

Robust Adaptive Beamformers Using Diagonal Loading

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Abstract—Diagonal loading imparts Robustness to the adaptive beamformer against signal mismatch due to low sample support. It helps to achieve desired sidelobe level and to improve Signal to Interference plus Noise Ratio (SINR). In this paper, variants of Minimum Variance Distortionless Response-Sample Matrix Inversion (MVDR-SMI) beamformer with diagonal loading are analyzed. A novel hybrid algorithm, MVDR-SMI beamformer with Colored Adaptive Diagonal Loading is also proposed in this paper. The simulation experiments show improvement in directivity and SINR compared to other existing MVDR-SMI beamformers with diagonal loading.

Index Terms— Smart Antenna, Adaptive beamforming, Uniform Linear Array, Minimum Variance Distortionless Response Beamformer (MVDR), Sample-Matrix Inversion (SMI), and Adaptive colored diagonal loading.

I. INTRODUCTION

Adaptive beamforming reveals to be a complementary means for signal-to-interference-plus-noise-ratio (SINR) optimization [6,7,10]. In this paper, at antenna array elements level, the formation of a lobe structure, that results from the dynamic variation of an element-space processing weight vector, is controlled by an adaptive algorithm, which is the MVDR-Sample Matrix Inversion algorithm [2, 7, 10]. It minimizes cost function reduction of a link's SINR by ideally directing beams toward the signal-of-interest (SOI) and nulls in the directions of interference. In optimum beamformers optimality can be achieved in theory if perfect knowledge of the second order statistics of the interference is available. It involves calculation of interference plus noise correlation matrix R_{i+n} . For real world scenarios, the adaptive methods are followed to obtain optimality. In adaptive beamformer, the correlation matrix is estimated from the collected data. In sample matrix Inversion technique a block of data is used to estimate the adaptive beamforming weight vector. The estimate \hat{R}_{i+n} is not really a substitute for true correlation matrix R_{i+n} . Hence there is loss in performance. The SINR which is a measure of performance of the beamformer degrades as the sample support (the number of

data) is low. The lower band on sidelobe levels of the beamformer when no interference sources were found at an angle is also calculated. Training issues like the presence of desired signal in the correlation matrix R_{i+n} is also dealt with.

The paper is organized as follows. In Section II, Problem formulation and general model is presented. In Section III Adaptive beamforming with various beamforming methods are presented along with the Novel Hybrid algorithm - Adaptive colored diagonal loading. In Section IV simulation experiments are presented. Section V contains Results and discussions. Section VI presents the conclusions.

II. PROBLEM FORMULATION AND GENERAL MODEL

An uniform linear array (ULA) of M elements or sensors is considered. Let a desired signal \tilde{S}_0 from a point source from a known direction θ_0 with steering vector 'a₀' and L number of J (jammer or) interference signals from unknown directions $[\theta_1, \theta_2, \theta_3 \dots \dots \theta_L]$, specified by the steering vectors $[\alpha_1, \alpha_2, \alpha_3, \dots \dots \alpha_L]$ respectively impinge on the array. The white or sensor or thermal noise is considered as 'n'.

A single carrier modulated signal $\tilde{S}_0(t)$ is given by

$$\tilde{S}_0(t) = S_0(t) \cos(2\pi F_c t) \quad (1)$$

It is arriving from an angle θ_0 and is received by the ith sensor. The signal $S_0(t)$ is a baseband signal having a deterministic amplitude and random uniformly distributed phase and F_c is the carrier frequency. The symbol $\tilde{}$ is used to indicate that the signal is a pass band signal. Let $X_1(k)$ be the single observation or measurement of this signal made at time instant k, at sensor 1, which is given as

$$X_1(k) = a_0 S_0(k) + [a_1 \ a_2 \ \dots \ a_L] [U_1(k) \ U_2(k) \ \dots \ U_L(k)]^T + n(k) \quad (2)$$

