO, the following configuration was set with inertial weight to ω = 0.729 and $c_1 = c_2$ = 1.49445, which is analogous to Clerc's settings with constriction factor [22] and a population size between 50 and 75 individuals, these settings were set after multiple simulations and a previous literature review [8],[21],[22]. For both optimizers, the fitness functions (2) and (3) were used.

A. GA vs PSO

The objective of the first simulation is to find the best fitness function for the problem of the antenna array synthesis under a constrained mask. For another hand, we want to test the optimizers (GA and PSO) with relaxing boundary conditions in the array factor.

For the above the following scenario was defined: a uniform linear array (ULA) of 15 elements is considered with a spacing between elements $d = \lambda/2$ and a relaxed mask in broadside mode (i.e. 90°) with an upper mask with -17 dB of uniform side lobe level and an interior mask with a width of 10°.

Twelve simulations with each optimization method were performed to study the effect of each fitness function with these boundary conditions applied to the radiation pattern.

The fitness functions have the purpose of limiting the radiation pattern generated by the antenna array within the mask. In this way, the main beam is contained between 2 masks with a limited isolation level.

The best results on average are taken from multiple simulations (in our case 12 simulations with each optimizer). The Fig. 4 and 5 show the influence of the function fitness for both optimizers.

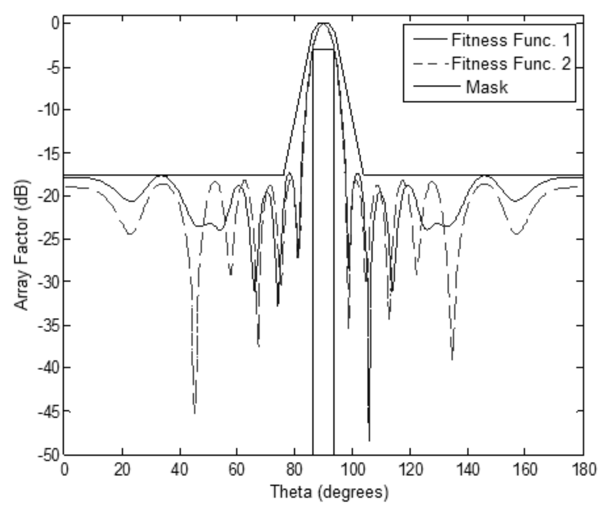

Fig. 4. Effect of the fitness function in the performance of GA.

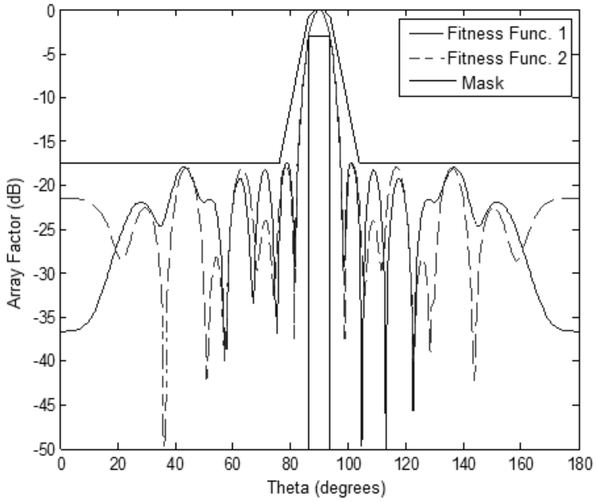

Fig. 5. Effect of the fitness function in the performance of PSO.

Fitness functions (2) and (3) obtained satisfactory results with a limited radiation pattern, these results are showed in Fig. 4 and 5. These statistics show the contained radiation pattern in the limits imposed by the upper and lower mask.

Thus, the selection of either fitness function to weight the quality of the solution is the one that registers a smaller time for convergence in average.

Therefore fitness function (3) was the selected function for the experiments since it showed a smaller time for convergence on average for both population based optimizers (see Table I) and was used in the rest of the simulations that involve a mask.

A second experiment with a more complex mask that involves an upper mask directed to 110°, isolation level at -20 dB, null insertion at an angle of 50° and a lower mask with 5° is proposed.

In order to confront this design problem, a uniform linear array with 30 antenna elements spaced $\lambda/2$ is used.

Given the stochastic nature of both algorithms twelve simulations were performed again for each algorithm and the best results on average are taken.

