The Dedicated Optical Path Protection in WDM Mesh Networks and the finite differences of Available Capacity and the Connection Groups

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Abstract- **In this article the problem of the traffic and the available network capacity of the WDM mesh networks are defined, analyzed and calculated by applying the difference equations. The requests for connection in a time interval are generated by difference equations. By these equations (linear, square or exponential) the available capacity is calculated. To be this problem more complete it is studied as dedicated optical path protection one. The requests for connection of a node pair in a time interval forms the connection group size of this node pair. The difference equation methods are arithmetical and solve successfully the problems.**

Key words: finite differences, linear, square, exponential , dedicated protection

I.INTRODUCTION

The WDM (Wavelength Division Multiplex) optical mesh networks are high capacity telecommunication networks based on optical technologies and carry enormous amount of data in each fiber link in the network. Research has been done [1]-[17] in relation to the methods and the problems associated with planning, protection and restoration of optical networks. The most used methods are special approaches based on ILP. In [1], there are finite difference issues. In [2], the book is an attempt to make the modeling and analysis of system performance more methodical and more realistic. The network survivability has been extensively studied. There are several approaches to ensure fiber network survivability [3] and [4]. In [5], the book attempts to bring together many of ideas of photonics in switching and set them into some general context. In [6], the authors write that optical communications has become the global highway system of the future and an engine of future economic growth. In [7] the authors write about the evolution of the OTN from operators view. In [8] a practical approach to operating survivable WDM networks is provided when the network operation is under dynamic traffic. In the [9] and [10] address issues in designing a survivable optical layer. In [11] a mesh based hybrid OMS / OCh protection /restoration scheme is suggested. In [12], a preplanned local repair restoration for Optical Transport Network is suggested.

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In [13] the authors propose a strategy for Protection and Restoration of Optical Paths in WDM Backbone Networks for Next Generation Internet Infrastructures. In [14], the modeling methods and simulation tools are described and used for the analysis of a new integrated restoration scheme operating at multilayer networks. In [15] deals with the modeling and simulation effectively help and validate the design of various components constituting the service delivery platform. In [16] deals with the modeling and simulation and gives practical advices for network designers and developers. In [17], the approach of path protection is examined, its wavelength capacity requirements, the routing and wavelength assignment of the primary and backup paths, as well as the protection switching time and the susceptibility of these schemes to failures.

The network topology and other parameters are known as WDM and optical fiber capacity, one optical fiber per link with an extension to a 1+1 fiber protection system. So this network is characterized by one working fiber per link, edges of two links, links of two optical fibers, one for working and one for protection. In this paper we are usually referred at the working optical fiber. The connections are lightpaths originating in the source nodes and terminating at the destination nodes proceeding from preplanned optical working paths. Additionally, the same number of optical paths are preselected for the preplanned fully disjoint backup paths,(1+1 dedicated protection connection). Thus the connections that have been set up are protected. The connections of the same node pair by same preplanned optical paths form a connection group along the network. Preplanned protection paths do the dedicated protection of the connection groups. So a suitable number of wavelengths per link along the network are used. The problem solution is to calculate the final available capacity of the network when the connection group size is generated by difference equations. The number of the node pair, the node pairs with their preplanned working and preplanned protection paths and the equations which generate the connection group size are contained in a given table.

The role of the Difference Calculus is in the study of the Numerical Methods. Computer solves these Numerical Methods. The subject of the Difference Equations is in the treatment of discontinuous processes. The network final available capacity and the connection group size are revealed as difference equation. The reduction of the available capacity of each working optical fiber is a discontinuous process when connection groups of several sizes pass through it. The reduction of the connection group size of each node pair is a discontinuous process depends of the time moment (m). This method is an accurate and arithmetical method to study the optical networks and their problems.

This paper is broken down in the following sections: Section II shows how the finite differences are used for this problem and illustrates the optical fiber final available capacity; Section III describes the problem and provides a solution, the algorithm synoptic description, an example, the proposals and a discussion; Section IV draws conclusions and finally ends with the references.

II. THE OPTICAL FIBER AND THE FINITE DIFFERENCES

A. The finite differences of the available capacity

Before studying finite differences and their use in optical WDM mesh networks survivability, it is necessary to provide a short comprehensive presentation of the finite differences computation. Let's assume that $y_1, y_2, \ldots, y_n$ is a sequence of numbers in which the order is determined by the index *n*. The number *n* is an integer and the y_n can be regarded as a function of *n,* an independent variable with function domain the natural numbers and it is discontinuous. Such a sequence shows the available capacity reduction of a telecommunication fibre network link between two nodes when the telecommunication traffic of 1,2, …, n source-destination node pairs pass through. It is assumed that the telecommunication traffic unit is the optical channel that is one wavelength (1λ). The telecommunications traffic includes optical connections with their protections. The total connections of a node pair form its connection group. The first order finite differences represent symbolically the connection group of each node pair that passes through a fiber. This connection group occupies the corresponding number of optical channels and it is the bandwidth that is consumed by connections of a node pair through this fiber. The first order finite differences are used to represent the connection groups in optical channels of the node pairs that pass through an optical fiber .An equation of the first order finite differences gives the available capacity of an optical fiber network link when a connection group passes through it. This available capacity is provided for the connection groups of the other node pairs that their connections will pass through this optical fiber. When the first connection group of Δy_1 connections passes through an optical fiber network link with installed capacity of y_1 optical channels the first order finite difference equation gives the available capacity y_2 $(y_1 +_1)$. The sequence $\Delta y_1, \Delta y_2, \Delta y_3, \ldots, \Delta y_n$ represents the connection groups that pass through this optical fiber network link .When Δy_1 subtracted from y_1, creates y_2, when Δy_2 subtracted from y_2, creates $y_3, \ldots$, when Δy_n subtracted from y_n creates y_{n+1} which is the total unused available capacity of this optical fiber. Thus the total unused available capacity of each network optical fiber is calculated after *n* connections groups pass through it. So these methods could be used to check each other. Table 1 gives a short comprehensive presentation of the computation of the finite differences for a given link that can be arranged quite simply.

