Adaptive Tree Search Detection with Variable Path Expansion Based on Gram-Schmidt Orthogonalization in MIMO Systems

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***Abstract*—This paper proposes new adaptive tree search detection with variable path expansion based on Gram-Schmidt (GS) orthogonalization (GSO) in MIMO systems. We adopt the GSO procedure to reduce the channel matrix instead of the QR-decomposition in the conventional QRM-MLD. This detection scheme combined the GSO reduction with the M-algorithm, what we call GSM-MLD, can achieve near-ML performance as the conventional QRM-MLD. The proposed detection method is a breadth-first algorithm and performs the adaptive tree search with variable path expansion in the GSM-MLD. In this paper, we introduce a path metric ratio function to evaluate the reliability for all the survived branches. The survived but lower reliable branches adopt parts of the constellation points as the candidates into the next detection layer. The proposed detection algorithm reduces the complexity by adaptively decreasing the computation of the path metric for the low reliable candidates. The numerical results exhibit that the proposed scheme achieves near-ML performance with relatively lower complexity compared to the conventional QRM-MLD.**

***Index Terms*—Adaptive signal processing; Gram-Schmidt (GS) orthogonalization (GSO); QRM-MLD; MIMO; tree search.**

I. Introduction

Multiple-input multiple-output (MIMO) technology has attracted attention in wireless communications, since it provides significant increases in data throughput and the high spectral efficiency [1]-[3]. MIMO systems employs multiply antennas at both ends of the wireless link, and hence can increase the data rate by transmitting multiple data streams. To exploit the potential gains offered by MIMO, signal processing involved in a MIMO receiver requires a large computational complexity in order to achieve the optimal performance. The maximum likelihood (ML) detection (MLD) is known as the optimal receiver in terms of minimizing bit error rate (BER). However, the complexity of MLD obstructs its practical implementation. The common linear detection such as zero forcing (ZF) or minimum mean squared error (MMSE) with the lattice-reduction (LR) technology can offer a remarkable complexity reduction with performance loss [4]-[7]. Numerous suboptimal detection techniques have been investigated to approximately approach the ML performance with relatively lower complexity, such as the sphere detection (SD) and the MLD with QR Decomposition and M-algorithm (QRM-MLD) [8]-[16]. To looking for the suboptimal detection algorithm with the near optimal performance and the affordable complexity costs for MIMO gains faces a major challenge.

The conventional QRM-MLD is one solution to relatively reduce the complexity while retaining the ML performance. The number of M in the QRM-MLD is defined as the number of the survived branches in each detection layer of the tree search, which is a tradeoff between the complexity and the performance. Furthermore, the value of M should be large enough to ensure that the correct symbols exist in the survived branches under the ill-conditioned channel, in particular for the large size MIMO and the high modulation order. Hence, the conventional QRM-MLD still requires high complexity in the high E_b/N_0 region [10]. To overcome this drawback, numerous methods with adaptively controlling the survived branch M have been proposed in [11]-[13]. These schemes still have the problem that needs to accurately and dynamically measure SNR for optimal setting of the number of survived branch in each layer.

In this paper, we first present a detection scheme combined the Gram-Schmidt (GS) orthogonalization (GSO) reduction with the M-algorithm, which we call the GSM-MLD. This scheme has such features that it achieves near-ML BER performance like the QRM-MLD with lower computational complexity. The channel matrix is reduced using the GSO procedure, and meanwhile a transform matrix is created. In contrast to the QR decomposition of the channel matrix in the QRM-MLD, which $\mathbf{R}$ retains the property of the channel matrix, the column vectors of the GS-reduced channel matrix are purely orthogonal for the GSM-MLD. The GS-reduced channel matrix spans the same subspace as the columns of the original channel matrix. The transform matrix is an upper triangular matrix with unity diagonal entries.

Based on the GSM-MLD, we propose novel adaptive tree search detection with variable path expansion based on GSO in the MIMO systems. The proposed algorithm retains the same breadth of the tree search as the GSM-MLD to achieve the

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near-ML performance, and however the number of the possible branches is adaptively controlled. The adaptive scheme avoids a large amount of the path metric evaluations and sorting to reduce the computational complexity. We also analyze the complexity of the proposed detection. The proposed detection can considerably decrease the complexity in the high E_b/N_0 region.

The remainder of this paper is organized as follows. Section II presents the system model and the conventional QRM-MLD algorithm. Section III explains the GSM-MLD algorithm. In Section IV, we propose an adaptive tree search scheme to the GSM-MLD in MIMO systems. Section V gives numerical results and discussions. Finally, we summarize and conclude the paper in Section VI.

Notations: Matrices and vectors are denoted by bold-face letters. $\mathbf{A}^{\mathrm{T}}$, $\mathbf{A}^{-1}$ and $\mathbf{A}^{\dagger}$ are used to denote the transpose, inverse, and pseudo-inverse of a matrix $\mathbf{A}$, respectively. The real and imaginary parts are denoted as $\mathrm{Re}[\cdot]$ and $\mathrm{Im}[\cdot]$. The operator $\mathcal{Q}[\cdot]$ is the quantization. $\|\cdot\|$ represents the Frobenius norm. $a_{i,j}$ denotes the entry at the i-th row and the j-th column of $\mathbf{A}$.

II. System Model and Conventional QRM-MLD

Consider a multiple antenna system with N_t transmit and N_r ($N_r \geq N_t$) receive antennas. The signals are transmitted over a rich scattering flat fading channel. Assume that the receiver has perfect knowledge of the channel state information (CSI). The received signal vector $\mathbf{y}^{\mathrm{c}} = [y_1^{\mathrm{c}}, \ldots, y_{N_r}^{\mathrm{c}}]^{\mathrm{T}} \in \mathbb{C}^{Nr \times 1}$ is expressed as

$$\mathbf{y}^{\mathrm{c}} = \mathbf{H}^{\mathrm{c}}\mathbf{s}^{\mathrm{c}} + \mathbf{z}^{\mathrm{c}} \tag{1}$$

where y_i^{c} is the received signal at the i-th receive antenna. The transmitted signal vector is denoted as $\mathbf{s}^{\mathrm{c}} = [s_1^{\mathrm{c}}, \ldots, s_{N_t}^{\mathrm{c}}]^{\mathrm{T}} \in \mathbb{Q}^{Nt \times 1}$, where each symbol s_j^{c} at the j-th transmit antenna is chosen from a finite subset of the complex-valued integer set $\mathbb{Q}$. Let $\mathbf{H}^{\mathrm{c}} = [\mathbf{h}_1^{\mathrm{c}}, \ldots, \mathbf{h}_{N_t}^{\mathrm{c}}]$ denote the $N_r \times N_t$ channel matrix. We assume that the entries of $\mathbf{H}^{\mathrm{c}}$ are of the i.i.d. complex Gaussian process with zero mean and unity variance. The noise vector $\mathbf{z}^{\mathrm{c}} = [z_1^{\mathrm{c}}, \ldots, z_{N_r}^{\mathrm{c}}]^{\mathrm{T}} \in \mathbb{C}^{Nr \times 1}$ is the additive white Gaussian noise (AWGN) vector, of which each entry is assumed to be zero mean and variance of N_0, the one-sided noise power spectral density.