$$= a_0 s_0(k) + \sum_{j=1}^L a_j j_j^*(k) + n(k) \quad (3)$$

Hence the single observation or measurement made at the array of elements at the time instant k, called array snapshot is given as a vector

$$X(k) = [X_1(k) X_2(k) X_3(k) \dots X_M(k)]^T \quad (4)$$

The general model of the steering vector [13] is given as

$$a_\theta = \frac{1}{\sqrt{M}} \begin{bmatrix} e^{-j(\frac{2\pi d}{\lambda}) \cos(\theta)} \\ e^{-j2(\frac{2\pi d}{\lambda}) \cos(\theta)} \\ \dots \\ e^{-j(M-1)(\frac{2\pi d}{\lambda}) \cos(\theta)} \end{bmatrix} \quad (5)$$

Also it is assumed that the desired signal, interference signals and noise are mutually uncorrelated.

III. ADAPTIVE BEAMFORMING

In optimum beamformer, a priori knowledge of true statistics of the array data is used to determine the correlation matrix which in turn is used to derive the beamformer weight vector. Adaptive Beamforming is a technique in which an array of antennas is exploited to achieve maximum reception in a specified direction by estimating the signal arriving from a desired direction while signals of the same frequency from other directions are rejected [17]- [21]. This is achieved by varying the weights of each of the sensors used in the array. Though the signals emanating from different transmitters occupy the same frequency channel, they still arrive from different directions. This spatial separation is exploited to separate the desired signal from the interfering signals. In adaptive beamforming the optimum weights are iteratively computed using complex algorithms based upon different criteria. For an adaptive beamformer, covariance or correlation matrix must be estimated from unknown statistics of the array snapshots to get the optimum array weights. The optimality criterion is to maximize the signal-to-interference-plus-noise ratio to increase the visibility of the desired signal at the array output. In this paper it is assumed that the angle of arrival of the desired signal is known.

A. Estimation of Correlation Matrix

The correlation matrix can be estimated [6,7,8,9] using different methods which would result in different performance and behavior of the algorithm. In block adaptive Sample Matrix Inversion technique, a block of snapshots are used to estimate the ensemble average of R^x and is written as [8]

$$R_x = E\{x(k)x^H(k)\} = \frac{1}{M} \sum_{k=1}^K x(k)x^H(k) \quad (6)$$

$$= M\sigma_s^2 a_0 a_0^H + R_j + R_n \quad (7)$$

where N is the number of snapshots used and k is the time index, σ_s^2 is the power of the desired signal and R_j and R_n are the jammer and noise correlation matrices, respectively and H is the complex conjugate transpose. The interference-plus-noise correlation matrix is the sum of these two matrices

$$R_{j+n} = R_j + R_n \quad (8)$$

Where $R_n = \sigma_n^2 I$, and σ_n^2 is the thermal noise power, I is the identity matrix. It is assumed that thermal noise is spatially uncorrelated.

B. Conventional Beamformer

The expectation values at the antenna elements is written as $E\{x(t)\} = [x_1(t) \dots x_M(t)]^T$ with $R = E\{x(t)x(t)^H\}$. The output signal $y(t) = w^H x(t)$. This is the conventional beamformer output signal with beamformer weight w. Maximizing the beamformer output problem will result in

$P = \text{Max}_w E\{|y|^2\} = \text{Max}_w (w^H R w)$. Solving this equation gives

$$w = \frac{a_\theta}{\sqrt{a_\theta^H a_\theta}} \quad \text{where } a_\theta \text{ is the steering vector.}$$