At the first column the node pairs are presented. At the second column the indexing or the numbering of the node pairs is presented. At the third column, the available capacity that offered to connection groups is presented. At the forth column, the number of the connection groups that pass through. At the fifth column, the differences between successive connection groups are represented. At the sixth column, the differences between the successive elements of the column five and called high order differences are presented. The unused available capacity of an optical fiber link after (*n*) connection groups have passed through it corresponding to the communication between (*n*) source– destination node pairs is the following.

$$y_{n+1} = y_1 - \sum_{j=1}^{n} \Delta y_j \tag{1}$$

y_{n+1} is the unused available capacity in optical channels (wavelengths) of the optical fiber network link for the n+1 nodes pair.

y_1 is the available working capacity in optical channels (wavelengths) of the optical fiber for the first node pair. It gives the installed capacity and it is a boundary condition.

$$\sum_{j=1}^{n} \Delta y_j$$

it is the sum of *n* first order finite differences and is the sum of all optical channels that pass through this optical fiber and correspond to *n* nodes pairs optical channels.

Second and higher order finite differences are used to represent other variations.

TABLE 1
THE FINITE DIFFERENCE SYNOPTIC TABLE OF THE REDUCTION OF THE FIBER CAPACITY

Node Pair	Node pair index	Available capacity connection group	Number of the optical connection group	Variation of the optical differences number	Higher order differences number
(S_1,D_1)	1	y_1			
			Δy_1		
(S_2,D_2)	2	y_2		$\Delta^2 y_1$	
			Δy_2		$\Delta^3 y_1$
(S_3,D_3)	3	y_3		$\Delta^2 y_2$	
			Δy_3		
(S_4,D_4)	4	y_4			
……					
(S_{n-1},D_{n-1})	n-1	y_{n-1}			
			$\Delta y_{,n-1}$		
(S_n,D_n)	n	y_{in}		$\Delta^2 y_{,n-1}$	
			Δy_n		
(S_{n+1},D_{n+1})	n+1	y_{n+1}			

The equation (1) is written as following with $n_w + n_{pr} = n$.

$$y_{n+1} = y_1 - \sum_{j=1}^{n_w} \Delta y_{w,j} - \sum_{j=1}^{n_{pr}} \Delta y_{pr,j} \tag{2}$$

The above equation of the available capacity is also written

$$y_{n+1} = y_1 - \sum_{l=1}^{n} \alpha_l * x_l \tag{3}$$

whereby α_l is a coefficient that takes the value one (*1*) if the node pair (1) passes all its connections from this fiber and zero (0) if no passes.

x_l is the total connections of the node pair (l) that is the connections groups and it is called connections group size.
n the total number of the node pairs.

B.The finite differences of the connection group size

It is assumed that the connection group size is produced by discrete time equations with scalar index denote (m), taking integer values of m=0,1,2,3,…and it is concerned with the values of variable equations which indexed by (m) and they are $x_0,x_1,x_2,\ldots,x_m$. The index (m) is referred as the time. The discrete time equations that produce the connection group size are showed in the table 2. x_m the connection group size for the time instance m. It is also greater than zero. The sign (-) means reduction and the sign (+) means increasing. The a,b are constant integer coefficients greater than zero.

TABLE 2
THE EQUATIONS OF CONNECTION GROUP SIZE

S/N	**EQUATION**
1	$\mathbf{x_m=a\text{-/+}b*m}$
2	$\mathbf{x_m=a\text{-/+}b*m^2}$
3	$\mathbf{x_m=a\text{-/+}b^m}$

TABLE 3
THE FINITE DIFFERENCE SYNOPTIC TABLE OF THE REDUCTION OF THE CONNECTION GROUP SIZE UP 2ND ORDER DIFFERENCES

Time moment index	Connection group size	Number of the change of the connections	Second order differences
0	x_0		
		Δx_1	
1	x_1		$\Delta^2 x_1$
		Δx_2	
2	x_2		$\Delta^2 x_2$
		Δx_3	
3	x_3		
………			
m-1	x_{m-1}		
		Δx_m	
m	x_m		$\Delta^2 x_m$
		Δx_{m+1}	
m+1	x_{m+1}		

The first order finite differences are used to represent the changes (reduction or increasing) of connection groups in optical channels of the node pairs that pass through an optical fiber. An equation of the first order finite differences gives the change (reduction or increment) of the connection group size of a node pair. When the first change (reduction or increment) of connection group of Δx_1 connections occurs of x_0 initial optical channels the first order finite difference equation gives the new connection group size x_1 (x_{0+1}) which is written as following

$$x_{0+1}= x_0 -/+ \Delta x_1 \qquad (4)$$

The sequence Δx_1, Δx_2, Δx_3,…, Δx_m represents the changes (reductions or increment) of connection groups of a node pair. When Δx_1 subtracted from or added to x_0, creates x_1, when Δx_2 subtracted from or added to x_1, creates $x_{2,\ldots,}$ when Δx_{m+1} subtracted from or added to x_m creates x_{m+1} which is the final connection group size of this node pair. Thus the final connection group size of the node pair is calculated after m changes (reductions or increments) occur. So changes (reduction or increment) of the connection group size for each node pair produces a table as Table 1 but with some changes and gives a short comprehensive presentation of the computation of the finite differences that can be arranged quite simply. This table is table 3. At the first column the time moment indexing is presented. At the second column the connection group size is presented. At the third column, the change (reduction or increment) of the connection group size is presented. At the forth column, the second order differences are presented. The equation of the available capacity is written as in equation (5) when the coefficient α_l takes only the value one (*1*) because the node pair (l) passes all its connections from this fiber.