As the system model in (1) is complex-valued, treating the real and imaginary parts separately, the system model can be rewritten as

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{z} \tag{2}$$

with the real-valued channel matrix and the real-valued vectors

$$\mathbf{H} = \begin{bmatrix} \mathrm{Re}(\mathbf{H}^{\mathrm{c}}) & -\mathrm{Im}(\mathbf{H}^{\mathrm{c}}) \\ \mathrm{Im}(\mathbf{H}^{\mathrm{c}}) & \mathrm{Re}(\mathbf{H}^{\mathrm{c}}) \end{bmatrix} \in \mathbb{R}^{n \times m} \tag{3}$$

$$\mathbf{s} = \begin{bmatrix} \mathrm{Re}(\mathbf{s}^{\mathrm{c}}) \\ \mathrm{Im}(\mathbf{s}^{\mathrm{c}}) \end{bmatrix}, \ \mathbf{y} = \begin{bmatrix} \mathrm{Re}(\mathbf{y}^{\mathrm{c}}) \\ \mathrm{Im}(\mathbf{y}^{\mathrm{c}}) \end{bmatrix}, \mathbf{z} = \begin{bmatrix} \mathrm{Re}(\mathbf{z}^{\mathrm{c}}) \\ \mathrm{Im}(\mathbf{z}^{\mathrm{c}}) \end{bmatrix} \tag{4}$$

Letting $n=2N_r$ and $m=2N_t$, we define the dimension of the real-valued channel matrix $\mathbf{H}$ to be $n \times m$. The dimensions of the vectors in (4) are given as $\mathbf{y} \in \mathbb{R}^n$, $\mathbf{z} \in \mathbb{R}^n$ and $\mathbf{s} \in \mathbb{Z}^m$, where $\mathbb{Z}$ denotes the finite set of the real-valued transmitted signals. This set is given by $\mathbb{Z} = \{\pm 1, \pm 3, \ldots, \pm(\sqrt{K}-1)\}$ for K-QAM (Quadrature Amplitude Modulation). Given $\mathbf{y}$ and the channel matrix $\mathbf{H}$, the ZF soft estimate of the transmitted signals is expressed as

$$\tilde{\mathbf{s}}^{(\mathrm{ZF})} = \mathbf{H}^{\dagger}\mathbf{y} \equiv (\mathbf{H}^{\mathrm{T}}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{y} \tag{5}$$

The concept of the QRM-MLD is to apply a tree search to detect the symbols in a sequential manner [10]. The channel matrix $\mathbf{H}$ applies the QR decomposition as $\mathbf{H}=\mathbf{Q}\mathbf{R}$, where $\mathbf{Q}$ is a unitary matrix: i.e., $\mathbf{Q}^{\mathrm{T}}\mathbf{Q}=\mathbf{I}_m$, and $\mathbf{R}$ is an $m \times m$ upper triangular matrix. The QR decomposition is executed by the modified GS algorithm (MGS) in [17]. The $\mathbf{R}$ retains the property of the channel matrix $\mathbf{H}$. Then, we pre-multiply both the hand sides of (2) by $\mathbf{Q}^{\mathrm{T}}$ as

$$\mathbf{y}' \triangleq \mathbf{Q}^{\mathrm{T}}\mathbf{y} = \mathbf{Q}^{\mathrm{T}}(\mathbf{Q}\mathbf{R}\mathbf{s} + \mathbf{z}) = \mathbf{R}\mathbf{s} + \mathbf{z}' \tag{6}$$

with expressing $\mathbf{R}$ as

$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1,m} \\ & r_{22} & \cdots & r_{2,m} \\ & & \ddots & \vdots \\ \mathbf{O} & & & r_{m,m} \end{bmatrix} \tag{7}$$

where $\mathbf{z}' \triangleq \mathbf{Q}^{\mathrm{T}}\mathbf{z}$. The ML detector searches over the whole set of transmitted signals $\mathbf{s} \in \mathbb{Z}^m$, and decides the transmitted signal $\hat{\mathbf{s}}^{(\mathrm{ML})}$ in terms of the minimum Euclidean distance (ED) to the received vector $\mathbf{y}$. The ML detection can be formulated as

$$\begin{aligned} \hat{\mathbf{s}}^{(\mathrm{ML})} &= \arg\min_{\mathbf{s} \in \mathbb{Z}^m} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2 = \arg\min_{\mathbf{s} \in \mathbb{Z}^m} \|\mathbf{y}' - \mathbf{R}\mathbf{s}\|^2 \\ &= \arg\min_{\mathbf{s} \in \mathbb{Z}^m} \left[\textstyle\sum_{i=1}^{m} | y_i' - \sum_{j=i}^{m} r_{i,j} s_j |^2 \right] \end{aligned} \tag{8}$$

where $\lambda_i \triangleq | y_i' - \sum_{j=i}^{m} r_{i,j} s_j |^2$ denotes the branch metric in the i-th layer. The accumulated branch metric $\Lambda_i \triangleq \sum_{j=i}^{m} \lambda_j$ is defined as the path metric from the m-th layer down to the i-th layer. For each detection layer of the tree search in the QRM-MLD, there are three major operations:

- Candidate Expansion: Expand the children nodes from each survived branch. The candidates for the children nodes consist of all the constellation points.

– Path metric evaluations: There are $M\sqrt{K}$ possible branches for K-QAM in each layer. Calculate the path metric for all the possible branches.
– Sorting and retaining: Sort the path metric and retain M branches with the smallest path metric from $M\sqrt{K}$ possible branches. The rest of branches discard.

Let $\Lambda_i^{(l)}$ denote the l-th smallest path metric of the survived path $\Pi_i^{(l)}$ after the operations of sorting and retaining, where $l\in[1,M]$ and $\Lambda_i^{(1)}\leq\Lambda_i^{(2)}\leq ... \leq\Lambda_i^{(M)}$. Correspondingly, the partial transmitted signal $\hat{\mathbf{s}}_i^{(l)}$ based $\Lambda_i^{(l)}$ is expressed as $\hat{\mathbf{s}}_i^{(l)}=[\hat{s}_i^{(l)},\ldots,\hat{s}_m^{(l)}]^{\mathrm{T}}$. The same operations are executed until the first layer. The output of the QRM-MLD is $\hat{\mathbf{s}}_1^{(1)}=[\hat{s}_1^{(1)},\ldots,\hat{s}_m^{(1)}]^{\mathrm{T}}$ as the final estimate of transmitted signal.

Although the exhaustive tree search of the QRM-MLD should visit $M\sqrt{K}$ nodes in each detection layer instead of $(\sqrt{K})^{m-q+1}$ nodes in the i-th layer for the full MLD. The conventional QRM-MLD reduces the exponentially growing complexity to a linear growing complexity while retaining the ML performance. However, the conventional QRM-MLD still requires high complexity in the high E_b/N_0 region.