C. MVDR BEAMFORMING

If 'M' number of sensors are used in a beamformer with spacing between them as $d=\lambda/2$, at any instant

$$y(n) = s_0(n) \cdot \sum_{k=0}^{M-1} W_k^* e^{-jk\theta_0} \quad (9)$$

where θ_0 is phase difference from the reference input and 'θ' may be written as $\theta=2\pi d/\lambda \sin \phi = \pi \sin \phi$ where ϕ is the angle of incidence. To protect all signals which are received from the wanted direction, a linear constraint may be defined as

$$\sum_{k=0}^{M-1} W_k^* e^{-jk\theta_0} = w^H(n) a(\theta_0) = g \quad (10)$$

where $a(\theta)$ is the steering vector. The constraint 'g' may be interpreted as gain at the look direction which is to be maintained as constant. A spatial filter that performs this function is called a linearly constraint minimum variance

beamformer (LCMV). If the constraint $g=1$ then the signal will be received at look direction with unity gain and the response at the look direction is distortionless. This special case of LCMV beamformer is known as minimum variance distortionless response (MVDR) beamformer. Mathematically, a weight vector 'w' is to be calculated for this constrained optimization problem

$$\min_w \mathbf{w}^H \mathbf{R} \mathbf{w} \quad \text{subject to} \quad \mathbf{w}^H \mathbf{a}_0 = 1 \quad (11)$$

Now the optimal weight vector may be written as

$$\mathbf{w} = \frac{\mathbf{R}_x^{-1} \mathbf{a}(\theta)}{\mathbf{a}^H(\theta) \mathbf{R}_x^{-1} \mathbf{a}(\theta)} \quad (12)$$

This beamforming method experiences the following drawbacks

- 1) Computational complexity in the order of $O(N^2)$ to $O(N^3)$.
- 2) In the case of large array, low sample support i.e. $(M \gg k)$, \mathbf{R}_x may result in singular matrix or ill-conditioned.

D. Sample Matrix Inversion (SMI)

Sample matrix Inversion techniques solve the equation $\mathbf{R}_x \mathbf{w}_0 = \mathbf{r}_{dx}$ directly by substituting the maximum likelihood estimates for the statistical quantities \mathbf{R}_x and \mathbf{r}_{dx} to obtain.

$$\mathbf{w}_0 = \mathbf{R}_x^{-1} \mathbf{r}_{dx} \quad (13)$$

The maximum likelihood estimates of the signal

correlation and cross correlation are

$$\hat{\mathbf{R}}_x = \sum_{k=0}^{M-1} x_k x_k^H$$

$$\text{and } \hat{\mathbf{r}}_{dx} = \sum_{k=0}^{M-1} \hat{d} x_k$$

When the input signal is stationary the estimates only need to be computed once. However in cases where the signal statistics are time varying the estimates must be continuously updated. In SMI the convergence performance is quantified in terms of number of statistically independent sample outer products that must be computed for the weight vector to be within 3dB of the optimum.

E. MVDR-SMI Beamformer

MVDR is an optimal minimum variance distortionless response beamformer. It is also referred as the full rank

solution as it uses all 'M' adaptive degrees of freedom. It resembles the Wiener filter of the form

$$\mathbf{w} = \mathbf{R}^{-1} \mathbf{r} \quad (14)$$

MVDR weight vector can be derived as

$$\mathbf{w}_{mvdr} = \frac{[\sigma_{i,n}^2]^{-1} \mathbf{a}_\theta}{\mathbf{a}_\theta^H [\sigma_{i,n}^2]^{-1} \mathbf{a}_\theta} \quad (15)$$

$$= \mathbf{s}$$

$$= \mathbf{w} \odot \mathbf{a}_\theta \quad (16)$$

where \mathbf{s} is unit norm i.e. $\mathbf{a}_\theta^H \mathbf{a}_\theta = 1$ and \odot is the Hadamard

product.