$$y_{n+1}= y_1 - \sum_{l=1}^{n} x_l \qquad (5)$$

For the difference equation (1) of the table 2 is proved that there are only of first order finite differences and no higher order ones. The equation of the available capacity (3) is also written

$$y_{n+1,m}= y_1 - \sum_{l=1}^{n} \alpha_l\, (a_l +/- m\, b_l\ ) \qquad (6)$$

For the difference equation (2) of the table 2 is proved that there are only of first and second order finite differences and no higher order ones. The equation of the available capacity (3) is also written

$$y_{n+1,m}= y_1 - \sum_{l=1}^{n} \alpha_l\, (a_l +/- m^2\, b_l\ ) \qquad (7)$$

For the difference equation (3) of the table 2 is proved that there are all orders finite differences. The equation of the available capacity (3) is also written

$$y_{n+1,m}= y_1 - \sum_{l=1}^{n} \alpha_l\, (a_l +/- b_l^{\ m}\ ) \qquad (8)$$

III. THE PROBLEM AND ITS SOLUTION

A. The problem

The network topology and other parameters are known as WDM and optical fiber capacity, one optical fiber per link with an extension to a 1+1 fiber protection system. So this network is characterized by one working fiber per link, edges of two links, links of two optical fibers, one for working and one for protection. In this paper we are usually referred at the working optical fiber. The connections are lightpaths originating in the source nodes and terminating at the destination nodes proceeding from preplanned optical working paths. Additionally, the same number of optical paths are preselected for the preplanned fully disjoint backup paths,(1+1 dedicated protection connection). Thus the connections that have been set up are protected. The connections of the same node pair by same preplanned optical paths form a connection group along the network. Preplanned protection paths do the dedicated protection of the connection groups. So a suitable number of wavelengths per link along the network is used. The problem solution is to calculate the final available capacity of the network when the connection group size is generated by difference equations. The number of the node pair, the node pairs with their preplanned working and preplanned protection paths and the equations which generate the connection group size are contained in a given table.

B. The formulation

The network is assumed to be an optical mesh network with the circuit switched as a graph. Each vertex represents the central telecommunications office (CO) with the OXC while each edge represents two links. Each edge link has a couple of optical fibers. All optical fibers have the same capacity as the WDM system. All nodes are identical.

A difference table (table 1) is calculated for each optical fiber and the problem is solved. The primary and backup connections use different optical fiber capacity.

The available capacity in optical channels for each optical fiber when n(i) groups of optical connections pass through it, is equal to the available capacity in optical channels that is offered for the first group of optical connections minus the total number of n(i) groups of optical connections that pass through it. The final available capacity of all optical fibers is written as a column matrix.

TABLE 4
THE SYMBOLS OF THIS PAPER

SN	Symbol	Comments
1	**q**	The node number
2	**p**	The edge number
3	**G(V,E)**	The network graph
4	**V(G)**	The network node set
5	**E(G)**	The network edge set
6	**2p**	The number of working and backup fiber for 1+1 line protection
7	**n**	The number of source – destination nodes pairs of the network
8	**Xnm**	Column matrix with dimension (nx1) and elements the connection group sizes of the corresponding source-destination nodes pairs at the time moment m
9	**n(i)**	The number of the connection groups that passes through the fiber (*i*) and means that each fiber has different number of connection groups pass through it
10	**$n(i)_w$**	The number of the working connection groups that passes through the fiber (*i*) and means that each fiber has different number of connection groups pass through it
11	**$n(i)_{pr,}$**	The number of the protection connection groups that passes through the fiber (*i*) and means that each fiber has different number of connection groups pass through it
12	**wdm**	The number of the wavelengths channels on each fiber that is the WDM system capacity
13	**Y_1**	Column matrix (2px1) with elements the installed capacity of fiber network links
14	**$Y_{2,m}$**	Column matrix (2px1) with elements the unused available capacity of each fiber network link of the linear function method at the time moment m
15	**A**	Matrix (2p x n) which shows the network active links that corresponding to working fibers
16	**$\alpha_{i,j}$**	Element of the matrix A and takes the value one if the node pair (j) passes all its working connections from the fiber (i) and zero (0) if no passes
17	**$\Delta y_{i,j}$**	First order of finite difference that corresponds to a group of optical connections that pass through the optical fiber *i* with serial number *j* and valid 1<=i<=2p and 1<=j<=n(i)
18	**$y_{i,j}$**	Unused available capacity of the optical fiber i that it is offered for optical connections group with serial number j and valid 1<=i<=2p and 1<=j<=n(i)
19	**Cav,m**	The total network available capacity at the time moment m
20	**Cb,m**	The total network busy capacity at the time moment m
21	Cinst	The total network installed capacity

The equation of the method is written as

$$Y_{2,m}=Y_1-A*Xnm \tag{9}$$

A is a matrix that shows the active optical fiber network links (*2p*) from which pass the (*n)* connection groups so its dimension is (2p x n) and its element are the coefficients $\alpha_{i,l}$, Y1 is the matrix with the installed capacity of each optical fiber network link and $Y_{2,m}$ the matrix which has elements the available capacity of each optical fiber network link at the time moment m. When all connections have been done then each element of $Y_{2,m}$ must be greater than or equal to zero. In other cases some connections are not set up.