III. GSM-MLD

Based on Fujino et al.'s previous work of the GSO based lattice-reduction aided detection in MIMO systems [4],[5], we introduce the GSM-MLD algorithm. The column vectors of channel matrix $\mathbf{H}$ are first sorted in ascending order in length. Then, they are weakly reduced using the GSO procedure shown in Table I. This algorithm transforms the channel matrix $\mathbf{H}$ to create the GS-reduced channel matrix $\hat{\mathbf{H}}$ and the transform matrix $\hat{\mathbf{T}}$. The column vectors of $\hat{\mathbf{H}}$ are mutually orthogonal, and the transform matrix $\hat{\mathbf{T}}$ is an upper triangular matrix with unity diagonal entries and det{$\hat{\mathbf{T}}$}=+1. Note that this algorithm in Table I is computationally-simple since it weakly reduces the column vectors of $\mathbf{H}$ without the size reduction in the LLL algorithm [4].

TABLE I. GRAM-SCHMIDT ORTHOGONALIZATION

(1) **Begin** Input $\mathbf{H}=[\mathbf{h}_1,\ldots,\mathbf{h}_m],\mathbf{T}:=\mathbf{I}_m=[\mathbf{t}_1,\ldots,\mathbf{t}_m]$.
Set $\hat{\mathbf{h}}_p=\mathbf{h}_p,\ p\in[1,m]$.
(2) **for** p:=2 to m
(3) **for** q:=p–1 down to 1
(4) $\mu_{p,q}=\dfrac{\hat{\mathbf{h}}_q^{\mathrm{T}}\hat{\mathbf{h}}_p}{\Vert\hat{\mathbf{h}}_q\Vert^2}$
(5) $\hat{\mathbf{h}}_p:=\hat{\mathbf{h}}_p-\mu_{p,q}\hat{\mathbf{h}}_q,\ \hat{\mathbf{t}}_p:=\hat{\mathbf{t}}_p-\mu_{p,q}\hat{\mathbf{t}}_q$
(6) **end**
(7) **end**
(8) **End**

The upper triangular matrix $\hat{\mathbf{T}}$ with unity diagonal entries is invertible. The column vectors of the matrix $\hat{\mathbf{H}}=\mathbf{H}\hat{\mathbf{T}}$ are orthogonal and span the same subspace as the columns of the original matrix $\mathbf{H}$. Using the GS-reduced channel matrix $\hat{\mathbf{H}}$ and $\hat{\mathbf{T}}$, we have

$$\mathbf{y}=\mathbf{Hs}+\mathbf{z}=(\mathbf{H}\hat{\mathbf{T}})(\hat{\mathbf{T}}^{-1}\mathbf{s})+\mathbf{z}\equiv\hat{\mathbf{H}}\mathbf{v}+\mathbf{z} \tag{9}$$

where $\hat{\mathbf{H}}\triangleq\mathbf{H}\hat{\mathbf{T}}$ and $\mathbf{v}\triangleq\hat{\mathbf{T}}^{-1}\mathbf{s}$ with expressing $\hat{\mathbf{T}}^{-1}$ as

$$\hat{\mathbf{T}}^{-1}=\begin{bmatrix} 1 & \tau_{12} & \cdots & \tau_{13} & \tau_{1,m} \\ & 1 & \cdots & \tau_{23} & \tau_{2,m} \\ & & \ddots & \vdots & \vdots \\ & & & 1 & \tau_{m-1,m} \\ \mathbf{O} & & & & 1 \end{bmatrix} \tag{10}$$

With the orthogonal column vectors of $\hat{\mathbf{H}}$, the soft estimate of $\mathbf{v}$ is derived as

$$\begin{aligned}\tilde{\mathbf{v}}&=\hat{\mathbf{T}}^{-1}\tilde{\mathbf{s}}^{(\mathrm{ZF})}=\hat{\mathbf{H}}^{\dagger}\mathbf{y}=(\hat{\mathbf{H}}^{\mathrm{T}}\hat{\mathbf{H}})^{-1}\hat{\mathbf{H}}^{\mathrm{T}}\mathbf{y}\\ &=\left[\frac{\hat{\mathbf{h}}_1}{\Vert\hat{\mathbf{h}}_1\Vert^2},\ \frac{\hat{\mathbf{h}}_2}{\Vert\hat{\mathbf{h}}_2\Vert^2},\ \cdots,\ \frac{\hat{\mathbf{h}}_m}{\Vert\hat{\mathbf{h}}_m\Vert^2}\right]^{\mathrm{T}}\mathbf{y}\end{aligned} \tag{11}$$

$$\text{or}\quad \tilde{v}_i=\left(\hat{\mathbf{h}}_i^{\mathrm{T}}\big/\Vert\hat{\mathbf{h}}_i\Vert^2\right)\mathbf{y},\ i\in[1,m] \tag{12}$$

Then, the soft estimate of $\hat{\mathbf{s}}$ is obtained by performing the following recursion as

$$\hat{s}_i=\begin{cases}\mathcal{Q}[\tilde{v}_i]: i=m\\ \mathcal{Q}[\tilde{v}_i-\sum_{j=i+1}^{m}\tau_{i,j}\hat{s}_j]: i=m-1,\ldots,1\end{cases} \tag{13}$$

A. Definition of Metric in GSM-MLD

The GSM-MLD applies a fixed number of M in each detection layer as the QRM-MLD, starting from the last entry of $\mathbf{s}$. Since $\hat{\mathbf{T}}^{-1}$ is an upper triangular matrix, the entry s_i depends on the decided estimates $\hat{s}_j$'s where $j\in[i+1,m]$. We define the branch metric λ_i : $i\in[1,m]$ in GSM-MLD as

$$\lambda_i=\begin{cases}\Vert\hat{\mathbf{h}}_i\Vert^2|\tilde{v}_i-\hat{s}_i|^2=\Vert\hat{\mathbf{h}}_i\Vert^2|\tilde{s}_i-\hat{s}_i|^2, i=m\\ \Vert\hat{\mathbf{h}}_i\Vert^2|\tilde{v}_i-\hat{s}_i-\sum_{j=i+1}^{m}\tau_{i,j}\hat{s}_j|^2=\Vert\hat{\mathbf{h}}_i\Vert^2|\tilde{s}_i-\hat{s}_i|^2, i\neq m\end{cases} \tag{14}$$

where $\tilde{s}_m\triangleq\tilde{v}_m$ and $\tilde{s}_i\triangleq\tilde{v}_i-\sum_{j=i+1}^{m}\tau_{i,j}\hat{s}_j$ for i=m–1,…,1. The path metric Λ_i: $i\in[1,m]$ is the accumulated branch metric, which is defined as

$$\begin{aligned}\Lambda_i&=\sum_{j=i}^{m}\lambda_j=\sum_{j=i}^{m}\Vert\hat{\mathbf{h}}_j\Vert^2|\tilde{s}_i-\hat{s}_i|^2\\ &=\begin{cases}\lambda_i, i=m\\ \lambda_i+\Lambda_{i+1}, i\neq m\end{cases}\end{aligned} \tag{15}$$

The Λ_i is the partial Euclidean distance (PED). In the GSM-MLD, $\Lambda_i^{(l)}$ still denotes the l-th smallest path metric. Correspondingly, the partial transmitted signal $\hat{\mathbf{s}}_i^{(l)}$ based on

$\Lambda_i^{(l)}$ should be expressed as $[\hat{s}_i^{(l)},\ldots,\ \hat{s}_m^{(l)}]^{\mathrm{T}}$. Three major operations are the same as the conventional QRM-MLD. The output of the GSM-MLD is $\hat{\mathbf{s}}_1^{(1)}=[\hat{s}_1^{(1)},\ldots,\ \hat{s}_m^{(1)}]^{\mathrm{T}}$ as the final estimate of transmitted signal.