A standard method of estimating the covariance matrix is by constructing the sample covariance matrix

$$\hat{\mathbf{R}}_{i,n} = \frac{1}{k} \sum_{k=1}^k x_{i,n}(k) x_{i,n}^H(k) \quad (17)$$

$x_{i,n}^H(k)$ is the k^{th} training sample and k is the total number of training samples that are available. The sample covariance matrix $\hat{\mathbf{R}}_{i,n}$ is the maximum likelihood estimate of the true covariance matrix $\mathbf{R}_{i,n}$. Now the approach is called sample matrix inversion with MVDR beam forming and the weights are calculated as

$$\mathbf{w}_{(MVDR-SMI)} = \frac{(\hat{\mathbf{R}}_{i,n}^{-1}) \mathbf{a}_\theta}{(\mathbf{a}_\theta^H \hat{\mathbf{R}}_{i,n}^{-1} \mathbf{a}_\theta)} \quad (18)$$

The MVDR method may suffer from significant performance degradation when there are even small array steering vector errors. Several approaches for increasing robustness to array steering vector errors have been proposed during the past few decades. Diagonal loading, linearly constrained minimum variance (LCMV) beam forming, quadratically constrained beamforming and second order cone programming (SOCP) are some of them. In this proposed work, adaptive colored diagonal loading is used to improve the SINR and to eliminate the steering vector errors.

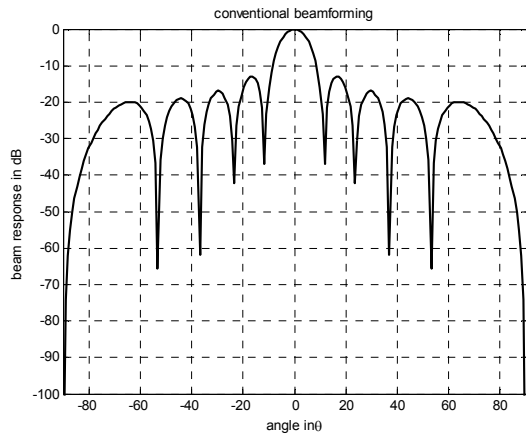


Fig. 1 Conventional beamforming showing the beampattern

F. Diagonal Loading (DL)

To overcome the above mentioned drawback no. 2 in section III. C, a small diagonal matrix is added to the covariance matrix. This process is called diagonal loading [15] or white noise stabilization which is useful to provide robustness to adaptive array beamformers against a variety of conditions such as direction-of-arrival mismatch; element position, gain, and/or phase mismatch; and statistical mismatch due to finite sample support [12], [14]. Because of the robustness that diagonal loading provides it is always desirable to find ways to add diagonal loading to beamforming algorithms. But little analytical information is available in the technical literature regarding diagonal loading [11].

To achieve a desired sidelobe level in MVDR-SMI beamformer sufficient sample support ‘k’ must be available. However due to non-stationarity of the interference only low sample support is available to train the adaptive beamformer. We know that the beam response of an optimal beamformer can be written in terms of its eigen values and eigen vectors. The eigen values are random variables that vary according to the sample support ‘k’. Hence the beam response suffers as the eigen values vary. This results in higher sidelobe level in adaptive beam pattern. A means of reducing the variation of the eigen values is to add a weighted identity matrix to the sample correlation matrix.

The result of diagonal loading of the correlation matrix is to add the loading level to all the eigen values. This in turn produces the bias in these eigen values in order to reduce their variation which in term produces side bias in the adaptive weights that reduces the output SINR. Recommended loading levels of $\sigma_n^2 \leq \sigma_L^2 \leq 10\sigma_n^2$ where σ_n^2 is the noise power and σ_L^2 is the diagonal loading level. The minimum loading level must be equal to noise power. Diagonal loading increases the variance of the

artificial white noise by the amount σ_L^2 . This modification forces the beamformer to put more effort in suppressing white noise rather than interference. When the SOI steering vector is mismatched, the SOI is attenuated as one type of interference as the beamformer puts less effort in suppressing the interferences and noise [17].