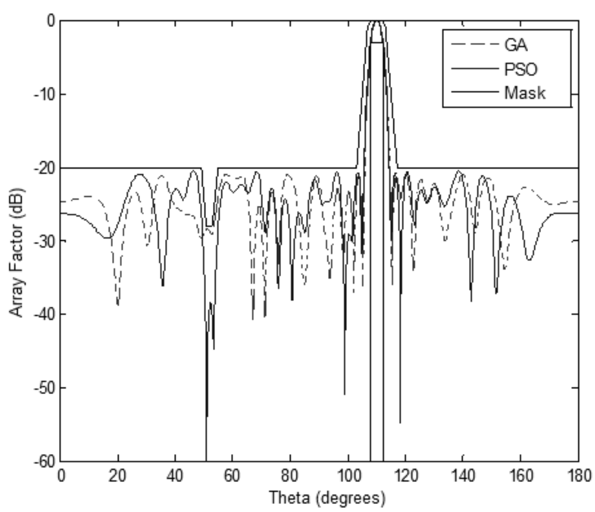

Fig. 6. GA vs PSO, application example with main beam width bounded and aiming at 110°, isolation level to -20 dB and null insertion in 50°.

In Fig. 6 both optimization methods achieved satisfactory results when fulfilling the imposed restrictions of the mask i.e. the radiation pattern is between the upper and lower mask.

Is important to note that for GA a population of 120 individuals was necessary and for PSO using the new scheme only 60 individuals were necessary which means only half as many calls to fitness function are required by PSO.

In order to confirm the performance advantages of PSO with asynchronous scheme compared to GA a typical antenna array design problem is proposed with the objective to observe the behavior of both optimization methods on a common electromagnetic optimization problem.

This third experiment involves a side lobe level and the beam width optimization which is a common tradeoff problem in antenna theory.

The experiment consists in a uniform linear array with 10 antenna elements in broadside mode without a mask. Twelve trials were performed for the new problem and for each optimizer (see Table I).

The three ULA problems discussed in this work are summarized below:

1) Side lobe level and the beam width optimization in broadside mode with 10 antenna elements.
2) Radiation pattern optimization through a mask with 90° aiming and side lobe level reduction in a 15 element antenna array.
3) Radiation pattern optimization through a mask with 110° aiming, isolation level to -20 dB and null insertion to 50°.

The processor used for the experiments was an Intel Core™2 Duo T5600 1.83GHz.

Table I. Convergence time in seconds on average for GA and PSO.

Optimization experiments	Optimization Method		PSO Convergence Improvement
	GA	PSO	%
(1)	10.286479	7.995501	22.2
(2)	176.63894	83.0760	47
(3)	943.26584	466.9157	49.5

Table I shows the computation time on average necessary for GA and PSO convergence for each of the optimization problems defined previously and the PSO convergence improvement.

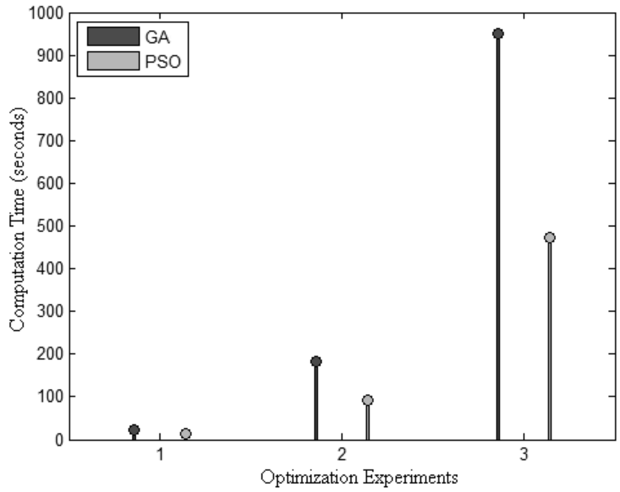

Fig. 7. Computation time on average necessary for the convergence of GA and PSO for the 3 optimization experiments.

In Fig. 7 the horizontal axis enumerates the 3 optimization experiments from minor to greater complexity (i.e. more restrictions to the radiation pattern implies more complexity). The vertical axis represents the computation time in seconds.

As a result the advantage of the PSO algorithm with global asynchronous scheme over GA when dealing with a series of problems with different levels of complexity is shown.

The advantage of PSO over GA is based in the reduction of the individuals used by the algorithm which means fewer calls to the fitness function and for another hand the PSO update scheme used allow to reduce the convergence time even over basic PSO algorithm. The above allow a significant reduction by PSO with global asynchronous scheme.

We can conclude that PSO with global asynchronous scheme shows better performance than GA in its application to linear antenna array synthesis with a constrained mask because of its simplicity in implementation and minor computing time.

V. CONCLUSION

In this paper the importance of using a scheme adapted for the linear array synthesis is emphasized for solving antenna array synthesis problems that involves a mask.

In the case of the global asynchronous PSO scheme used a better performance over GA in all problems treated according to the shown statistics can be obtained.

The comparative analysis demonstrates that both optimization methods satisfactorily solve the problems of linear array synthesis but the simplicity in the implementation and fast convergence time for each of the problems here exposed demonstrates that PSO with global asynchronous scheme has better performance than GA in the specific problem of the linear antenna array synthesis under constrained conditions.

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