B. Computational Complexity

We here use the floating point operations (flops) for the measure of the complexity, which defines one addition, one subtraction, one multiplication, and one division for real-valued number to take one flop. For the m-th layer, expanding $\sqrt{K}$ branches, λ_m in (14) requires two multiplications and one subtraction, and it consumes 3 flops expressed by $\mathcal{N}(\lambda_m)=3$. For the $(m-1)$-th layer down to the first layer, M branches are retained from $M\sqrt{K}$ possible branches in the i-th layer, where $i\in[1,m-1]$. For a survived branch, $\tilde{s}_i$ in (14) requires $(m-i)$ multiplications and $(m-i)$ subtractions. Hence, the complexity for the computing of $\tilde{s}_i$ is expressed as $\mathcal{N}(\tilde{s}_i)=2(m-i)$. For a possible branch, Λ_i in (15) requires one addition, which we express the complexity as $\mathcal{N}(\Lambda_i)=1$. $\mathcal{N}(\lambda_i,\Lambda_i)=\mathcal{N}(\lambda_i)+\mathcal{N}(\Lambda_i)=4$ denotes the total complexity for the computations of the branch metric λ_i in (14) and the path metric Λ_i in (15).

The complexity of the GSM-MLD $\mathcal{N}_{\text{GSM-MLD}}$ which excludes the complexity of the GSO reduction and the computation of $\tilde{\mathbf{v}}$ in (11) can be derived as

$$\begin{aligned}\mathcal{N}_{\text{GSM-MLD}} &= \underbrace{\sqrt{K}\cdot\mathcal{N}(\lambda_m)}_{m\text{-th layer}}+\sum_{i=1}^{m-1}\Big[\underbrace{M\cdot\mathcal{N}(\tilde{s}_i)}_{\text{Survived Branches}}+\underbrace{M\sqrt{K}\cdot\mathcal{N}(\lambda_i,\Lambda_i)}_{\text{Path Expansion}}\Big]\\ &=3\sqrt{K}+\sum_{i=1}^{m-1}\left[M\cdot 2(m-i)+M\sqrt{K}\cdot 4\right]\\ &=3\sqrt{K}+M(m^2-m)+4M\sqrt{K}(m-1)\end{aligned}\qquad(16)$$

IV. Proposed Adaptive Tree Search Scheme in GSM-MLD

In this section, we propose an adaptive tree search scheme in the GSM-MLD. The proposed algorithm retains the same breadth of the tree search as the GSM-MLD to achieve the near-ML performance. On the other hand, we perform adaptive tree search scheme to reduce the complexity, and to overcome the drawback which the fixed number of tree search algorithm requires high complexity in the high E_b/N_0 region. In the adaptive tree search scheme, we introduce a path metric ratio without the necessary to accurately and dynamically measure SNR. According to the reliability of each survived branch, assign a suitable candidates expansion from a parent node. To decrease the number of lower reliable possible branch, thereby avoid a large amount of the path metric evaluations and sorting.

A. Reliability Evaluation

In this subsection, we derive the reliability evaluation (RE) for all the survived branches in each layer. As above mentioned, the estimate of entry s_i depends on the decided estimates $\hat{s}_j$'s where $j\in[i+1,m]$. Hence, the wrong estimate existing in the decided estimates may cause more wrong estimates of the transmitted signal in the following recursion detection. According to MLD, the final estimate of transmitted signal is determined by the path with the smallest path metric. To a certain degree, we can apply the PED to evaluate the reliability of all the survived paths in a detection layer of the tree search. In that sense, we introduce a ratio function among the path metrics in the i-th layer, where $i\in[1,m]$, defined as

$$\beta_i(l)=\frac{\Lambda_i^{(l)}}{\Lambda_i^{(1)}},\, l\in[1,M] \qquad (17)$$

where $\Lambda_i^{(1)}$ denotes the smallest path metric after sorting the survived branch in the i-th layer. Note that the layer number i is decreased such that $i:=m$ down to 1 successively. In general, the survived path $\Pi_i^{(1)}$ with the high probability should be the correct path if the channel is better-conditioned. Hence, we assume that the survived path $\Pi_i^{(1)}$ has the most possible to be correct path. In terms of the path metric ratio $\beta_i(l)$ in (17), indirectly evaluate the reliability of the l-th branch in the i-th layer. That is, if $\Lambda_i^{(l)}$ is much larger than $\Lambda_i^{(1)}$ and thus $\beta_i(l)$ is larger, it illustrates that the correct path with lower reliability is the l-th path.

The ratio function $\beta_i(l)$ can be the measure of evaluating the reliability for the l-th branch. In order to adaptively control the candidates expansion according to $\beta_i(l)$, we assume that the number of the candidates should be a integer between 1 and $\sqrt{K}$ in the $(i-1)$-th layer. That means the number of candidates from a parent node is determined by the path metric ratio in the previous layer. We define the number of the candidates as $\rho_{i-1}(l)$ for the l-th survived branch in the $(i-1)$-th layer. In order to find a proper rule to adaptively assign the candidates for a survived branch, we consider a decision function of the i-th layer as

$$\alpha(i)=\frac{i}{m}\cdot C,\, i\in[1,m] \qquad (18)$$

where C is a constant to be predetermined, which is the tradeoff between the BER performance and the computational complexity. The parameter $\alpha(i)$ is depended on the detection layer i. Since the tree search starts with the last entry of $\mathbf{s}$, the path metric at first in the larger numbered layer is insufficient to reflect the whole channel condition. To retain the correct path, the parameter $\alpha(i)$ is defined to be proportional to the value of the detection layer i. Using the variable decision function, the value $\alpha(m)$ is maximum as C. Correspondingly, the value $\alpha(1)$ is minimum as C/m. The decision value becomes strict as the detected layers increase. The various decision based on the layer number significantly reduces the number of candidates in the smaller numbered layer seen in the Section V.