Every element of matrix $Y_{2,m}$ for every equation of table 2 is written

$$y_{i,n(i)+1,m}= y_{i,1} - \sum_{j=1}^{n} \alpha_{i,j} (a_j -/+mb_j) \tag{10}$$

$$y_{i,n(i)+1,m}= y_{i,1} - \sum_{j=1}^{n} \alpha_{i,j} (a_j -/+m^2b_j) \tag{11}$$

$$y_{i,n(i)+1,m}= y_{i,1} - \sum_{j=1}^{n} \alpha_{i,j} (a_j -/+ b_j^{\ m}) \tag{12}$$

for every (j) and for every time moment (m) then $\alpha_{i,j}$=1 and the corresponded $x_{j,m}$>0 but for every (j) that $\alpha_{i,j}$=0, the corresponded $x_{j,m}$=0.

The equations (10),(11) and (12) must be greater or equal than zero because each shows capacity.

It is also valid

$$y_{i,1} - \sum_{j=1}^{n}\alpha_{i,j} (a_j +mb_j)<= y_{i,1} - \sum_{j=1}^{n}\alpha_{i,j} (a_j -mb_j) \tag{13}$$

$$y_{i,1} - \sum_{j=1}^{n}\alpha_{i,j} (a_j +m^2b_j)<=y_{i,1} - \sum_{j=1}^{n}\alpha_{i,j} (a_j -m^2b_j) \tag{14}$$

$$y_{i,1} - \sum_{j=1}^{n}\alpha_{i,j} (a_j + b_j^{\ m})<=y_{i,1} - \sum_{j=1}^{n}\alpha_{i,j} (a_j -b_j^{\ m}) \tag{15}$$

The symbol <= means less than.

The total capacity of the busy (working and protection) connection groups depends on the number of their first order finite differences that are used by the network. Thus, the total busy capacity (C_b) is given below

$$C_b = \sum_{i=1}^{2p} \sum_{j=1}^{n(i)_w} \Delta y_{w,i,j}+\sum_{i=1}^{2p} \sum_{j=1}^{n(i)_{pr}} \Delta y_{pr,i,j} \tag{16}$$

The total final available capacity of the network is the following

$$\sum_{i=1}^{2p} y_{i,n(i)+1,m}= \sum_{i=1}^{2p}y_{i,1}-\sum_{i=1}^{2p} \sum_{j=1}^{n}\alpha_{i,j}*X_{jm} \tag{17}$$

$y_{i,n(i)+1,m}>=0$, $y_{i,1} >=0$, $\alpha_{i,j} >=0$, $X_{jm}>0$

It is also valid

$$\sum_{i=1}^{2p}y_{i,1}-\sum_{i=1}^{2p} \sum_{j=1}^{n}\alpha_{i,j}(a_j+mb_j)<=\sum_{i=1}^{2p}y_{i,1}-\sum_{i=1}^{2p} \sum_{j=1}^{n}\alpha_{i,j}(a_j -mb_j) \tag{18}$$

$$\sum_{i=1}^{2p}y_{i,1}-\sum_{i=1}^{2p} \sum_{j=1}^{n} \alpha_{i,j} (a_j +m^2b_j)<=\sum_{i=1}^{2p}y_{i,1}-\sum_{i=1}^{2p} \sum_{j=1}^{n} \alpha_{i,j}(a_j -m^2b_j) \tag{19}$$

$$\sum_{i=1}^{2p}y_{i,1}-\sum_{i=1}^{2p} \sum_{j=1}^{n} \alpha_{i,j} (a_j+b_j^{\ m})<= \sum_{i=1}^{2p}y_{i,1}-\sum_{i=1}^{2p} \sum_{j=1}^{n} \alpha_{i,j} (a_j -b_j^{\ m}) \tag{20}$$

C. Synoptic description of the method

The value of models as tools to design and plan networks, aid the decision making process and as methods to conceptualize abstract ideas is well known. These describe the operation of the WDM optical fiber mesh network with 1+1 optical fiber protection, the working connections passed through preplanned optical paths but when a failure occurs the traffic is passed through preplanned protection paths. It has two parts, the first part or the planning and designing phase and the second part or network with failure phase. So when a cut occurs, the network has failure and the preplanned protection method is activated. The network has links with a finite, nonzero capacity and the link capacity is not exceeded. This method is driven by suitable data and then simulates the actual dynamic behavior of the network. Simulation language is critical to the economic feasibility of this entire investigation. TURBO PASCAL is used to program the model. A working connection starts from the source node and progresses through the network occupying a wavelength on each optical fiber and switch to another fiber on the same or other wavelength by OXC, according to its preplanned working optical path up to arrive at the destination node. Simultaneously, the protection connection starts from the source node and progresses through the network occupying a wavelength on each optical fiber and switch to another fiber on the same or other wavelength by OXC, according to its preplanned protection optical path up to arrive at the destination node. So there is full and dedicated protection for this connection.

The number of connections of each node pair is equal to its connection group size. The connection group size is produced by a difference equation. After a connection (working as well as protection) has been established, the available capacity is also calculated. When all connections are done the results are calculated.

TABLE 5
THE SYNOPTIC PRESENTATION OF THE METHOD

FIRST PART(*Planning and designing Phase*)

First step, network parameters reading

(q, p, V(G), E(G), G(V,E), 2,2p, k)

Second step, connection selections

(n, (S_n,D_n), Xnm, Preplanned working and protection lightpaths)

Third step, checking and wavelength allocation

(Routing and wavelength assignment method)

Forth step, results

($C_{av,m}$,$C_{b,m}$, C_{inst})

SECOND PART(*Network with failure Phase)*

Fifth step. Network parameter modifications

(cut link, q, p, V(G'), E'(G'), G'(V,E'), 2,2p-1, k)

Sixth step. Traffic passing from protection path

(Dedication protection method)

Seventh step. New Results

($C'_{av,m}$, $C'_{b,m}$,C'_{inst})

The algorithm permits to investigate the impact of the connection group size at the link capacity when the connection group size is produced by difference equations of table 2. The complexity of this method for the node number q depends on the square of the node number and the total number of the requests for connection (s) so it is written as O $(s*q^2)$. The time complexity of that algorithm is 'order q^2, $O(s*q^2)$. Thus for the below example, on a 133MHz computer, q=6 and s=12, the time is 4 hundredths of second. It means that worst time consuming depends by the network size for the same computer. The symbols with tone mean modifications. The synoptic description of the method is shown in the table 5.