However when σ_L^2 is too large, the beamformer fails to suppress strong interference because it puts more effort to suppress the white noise. Hence, there is a tradeoff between reducing signal cancellation and effectively suppressing interference. For that reason, it is not clear how to choose a good diagonal loading factor σ_L^2 in the traditional MVDR beamformer.

This conventional diagonal loading can be thought of as a gradual morphing between two different behavior, a fully adaptive MVDR solution ($L = 0$, no loading) and a conventional uniformly weighted beampattern ($L = \infty$, infinite loading) [5]. The conventional DL weight vector can be calculated as

$$W_{MVDR-DL} = \alpha_{MVDR-DL} [\bar{R} + \sigma_L^2 I]^{-1} a(\theta) \quad (19)$$

where $\alpha_{MVDR-DL}$ is the normalization constant given by

$$\alpha_{MVDR-DL} = a(\theta)^H [\bar{R} + \sigma_L^2 I]^{-1} a(\theta) \quad (20)$$

and σ_L^2 reduces the sensitivity of the beampattern to unknown uncertainties and interference sources at the expenses of slight beam broadening [3]. The choice of loading can be determined from L-Curve approach [12] or adaptive diagonal loading.

G. Colored Diagonal Loading (CDL)

In the presence of colored noise, DL can be applied which is termed as colored diagonal loading (CDL) and the morphing process may result in a beampattern of our choosing. The colored diagonal loading is similar to $W_{MVDR-DL}$ but the diagonal loading level of $\sigma_L^2 = \infty$, end point, can be altered by the term [5]

$$W_{MVDR-CDL} = \alpha_{MVDR-CDL} [\bar{R} + \sigma_L^2 R_{dq}]^{-1} a(\theta) \quad (21)$$

where R_{dq} is the covariance matrix that captures the desired quiescent structure. It may be determined directly 1) based on a priori information – where R_{dq} , need not be a diagonal or 2) desired weight vector – where R_{dq} must be diagonal. It is given as

$$R_{dq} = \text{diag} \left([\text{diag}(W_{dq})]^{-1} a(\theta) \right) \quad (22)$$

where W_{dq} is the desired quiescent weight vector. The colored diagonal loading shows no improvement in pattern shape

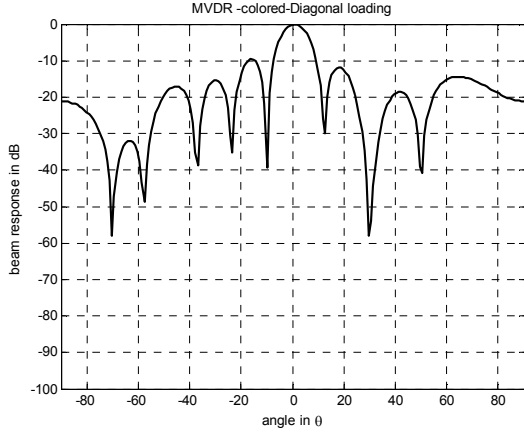


Fig. 2 MVDR-Colored Diagonal Loading

H. Adaptive Diagonal Loading (ADL)

In this method the loading level is calculated assuming the a priori information about the Signal to Noise Ratio (SNR) is available. The SNR can be estimated from link budget or using some SNR estimation algorithm. A variable loading MVDR.(VL-MVDR) is proposed in [16] in which the loading level is chosen as $(\sigma^2 2 R^*)$

$$W_{MVDR-ADL} = \alpha_{MVDR-DL} [\bar{R} + \sigma_{ADL}^2 I]^{-1} a(\theta) \quad (23)$$

where $\sigma_{ADL} = M \cdot SNR$ [4]