For K-QAM, the number of the finite set for the real-valued transmitted signals is $\sqrt{K}$ .We compare $\beta_i(l)$ with $\{\alpha(i),\ 2\alpha(i),\ \ldots,\ (\sqrt{K}-1)\cdot\alpha(i)\}$. Then we have

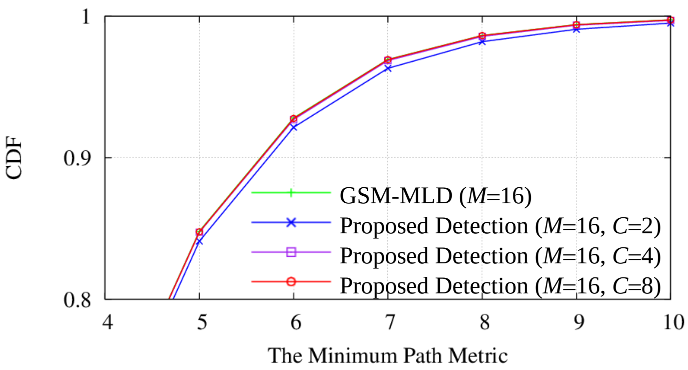

Fig. 1 The CDF of the minimum path metric at E_b/N_0=10dB for 16QAM.

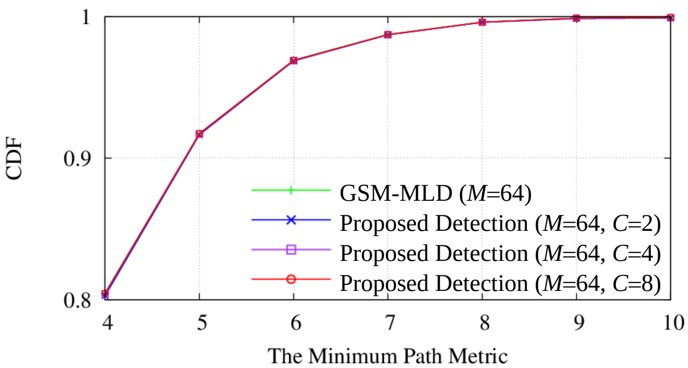

Fig. 2 The CDF of the minimum path metric at E_b/N_0=15dB for 64QAM.

$$\rho_{i-1}(l) = \begin{cases} \sqrt{K} & \text{for } \beta_i(l) \in [0, \alpha(i)] \\ \sqrt{K} - x & \text{for } \beta_i(l) \in (x \cdot \alpha(i), (x+1) \cdot \alpha(i)] \\ 1 & \text{for } \beta_i(l) > (\sqrt{K} - 1) \cdot \alpha(i) \end{cases} \quad (19)$$

where $i \in [2,m]$ and $x \in [1, \sqrt{K} - 2]$. Let $\alpha(i)$ denote the basic unit to divide $\beta_i(l)$ into $\sqrt{K}$ regions. Then, according to $\beta_i(l)$ in which region resolves the number of candidates $\rho_{i-1}(l)$. Ranking the constellation points with the nearest distance to $\tilde{s}_{i-1}^{(l)}$ obtained in (14), the candidates in the $(i-1)$-th layer consist of the nearest constellation point up to the $\rho_{i-1}(l)$-th nearest constellation point. In the case of 16QAM, if $\tilde{s}_{i-1}^{(l)}$=2.5, the order of candidates is {3,1,–1,–3}. If $\rho_{i-1}(l)$=2, the candidate selection from the constellation points is {3, 1}.

Due to the definitions of the branch metric and the path metric in the GSM-MLD, the ED can be expressed as

$$\| \mathbf{y} - \mathbf{Hs} \|^2 = \sum_{i=1}^{m} \left[\| \hat{\mathbf{h}}_i \|^2 | \tilde{s}_i - \hat{s}_i |^2 \right] \quad (20)$$

The maximum likelihood detection is very simple to implement since the decision criterion depends on the ED. This detection scheme minimizes the probability of bit error when the transmitted messages are equally likely. Since the proposed detection expects to achieve the near-ML performance as GSM-MLD, we first investigate the cumulative distribution function (CDF) of the minimum path metric. In Figs. 1 and 2, we plot the CDF of the minimum path metric compared the GSM-MLD with the proposed detection with C={2, 4, 8} for 16QAM and 64QAM, respectively. The results illustrate that the CDF curve of the proposed detection closely approaches the that of GSM-MLD as the value of constant C increases. The constant C=4 is almost optimal value between the BER performance and the complexity.

B. *Proposed Detection Scheme*

As an example, Fig. 3 illustrates an adaptive tree search scheme from the i-th layer to the $(i-1)$-th layer. In the $(i-1)$-th layer, first perform the path expansion from M survived branches in the i-th layer. Since the adaptive tree search scheme is executed, the branch metric and the path metric can be expressed as $\lambda_{i-1}^{(1,1)}, \ldots, \lambda_{i-1}^{(1,\rho_{i-1}(1))}, \lambda_{i-1}^{(2,1)}, \ldots, \lambda_{i-1}^{(2,\rho_{i-1}(2))}, \lambda_{i-1}^{(M,1)}, \ldots, \lambda_{i-1}^{(M,\rho_{i-1}(M))}$ and $\Lambda_{i-1}^{(1,1)}, \ldots, \Lambda_{i-1}^{(1,\rho_{i-1}(1))}, \Lambda_{i-1}^{(2,1)}, \ldots, \Lambda_{i-1}^{(2,\rho_{i-1}(2))}, \Lambda_{i-1}^{(M,1)}, \ldots, \Lambda_{i-1}^{(M,\rho_{i-1}(M))}$, respectively. Note that $\lambda_{i-1}^{(l,k)} : k \in [1, \rho_{i-1}(l)]$ represents the branch metric expanded from the l-th branch in the $(i-1)$-th layer. We calculate the path metric for the possible branches as $\Lambda_{i-1}^{(l,k)} = \lambda_{i-1}^{(l,k)} + \Lambda_i^{(l)}$. Hence, $\sum_{l=1}^{M} \rho_{i-1}(l)$ denotes the total number of all the children nodes in the $(i-1)$-th layer, which should be equal to or less than $M\sqrt{K}$. Next, sort $\sum_{l=1}^{M} \rho_{i-1}(l)$ path metrics and select M with the smallest path metric. Based on the sorted $\Lambda_{i-1}^{(l)}$, calculate the number of candidates expansion $\rho_{i-2}(l)$, $l \in [1,M]$, for the next layer. The proposed adaptive tree search scheme is summarized as follows:

Step 1: Set a fixed value of M. For K-QAM, if $\sqrt{K} < M$, define a layer number q such that $(\sqrt{K})^{m-q+1}$ should be equal to or more than M in order to select M branches with the smallest path metric among all of the possible branches. Then, the candidates from the m-th layer down to the q-th layer are all the constellation points.

Step 2: Start the adaptive candidate selection scheme from the q-th layer. According to $\beta_q(l)$ and $\alpha(q)$, the number of the candidates $\rho_{q-1}(l)$ for the l-th survived branch in the $(q-1)$-th layer is obtained in (19). Hence, the number of the possible branches in the $(q-1)$-th layer is from M to $M\sqrt{K}$.

Step 3: Proceed to the next stage of the $(q-1)$-th layer. Rank the constellation points for the l-th survived branches with the nearest distance to $\tilde{s}_{q-1}^{(l)}$ in (14). According to $\rho_{q-1}(l)$, we select the candidates from the constellation points and calculate the path metric for the possible branches. M branches are retained with the smallest path metric to the next layer. The same operations are executed until the first layer.