D. Example

The network of this example is selected because is more suitable. It is because for a larger network is difficult to present the results as well as the number, the size of tables is larger the dimensions of the matrices are also larger. A network of eleven (11) nodes, twenty (20) edges and forty (40) single working fiber links in which the half of its node pairs communicate each other needs larger tables. The presentation of the finite differences is more difficult because the number of the table 1 is also larger. For this method, they need matrices with dimensions 40x1, 40x55 and 55x1. It is assumed that the topology of the network is presented by the graph G (V, E). This mesh topology is used because it is a simple, palpable and an analytical example of the finite differences and it is easy to expand to any mesh topology. The vertex set has q=6 elements which are V={v_1, v_2, v_3, v_4, v_5, v_6} and the edge set has p=9 elements which are E={ e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9}. Each edge is an optical link of two directions with one working fiber for each direction. Thus there are 2*p=2*9=18 optical fibers. Connection groups transverse the mesh network and correspond to *n* source-destination node pairs. Figure 1 below presents the mesh topology.

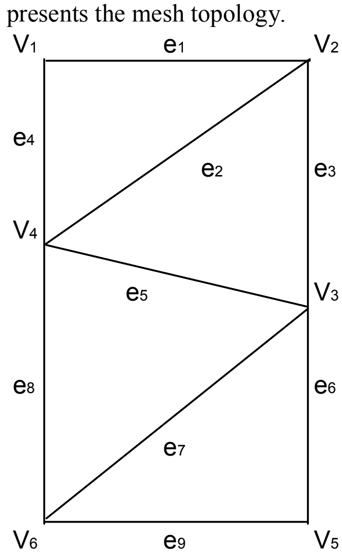

Figure1.The mesh topology of the network.

Table 6 presents the network parameters.

TABLE 6
THE NETWORK PARAMETERS

S/N	Network parameters	Amount
1	**Node number**	**6**
2	**Edge number**	**9**
3	**Working fiber per edge**	**2**
4	**Working fiber per link**	**1**
5	**Network working fiber**	**18**
6	**Protection fiber per edge**	**2**
7	**Protection fiber per link**	**1**
8	**Network protection fiber**	**18**
9	**WDM system capacity**	**30**

The problem is solved for *n=12 of 30 possible* connection groups. These have their order for each source-destination node pair, their working paths and their protection paths as shown in table 7. The results with finite differences are also showed. It is obvious that the dedicated path protection mechanisms use more than 100% redundant capacity because their lengths are longer than their working paths. The total length of working paths is nineteen, (19) and the total length of protection paths is twenty-seven, (27). Similarly for the same connections requested group size the capacity that is used by the protection paths is larger than the corresponding working paths. The optical fiber link numbering and the connection groups of each fiber link are showed in the table 8. The connection group size function of node pairs that pass through each optical fiber (both signs (-/+)) is showed in the table 9. The table 9 is transformed to the table 10 for a(1)=a(2)= …=a and b(1)=b(2)=…=b. The table 1 (finite difference table) of each fiber is not presented because the number of these tables is eighteen, (18). The finite difference of each optical fiber depends of the connection group size function that is a compound one. The order of finite difference depends of the connection group size functions that pass through.

TABLE 7
THE ORDER, WORKING PATH, PROTECTION PATH OF EACH NODE PAIR

Node Pair [Si,Di]	Node pair $[v_i, v_j]$	Working Path	Protection Path	Size x_m
$[S_1, D_1]$	$[v_1, v_2]$	v_1, v_2	v_1, v_4, v_2	a(1)-/+ b(1)*m
$[S_2, D_2]$	$[v_1, v_3]$	v_1, v_2, v_3	v_1, v_4, v_3	a(2)-/+b(2)*m^2
$[S_3, D_3]$	$[v_1, v_5]$	v_1, v_2, v_3, v_5	v_1, v_4, v_6, v_5	a(3)-/+b(3)^m
$[S_4, D_4]$	$[v_2, v_3]$	v_2, v_3	v_2, v_4, v_3	a(4)-/+ b(4)*m^2
$[S_5, D_5]$	$[v_2, v_4]$	v_2, v_4	v_2, v_3, v_4	a(5)-/+b(5)^m
$[S_6, D_6]$	$[v_2, v_5]$	v_2, v_3, v_5	v_2, v_4, v_6, v_5	a(6)-/+ b(6)*m^2
$[S_7, D_7]$	$[v_3, v_4]$	v_3, v_4	v_3, v_6, v_4	a(7)-/+b(7)^m
$[S_8, D_8]$	$[v_3, v_6]$	v_3, v_6	v_3, v_5, v_6	a(8)-/+ b(8)*m
$[S_9, D_9]$	$[v_4, v_1]$	v_4, v_1	v_4, v_2, v_1	a(9)-/+ b(9)*m^2
$[S_{10}, D_{10}]$	$[v_4, v_5]$	v_4, v_6, v_5	v_4, v_3, v_5	a(10)-/+ b(10)^m
$[S_{11}, D_{11}]$	$[v_5, v_4]$	v_5, v_3, v_4	v_5, v_6, v_4	a(11)- /+b(11)*m
$[S_{12}, D_{12}]$	$[v_6, v_1]$	v_6, v_4, v_1	v_6, v_3, v_2, v_1	a(12)- /+b(12)*m^2