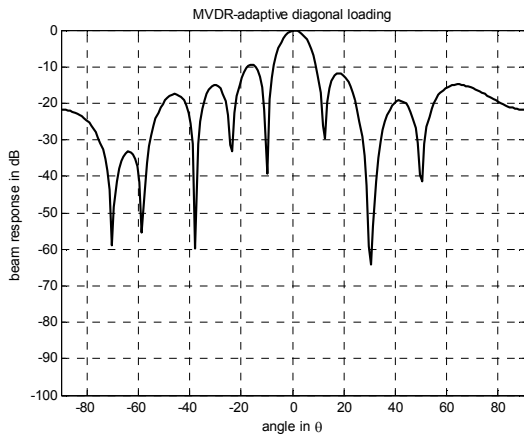


Fig. 3 MVDR- Adaptive Diagonal Loading beampattern

I. Adaptive Colored White Noise Stabilization (ACDL)

As already discussed, white noise stabilization is nothing but diagonal loading in which the adaptive colored loading technique is embedded to get a novel hybrid method which is proposed as

$$W_{MVDR-ACDL} = \alpha_{MVDR-DL} [\bar{R} + K_{dq}]^{-1} a(\theta) \quad (24)$$

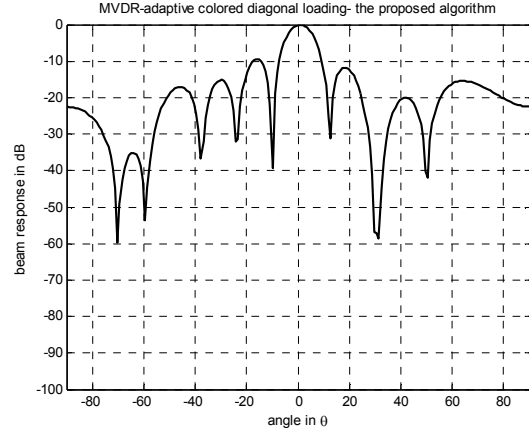


Fig. 4 MVDR- Adaptive Colored Diagonal Loading beampattern

IV. SIMULATIONS AND EXPERIMENTS

For the proposed hybrid algorithm, a 10 element Uniform Linear Array is considered with SNR of 20 dB for the desired signal coming from $\theta_s = 0^\circ$ and INR of 70 dB for three jammer signals coming from the directions $\theta_i = -70^\circ, -10, 30^\circ$ and 50° . The element spacing is $d = 0.5 \lambda$. The beampatterns for various methods of beamforming are obtained to compare with the performance of MVDR-Adaptive colored Diagonal Loading. Fig 1 shows conventional beamformer. It is observed that the conventional beamformer performs well to get the maximum gain in the desired look direction of 0° . But its performance is worst regarding the cancellation of interferences.

Fig 2 shows the MVDR Colored Diagonal Loading beampattern which performs much better than the conventional beamformer. This shows a greater improvement in SINR than the conventional. The null is placed properly with out any angle deviation. Fig 3 shows MVDR-ADL beampattern. Fig 4 shows MVDR-ACDL beampattern. This beampattern gives improvement in SINR when compared to other diagonal loading methods. The interferers' angle and their corresponding beam responses are given below.

- Interferer 1 at angle -70° : -60dB
- Interferer 2 at angle -10° : -40dB
- Interferer 3 at angle 30° : -60dB
- Interferer 4 at angle 50° : -42dB