Step 4: Obtain the detection result of the estimate $\hat{\mathbf{s}}_1^{(1)} = [\hat{s}_1^{(1)}, \ldots, \hat{s}_m^{(1)}]^{\mathrm{T}}$.

C. *Complexity Analysis*

The proposed detection reduces the complexity of the path metric evaluations with less possible branches. The additional

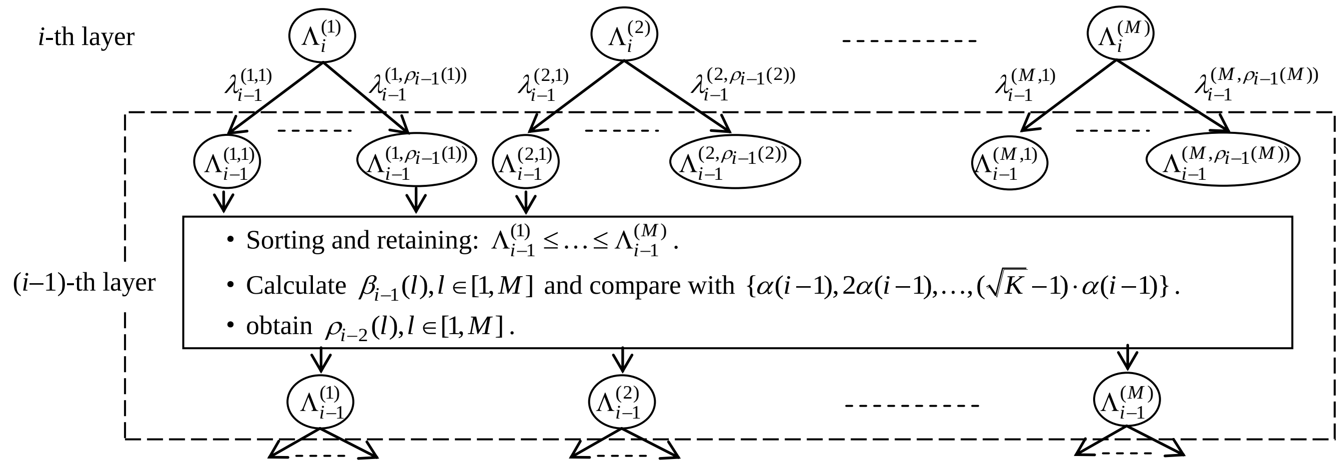

Fig. 3 Example of the adaptive tree search scheme from the *i*-th layer to the (*i*–1)-th layer.

complexity $\mathcal{N}_{\mathrm{A}}$ is the computations for the path metric ratio in (17), which require a complexity of (M–1)(q–1) flops. If we fix the value of the constant C, $\alpha(i)$ in (18) is predetermined. Hence, the computational complexity of $\alpha(i)$ is neglect. The complexity of the proposed detection consists of three parts: the fixed complexity from the m-th layer down to the q-th layer, the various complexity from the (q–1)-th layer down to the first layer, and the above additional complexity. The fixed complexity of the proposed detection $\mathcal{N}_{\mathrm{F}}$ can be derived as

$$\begin{aligned}\mathcal{N}_{\mathrm{F}} &= \sqrt{K}\cdot\mathcal{N}(\lambda_m) + \textstyle\sum_{i=q}^{m-1}\begin{bmatrix}(\sqrt{K})^{m-i}\cdot\mathcal{N}(\tilde{s}_i) \\ +(\sqrt{K})^{m-i+1}\cdot\mathcal{N}(\lambda_i,\Lambda_i)\end{bmatrix} \\ &= 3\sqrt{K} + \textstyle\sum_{i=q}^{m-1}\left[2(m-i)\cdot(\sqrt{K})^{m-i} + 4\cdot(\sqrt{K})^{m-i+1}\right]\end{aligned} \quad (21)$$

The various complexity of the proposed detection $\mathcal{N}_{\mathrm{V}}$ is varied with the number of the children nodes, derived as

$$\begin{aligned}\mathcal{N}_{\mathrm{V}} &= \textstyle\sum_{i=1}^{q-1}\left\{M\cdot\mathcal{N}(\tilde{s}_i) + \left[\sum_{l=1}^{M}\rho_i(l)\right]\cdot\mathcal{N}(\lambda_i,\Lambda_i)\right\} \\ &= \textstyle\sum_{i=1}^{q-1}\left[2M(m-i) + 4\cdot\sum_{l=1}^{M}\rho_i(l)\right]\end{aligned} \quad (22)$$

where $\sum_{l=1}^{M}\rho_i(l)$ denotes the total number of the children nodes in the i-th layer.

As a result, the complexity of the proposed detection $\mathcal{N}_{\mathrm{Prop.}}$ which excludes the complexity of the GSO reduction and the computation of $\tilde{\mathbf{v}}$ in (11) can be derived as

$$\begin{aligned}\mathcal{N}_{\mathrm{Prop.}} &= \mathcal{N}_{\mathrm{A}} + \mathcal{N}_{\mathrm{V}} + \mathcal{N}_{\mathrm{F}} \\ &= (M-1)(q-1) + \textstyle\sum_{i=1}^{q-1}\left[2M(m-i) + 4\cdot\sum_{l=1}^{M}\rho_i(l)\right] \\ &\quad + 3\sqrt{K} + \textstyle\sum_{i=q}^{m-1}\left[2(m-i)\cdot(\sqrt{K})^{m-i} + 4\cdot(\sqrt{K})^{m-i+1}\right]\end{aligned} \quad (23)$$

V. NUMERICAL RESULTS

The computer simulations were carried out for 16QAM and 64QAM in the 4×4 MIMO system, respectively. We assume the channel is the typical flat Rayleigh fading. The performances of the different detection algorithms are measured by the BER characteristics and the complexity. The complexity of the tree search detection is determined by the amount of the path metric evaluations.

A. BER with Perfect CSI

Figs. 4 and 5 show the BER characteristics versus E_b/N_0 using the full MLD, the conventional QRM-MLD, the GSM-MLD and the proposed detection, respectively. The value of M in the proposed detection is the same as that in the QRM-MLD and the GSM-MLD, i.e. M=16 for 16QAM and M=64 for 64QAM, respectively. The constant C in the decision function is assigned as C={2, 4, 8}.

As seen in Fig. 4, we chose M=16, which is large enough for the 16QAM in the 4×4 MIMO system, and hence the BER curves of the GSM-MLD and the QRM-MLD totally achieve the ML performance. For the proposed detection, the BER curve with C=8 is almost equivalent to the BER characteristics of the GSM-MLD or the QRM-MLD. The proposed detection with C=2 has less possible branches in each layer, and hence the BER curve is about 1dB worse than the BER of the QRM-MLD at a BER of 10^{-5}.