TABLE 8
THE OPTICAL FIBER LINK NUMBERING AND THE CONNECTION GROUPS OF EACH FIBER LINK

Fiber,i	Optical fiber link	$n_w(i)$	$n_{pr}(i)$	n(i)
1	$<v_1, v_2>$	3	0	3
2	$<v_2, v_1>$	0	2	2
3	$<v_2, v_4>$	1	2	3
4	$<v_4, v_2>$	0	2	2
5	$<v_2, v_3>$	4	1	5
6	$<v_3, v_2>$	0	1	1
7	$<v_1, v_4>$	0	3	3
8	$<v_4, v_1>$	2	0	2
9	$<v_4, v_3>$	0	3	3
10	$<v_3, v_4>$	2	1	3
11	$<v_5, v_3>$	1	0	1
12	$<v_3, v_5>$	2	2	4
13	$<v_3, v_6>$	1	1	2
14	$<v_6, v_3>$	0	1	1
15	$<v_4, v_6>$	1	2	3
16	$<v_6, v_4>$	1	2	3
17	$<v_5, v_6>$	0	2	2
18	$<v_6, v_5>$	1	2	3

For the linear function (9), $Y_{2,m}$ has a dimension of (18x1), Y_1 has a dimension of (18x1), A has a dimension of (18x12), and Xnm has a dimension of (12x1). Matrix A is a known matrix (18x12) that is always constant because it depends on the optical paths that are constant for all examples. So the linear function provides for sign plus (+) the contents of table 11 and the corresponded sign minus (-) of table 12. The columns between Y1(i) and Y2(i),m are the values of the connection group size functions that pass through each fiber. The connection group size functions must be always greater or equal to zero. The total available capacity for each optical fiber is positive or zero so there is no possible connection problem. The total final available capacity for functions of sign plus (+) of table 11 is

$$C_{av+,m} = \sum_{i=1}^{18} y_{i,n(i)+1,m} = 1580 \text{ wavelengths}$$

The network installed capacity is Cinst=18*128=2304 wavelengths. So the following sum is valid $C_{b+,m} + C_{av+,m} =$ =Cinst or 724 +1580 = 2304.

The total final available capacity for functions of sign minus (-) of table 12 is

$$C_{av-,m} = \sum_{i=1}^{18} y_{i,n(i)+1,m} = 2108 \text{ wavelengths}$$

The network installed capacity is the same as previous. So the following sum is valid $C_{b-,m} + C_{av-,m}$=Cinst or 196 +2108 = =2304.

TABLE 9
THE BUSY CAPACITY PLUS AND MINUS OF EACH OPTICAL FIBER LINK

Optical fiber link	Plus(+) and minus(-) function
$<v_1, v_2>$	a(1)-/+b(1)*m + a(2)-/+b(2)*m^2 + a(3)-/+b(3)^m
$<v_2, v_1>$	a(9)-/+b(9)*m^2+ a(12)-/+b(12)*m^2
$<v_2, v_4>$	a(4)-/+b(4)*m^2+a(5)-/+b(5)^m+a(6)-/+b(6)*m^2
$<v_4, v_2>$	a(1)-/+b(1)*m+a(9)-/+b(9)*m^2
$<v_2, v_3>$	a(2)-/+b(2)*m^2+a(3)-/+b(3)^m+a(4)-/+ b(4)*m^2+a(5)-/+b(5)^m+a(6)-/+b(6)*m^2
$<v_3, v_2>$	a(12)-/+b(12)*m^2
$<v_1, v_4>$	a(1)-/+b(1)*m+a(2)-/+b(2)*m^2+a(3)-/+b(3)^m
$<v_4, v_1>$	a(9)-/+ b(9)*m^2+a(12)-/+b(12)*m^2
$<v_4, v_3>$	a(2)-/+b(2)*m^2+a(4)-/+b(4)*m^2+a(10)-/+b(10)^m
$<v_3, v_4>$	a(5)-/+b(5)^m+a(7)-/+b(7)^m+a(11)-/+b(11)*m
$<v_5, v_3>$	a(11)-/+b(11)*m
$<v_3, v_5>$	a(3)-/+b(3)^m+a(6)-/+b(6)*m^2+a(8)-/+ b(8)*m+a(10)-/+b(10)^m
$<v_3, v_6>$	a(7)-/+b(7)^m+a(8)-/+b(8)*m
$<v_6, v_3>$	a(12)-/+b(12)*m^2
$<v_4, v_6>$	a(3)-/+b(3)^m+a(6)-/+b(6)*m^2+a(10)-/+b(10)^m
$<v_6, v_4>$	a(7)-/+b(7)^m+a(11)-/+b(11)*m+a(12)-/+ b(12)*m^2
$<v_5, v_6>$	a(8)-/+ b(8)*m+a(11)-/+b(11)*m
$<v_6, v_5>$	a(3)-/+b(3)^m+a(6)-/+b(6)*m^2+a(10)-/+ b(10)^m

It is noted if all values of (a) and (b) are equals (a(1)=a(2)=…=a and b(1)=b(2)=…=b) then the total busy capacity equations are given by

$$C_{b+,m} = 46a+10bm+20bm^2+ 16b^m \qquad (21)$$

and

$$C_{b-,m} = 46a -10bm-20bm^2 - 16b^m \qquad (22)$$

respectively. The available capacity for these cases is

$$C_{av+,m}=18*wdm- C_{b+,m} \qquad (23)$$

and

$$C_{av-,m}=18*wdm-C_{b-,m} \qquad (24)$$

respectively. The difference

$$\Delta C_b=C_{b+,m}- C_{b-,m}=20bm-+40bm^2+32b^m \qquad (25)$$

and the difference

$$\Delta Cav= C_{av-,m} - C_{av+,m}=\Delta Cb \tag{26}$$

. In the table 13 the total final busy capacity versus m of signs plus (+) and minus (-) is showed for a(1)=a(2)=…=a and b(1)=b(2)=…=b. The constraints that exist in a real network limit the maximum value of m. In table 14 the available capacity versus b and the maximum value of m for both signs when WDM system capacity is 128λ and a=10λ, are showed. From the table 14 is showed that the maximum value of m is 3 when wdm=128λ, a=10λ and b=2λ. So in the table 15, the total available capacity versus m is showed when WDM system capacity is 128λ, a=10λ and b=2λ.