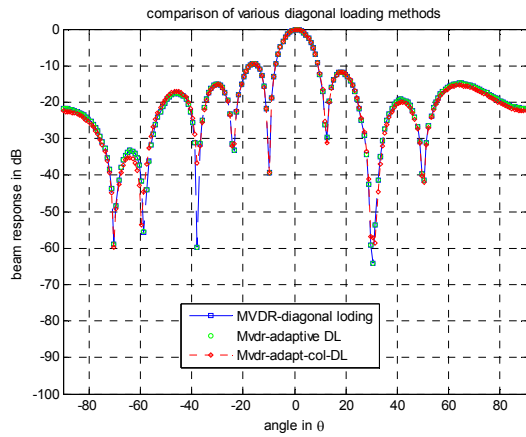


Fig. 5 Beampattern of various diagonal loading methods

V. RESULTS AND DISCUSSION

A. Number of Elements

For the ULA which is considered for simulation work, the beampatterns were analyzed by changing the number of elements as 4, 8, 12, 16, 24, 50 and 100. As the number of elements increases, the beampattern shows higher resolution i.e the 3 dB beamwidth becomes much narrower from 26° to 1° for conventional beamformer and 17° to 1° for adaptive diagonal loading beamformer. The detailed results are shown in Fig [1]-Fig [5]. Finer or sharper beams are obtained when more number of elements are used. Sharper the beam, the beamformer is not susceptible to jammers. But the numbers of side lobes are also increased. The 3-dB beamwidth of the different beamformers are tabulated in Table 1. A trade off can be obtained to reduce the cost and to have a compact size Hence a maximum of 16 elements are chosen for further analysis.

B. Noise Effect

An ULA with 16 elements is considered for analyzing the effect of noise on the peaks of the signal power. Signal to noise ratio (SNR) was varied in steps of 10 dB starting from 10 dB till 60 dB. As SNR increases the peak becomes sharper. It also showed that the interference sources are suppressed to a maximum extent, so that it will not be a disturbance while extracting the signal even in the presence of strong interferers.

C. Training issues with the Number of Array Snapshots

Increasing the number of array snapshots lead to complexity and computational cost but the performance of the beamformer increases. It is a trade off between the cost and the performance. This is shown in Fig 6.

D. Element Spacing

The spacing between the elements for an 16 element ULA was varied as $\lambda/4$, $\lambda/2$, $3\lambda/4$ and λ which in turn vary the effective aperture length of the array. Among the four choices $\lambda/2$ showed the best performance for the particular frequency used for simulation. When the distance between the elements is increased beyond $\lambda/2$, it results in spatial aliasing i.e a lot of spurious peaks are obtained that correspond to different frequencies. Below $\lambda/2$ the resolution of the beams are not satisfactory.

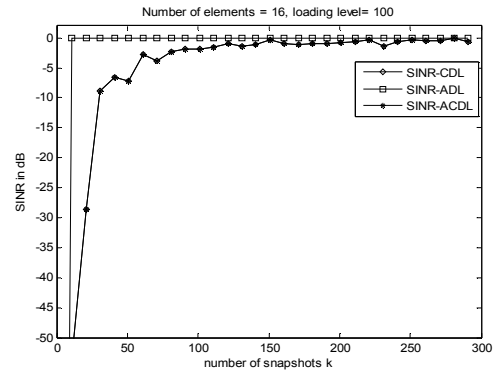


Fig. 6 Training issues with the number of snapshots

VI. CONCLUSION

In this paper diagonal loading method for MVDR-SMI beamformer and its variants are discussed. Also new Hybrid Robust adaptive beamforming algorithm is proposed with Adaptive Colored diagonal loading based on data dependent approach. This method is computationally efficient and proved its effectiveness by lowering the null below -40 in all the cases considered. This shows greater SINR improvement over the other methods discussed in this paper. Simulation results show that the proposed method provide robustness against steering vector errors.

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TABLE I
EFFECT OF CHANGING NUMBER OF ANTENNA ELEMENTS

No. of elements	3dB beamwidth						
	conventional	MVDR	MVDR-SMI	Diagonal loading	Colored DL	Adaptive DL	Adaptive CDL
4	26.2	19.5	17.1	17	17	17	16
8	12.8	15.47	13.3	14.8	14.8	14.8	25.5
12	8.4	8.7	6.9	8.4	8.5	8.7	8.5
16	6.25	6.4	6.4	6.4	6.4	6.6	13.2
20	5.1	5.2	6	5.3	5.3	5.2	6.8
24	4.4	4.5	4.5	4.3	4.3	4.3	4.3
50	2	2	2	2	2	2	2
100	1	1	1	1	1	1	1