The BER curves of the QRM-MLD and the GSM-MLD with M=64 for 64QAM are equivalent to the BER characteristics of the full MLD in Fig. 5. For the proposed detection, the BER curves with C={4,8} achieve a near-ML performance. The proposed detection with C=2 remarkably reduces the possible branches in each layer, and hence the BER curve is about 0.5dB worse than that of the QRM-MLD at a BER of 10^{-5}.

B. Computational Complexity

We evaluated the average number of possible branches in each layer for the proposed detection with C={2, 4, 8}, seen in Figs. 6 and 7. For the QRM-MLD or GSM-MLD, the number of the possible branches in each layer is fixed to 64 if M=16 for 16QAM and 512 if M=64 for 64QAM, respectively. In Fig. 6, the average number of the possible branches in the adaptive stage is varied within a certain range from 16 to 64 for 16QAM. In particular, for the curve with C=2 in Fig. 6(a), the average

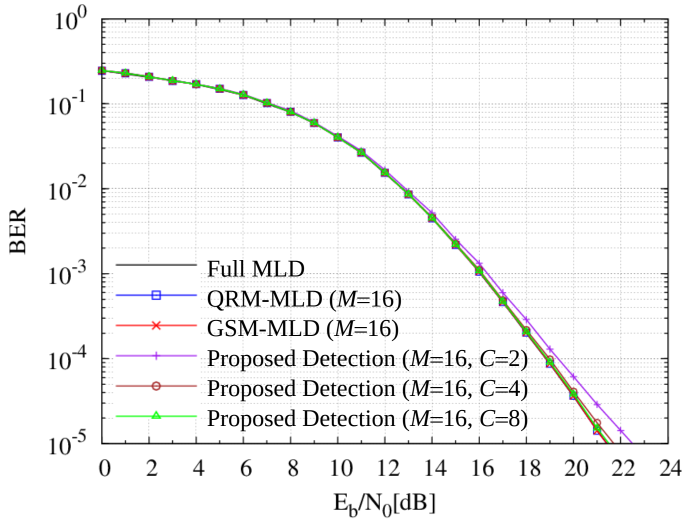

Fig. 4 The E_b/N_0 vs. BER characteristics: 16QAM and m=n=8.

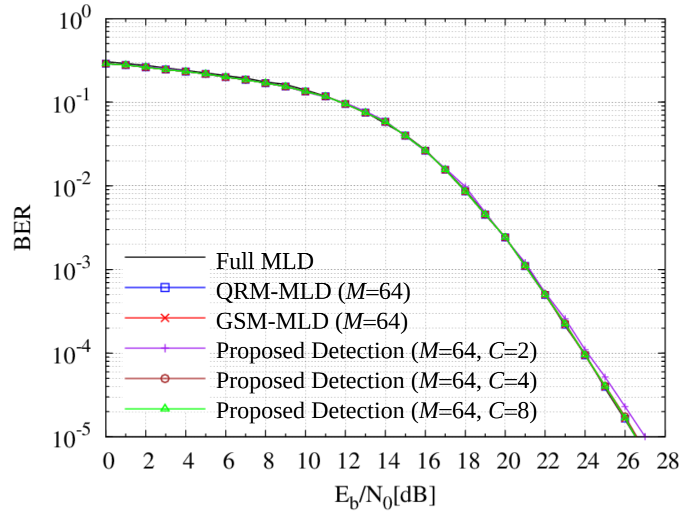

Fig. 5 The E_b/N_0 vs. BER characteristics: 64QAM and m=n=8.

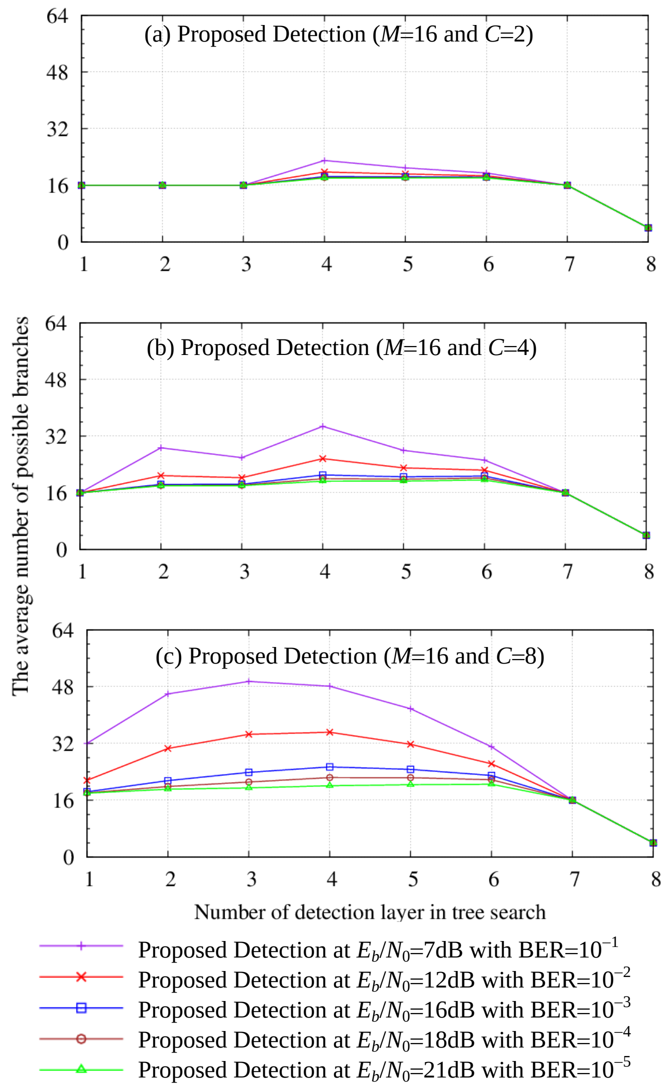

Fig. 6 The average number of possible branches in each layer in tree search at various E_b/N_0: 16QAM.