TABLE 10
THE BUSY CAPACITY PLUS AND MINUS FOR a(1)=a(2)=…=a AND b(1)=b(2)=…=b OF EACH OPTICAL FIBER LINK

Optical fiber link	Plus(+) function	**Minus(-) function**
$<v_1, v_2>$	**3*a+b*m+b*m^2+b^m**	3*a-b*m-b*m^2-b^m
$<v_2, v_1>$	**2*a+2*b*m^2**	2*a-2*b*m^2
$<v_2, v_4>$	**3*a+2*b*m^2+b^m**	3*a-2*b*m^2-b^m
$<v_4, v_2>$	**2*a+b*m+b*m^2**	2*a-b*m-b*m^2
$<v_2, v_3>$	**5*a+3*b*m^2+2*b^m**	5*a-3*b*m^2-2*b^m
$<v_3, v_2>$	**a+b*m^2**	a-b*m^2
$<v_1, v_4>$	**3*a+b*m+b*m^2+b^m**	3*a-b*m-b*m^2-b^m
$<v_4, v_1>$	**2*a+ 2*b*m^2**	2*a-2*b*m^2
$<v_4, v_3>$	**3*a+2*b*m^2+b^m**	3*a-2*b*m^2-b^m
$<v_3, v_4>$	**3*a+2*b^m+b*m**	3*a-2*b^m-b*m
$<v_5, v_3>$	**a+b*m**	a-b*m
$<v_3, v_5>$	**4*a+2*b^m+b*m^2+b*m**	4*a-2*b^m-b*m^2-b*m
$<v_3, v_6>$	**2*a+b^m+b*m**	2*a-b^m-b*m
$<v_6, v_3>$	**a+b*m^2**	a-b*m^2
$<v_4, v_6>$	**3*a+b*m^2+2*b^m**	3*a-b*m^2-2*b^m
$<v_6, v_4>$	**3*a+b^m+b*m+ b*m^2**	3*a-b^m-b*m- b*m^2
$<v_5, v_6>$	**2*a+ 2*b*m**	2*a-2*b*m
$<v_6, v_5>$	3*a+b*m^2+2* b^m	3*a-b*m^2-2* b^m

TABLE 11
THE AVAILABLE CAPACITY OF OPTICAL FIBRE LINK FOR SIGN PLUS (+)

$x(i)_m$	a	b	m	Y1(i)						Y2(i),m	Fiber,i
$x(1)_2$	10	2	2	128	14	18	14			82	1
$x(2)_2$	10	2	2	128	18	18				92	2
$x(3)_2$	10	2	2	128	18	14	18			78	3
$x(4)_2$	10	2	2	128	14	18				96	4
$x(5)_2$	10	2	2	128	18	14	18	14	18	46	5
$x(6)_2$	10	2	2	128	18					110	6
$x(7)_2$	10	2	2	128	14	18	14			82	7
$x(8)_2$	10	2	2	128	18	18				92	8
$x(9)_2$	10	2	2	128	18	18	14			78	9
$x(10)_2$	10	2	2	128	14	14	14			86	10
$x(11)_2$	10	2	2	128	14					114	11
$x(12)_2$	10	2	2	128	14	18	14	14		68	12
				128	14	14				100	13
				128	18					110	14
				128	14	18	14			82	15
				128	14	14	18			82	16
				128	14	14				100	17
				128	14	18	14			82	18
			Cinst	2304			Cav+,m			1580	

It is studied from the tables 8, 9, 10, 11 and 12 that the fibre number 5 passes through the larger traffic that corresponds to the five (5) node pairs. So the values of the parameters of this fibre are effect most than another of the network. The busy capacity and the available one of each fibre must be greater or equal to zero. The equation of the available capacity with plus sign that corresponds to the equation of the fibre 5 of the table 10 is written bellow.

$$Cav+,m=wdm-(5*a+3*b*m^2+2*b^m) \tag{26}$$

The equation of plus sign is selected because it obtains most suitable values. The previous equation has its maximum value when the Cb+,m is equal to zero that means that there is not any traffic. This equation is linear to (a). The maximum value of (a) is 50 for wdm capacity equal to 256 because this is the maximum integer number that satisfies the previous equation when b=1 and m=0.The minimum value of (a) is zero. The maximum value of equation (26) with traffic is for a=0,b=1 and m=0 that gives Cav+,0=254. The corresponded value of Cav-,0=258 that is rejected. The minimum value of the equation (26) is obtained by a=50, b=1,m=1 and is one (1).