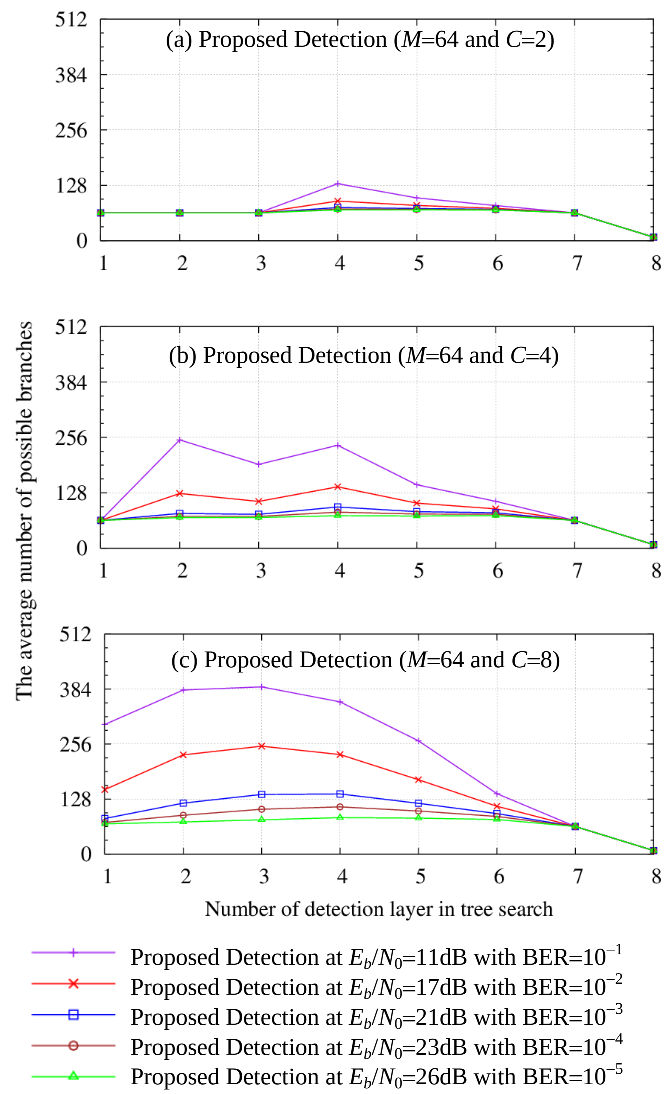

Fig. 7 The average number of possible branches in each layer in tree search at various E_b/N_0: 64QAM.

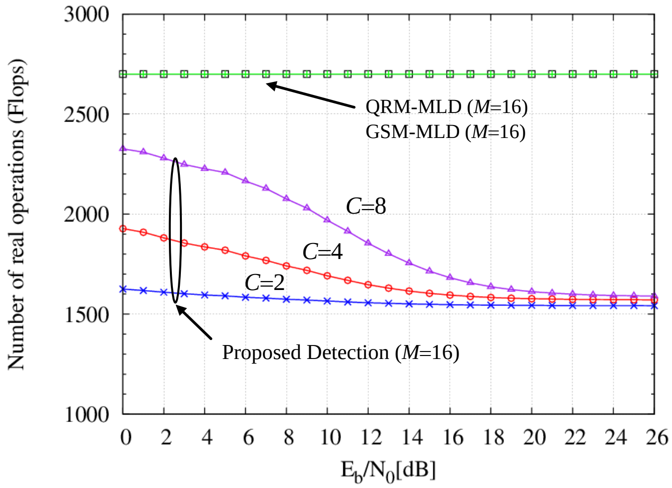

Fig. 8 The average complexity comparison for three detection schemes: 16QAM and m=n=8

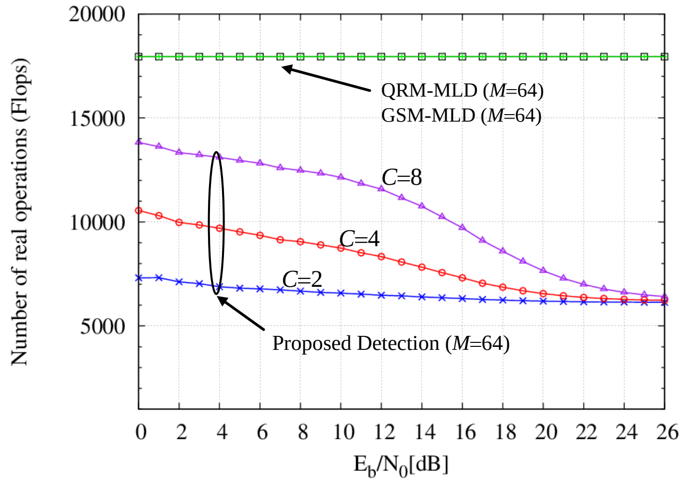

Fig. 9 The average complexity comparison for three detection schemes: 64QAM and m=n=8.

number of the possible branches is close to 16 if the BER characteristics are less than 10^{-2}. Furthermore, the BER curve of the proposed detection with C=2 is about 1dB worse than that of the full MLD at a BER of 10^{-5}. It should be noticed that the number of the low reliable possible branches in the proposed detection with C=4 in Fig. 6 (b) is halved or more reduced, compared to the fixed number of 64. The BER curve of the proposed detection with C=4 is about 0.2dB worse than that of the full MLD at a BER of 10^{-5}. In addition, the BER of the proposed detection with C=8 shown in Fig. 4 can retain the near-ML performance. The number of the possible branches can remarkably reduce in the high E_b/N_0 region.

Fig. 7 shows the average number of possible branches in each layer for the proposed detection for 64QAM. The average number of the possible branches in the adaptive stage is varied within a certain range from 64 to 512. Similar to 16QAM, the curves with C=2 in Fig. 7(a) are close to 64 if the BER characteristics are less than 10^{-2}, and correspondingly the BER curve with C=2 in Fig. 5 has about 0.5dB performance loss compared to the full MLD at a BER of 10^{-5}. If the channel is better-conditioned, the average numbers of the possible branches with C={4,8} in the adaptive stage are in the range from 64 to 128, which is much smaller than the fixed number of 512. Meanwhile, the BER curves with C={4,8} achieve a near-ML performance. From Figs. 6 and 7, the adaptive decision threshold in each detection layer is determined by the constant C.

According to the average number of possible branches, we present the computational complexity of the proposed detection in Figs. 8 and 9 for 16QAM and 64QAM, respectively. Due to M=16 for 16QAM and M=64 for 64QAM, the layer number q in (21)-(23) is set as q=m−1. We calculate the complexity of QRM-MLD excluding the complexity of QR-decomposition and the computation of $\mathbf{Q}^T\mathbf{y}$ in (5) [10]. From the numerical results, the GSM-MLD has the same complexity with the conventional QRM-MLD. Since the GSO reduction is computationally-simple and the transform matrix is with unity diagonal entries, the soft estimate of $\tilde{\mathbf{s}}$ is directly obtained in (14) with no division operation. It is convenient to rank the constellation points according to $\tilde{\mathbf{s}}$ in the adaptive stage. The adaptive tree search scheme is performed using the path metric ratio function, and thus the number of the possible branches in each layer of adaptive stage is remarkably reduced. Hence, the computational complexity of the proposed detection is much lower than the conventional QRM-MLD, especially in the high E_b/N_0 region. From Figs. 8 and 9, the complexity of the proposed detection at a BER of 10^{-5} is about 40% and 64% smaller than that of the QRM-MLD for 16QAM and 64QAM, respectively.

VI. Conclusions

In this paper, introducing the Gram-Schmidt Orthogonalization procedure to reduce the channel matrix, we proposed a MIMO detection scheme using the adaptive tree search with variable path expansion in the GSM-MLD algorithm. The adaptive tree search scheme is to adaptively control the candidates for each survived branch in the tree search. We adopted a path metric ratio function to evaluate the reliability for all the survived branches. To decrease the number of the low reliable candidates in each layer, a large amount of the computation for the path metric is avoided. Hence, the complexity of the proposed detection should be reduced. In particular in the high E_b/N_0 region, the complexity of the proposed detection is about 60% and 36% of that of the QRM-MLD for 16QAM and 64QAM, respectively. The proposed detection can provide the near-ML performance with relatively lower complexity. As a result, it is worthy for applying even to the high modulation order.

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