TABLE 12
THE AVAILABLE CAPACITY OF OPTICAL FIBRE LINK FOR SIGN MINUS (-)

$x(i)_m$	a	b	m	Y1(i)						Y2(i),m	Fiber,i
$x(1)_2$	10	2	2	128	6	2	6			114	1
$x(2)_2$	10	2	2	128	2	2				124	2
$x(3)_2$	10	2	2	128	2	6	2			118	3
$x(4)_2$	10	2	2	128	6	2				120	4
$x(5)_2$	10	2	2	128	2	6	2	6	2	110	5
$x(6)_2$	10	2	2	128	2					126	6
x(7)6	10	2	2	128	6	2	6			114	7
x(8)2	10	2	2	128	2	2				124	8
x(9)6	10	2	2	128	2	2	6			118	9
$x(10)_2$	10	2	2	128	6	6	6			110	10
$x(11)_2$	10	2	2	128	6					122	11
$x(12)_2$	10	2	2	128	6	2	6	6		108	12
				128	6	6				116	13
				128	2					126	14
				128	6	2	6			114	15
				128	6	6	2			114	16
				128	6	6				116	17
				128	6	2	6			114	18
		Cinst		2304				Cav-,m		2108	

TABLE 13
THE TOTAL BUSY CAPACITY VERSUS m FOR a(1)=a(2)=…=a AND b(1)=b(2)=…=b, SIGNS PLUS (+) AND MINUS (-)

m	Cb+,m=46a+10bm+20bm^2+16b^m	Cb-,m=46a-10bm-20bm^2-16b^m
0	Cb+,0=46a+16	Cb-,0=46a-16
1	Cb+,1=46a+46b	Cb-,1=46a-46b
2	Cb+,2=46a+100b+16b^2	Cb-,2=46a-100b-16b^2
3	Cb+,3=46a+210b+16b^3	Cb-,3=46a-210b-16b^3
4	Cb+,4=46a+360b+16b^4	Cb-,4=46a-360b-16b^4
5	Cb+,5=46a+550b+16b^5	Cb-,5=46a-550b-16b^5
6	Cb+,6=46a+780b+16b^6	Cb-,6=46a-780b-16b^6
7	Cb+,7=46a+1050b+16b^7	Cb-,7=46a-1050b-16b^7
8	Cb+,8=46a-1360b+16b^8	Cb-,8=46a-1360b-16b^8
9	Cb+,9=46a+1710b+16b^9	Cb-,9=46a-1710b-16b^9
10	Cb+,10=46a+2100b+16b^{10}	Cb-,10=46a-2100b-16b^{10}

These values of (wdm, a, b, m) are valid for all network. So for wdm=256, a=0, b=1 (as well as any value of b>0, it is showed from table 13) and m=0 the equation (23) gives its maximum value which is 4592 and the corresponded of (24) 4624 that is rejected because is larger than WDM system capacity which is 4608. The values of these equations that are obtained for wdm=256, a=50, b=1 and m=1, for the equation (23) its minimum value which is 2262 and for the (24) is 2354.

TABLE 14
THE AVAILABLE CAPACITY VERSUS b AND MAXIMUM VALUE OF M FOR BOTH SIGN CASES, AND WDM=128λ, a=10λ

b	m_{max+}	Cav+,m	m_{max-}	Cav-,m
1	5	1278	3	2070
2	3	1296	2	2108
3	2	1400	1	1982
4	1	1660	1	2028
5	1	1614	1	2074
6	1	1568	1	2120
7	1	1522	1	2166
8	1	1476	1	2212
9	1	1430	1	2258
10	1	1384	1	2304

TABLE 15
THE AVAILABLE CAPACITY VERSUS m FOR a=10 and b=2

m	Cav+,m	**Cav-,m**
0	**1828**	1860
1	**1752**	1936
2	**1580**	2108
3	1296	

E. Discussion and Proposals

Protection and restoration strategies are critical for optical mesh networks. Although dedicated path protection mechanisms are simple and fast, they use 100% or more redundant capacity. On the other hand, shared mesh restoration mechanisms uses spare capacity much more efficiently but are slower than dedicated protection methods to react to a failure. There are also the p-cycles that are an efficient way of obtaining mesh like efficiency with ring like speeds.

For this protection scheme any failure on working path can overcome by dedicated protection path. The failure may be link cut, fiber cut, partial node failure, wavelength crash etc. This dedicated optical path protection was selected because it is more representative and full one. In this paper, each connection of each source destination node pair is generated by a function. The function of each connection group size may be exponential one, square one or linear one. These functions are representatives. These functions must be always greater or equal to zero. The functions are used with the sign plus (+) and minus (-) that are boundary values. The use of the finite differences is possible for the study of the problems related to the protection and restoration of connections.

For a better presentation of this study an example is used that depicts the results in these methods. The algorithm provides for each source-destination node pair, a desired connection group size function for each sign, plus (+) and (-), and a corresponded value of the total available capacity of the network. The connection length depends on the number of hops. The network is a switch circuit network so that one lightpath corresponds to one optical connection. Different wavelengths may be used for each connection in each hop, so that wavelength conversion is used at each node. The routing and wavelength assignment is done by preplanned working and protection lightpaths. These methods solve problems with small networks when the connections groups that pass through a link increases and it is difficult to represent and solve.

IV. CONCLUSIONS

By using WDM the optical networks are capable of carrying many independent channels, which are carried on different wavelengths, over a single optical fiber. This allows the network to transport huge amounts of data that are needed for many current and future communication services, which play a very important role in many of our daily activities. The main drawback is the failures that can lead to the loss of a large amount of data. Thus the suitable strategy must use to minimize all such failures effects.

In this article the problems of the traffic and the available network capacity of the WDM mesh networks are defined, analyzed and calculated by applying the difference equations. The dedicated protection optical path method has also used and researched on the basis of the finite differences and its impact on available capacity reduction when the connection group size function change (reduction or increasing). The finite differences make it possible to research and study of these problems. They provide suitable, accurate methods to solve the problems of planning of a completely protected network, complete protection for any failure occurs on optical path, node or link e. t. c.

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