Optimal Power Allocation for Correlated Rayleigh Fading MIMO Wireless Systems with Statistic Information Feedback

Feng Li, *Member, IEEE*

***Abstract*—In telecommunication field, channel capacity is the tightest upper bound on the amount of information that can be reliably transmitted over a communications channel. In this paper, we investigate how to increase information throughput to approach the capacity of a wireless communications system using multiple-input multiple-output (MIMO) technique. When statistic information is available at the transmitter, we can improve the information throughput by optimizing the input covariance matrix of the transmitter. Efficient design of the optimal input covariance matrix, however, remains unavailable although its eigenvector structure was clearly revealed. In this paper, we obtain an upper bound of the capacity by matrix operation and concavity of the capacity function. Then by optimizing this upper bound using Lagrange multiplier method and matrix differentiation, we develop an efficient algorithm for determining the optimal eigenvalues, which denote the optimal power allocation for the transmitter. The technique is illustrated through numerical examples.**

***Index Terms*—Channel capacity, multiple-input-multipleoutput (MIMO) channel, covariance matrix, concavity, Cramer rule.**

I. Introduction

MULTIPLE-input multiple-output (MIMO) technology is an antenna technology for wireless communications in which multiple antennas are used at both the source (transmitter) and the destination (receiver) [1]–[6]. The antennas at each end of the communications circuit are combined to minimize errors and optimize data speed. MIMO is one of several forms of smart antenna technology, the others being MISO (multiple input, single output) and SIMO(single input, multiple output). In conventional wireless communications, a single antenna is used at the source, and another single antenna is used at the destination. In some cases, this gives rise to problems with multipath effects. When an electromagnetic field (EM field) is met with obstructions such as hills, canyons, buildings, and utility wires, the wavefronts are scattered, and thus they take many paths to reach the destination. The late arrival of scattered portions of the signal causes problems such as fading, cut-out (cliff effect), and intermittent reception (picket fencing). In digital communications systems such as wireless Internet, it can cause a reduction in data speed and an increase in the number of errors. The use of two or more antennas, along with the transmission of multiple signals (one for each antenna) at the source and the destination, eliminates the trouble caused by multipath wave propagation, and can even take advantage of this effect [7]–[11].

MIMO technology has aroused interest because of its possible applications in digital television (DTV), wireless local area networks (WLANs), metropolitan area networks (MANs), and mobile communications [12]–[17]. MIMO systems are a natural extension of developments in antenna array communication. While the advantages of multiple receive antennas, such as gain and spatial diversity, have been known and exploited for some time [18, 19, 20], the use of transmit diversity has only been investigated recently [21, 22]. The advantages of MIMO communication, which exploits the physical channel between many transmit and receive antennas, are currently receiving significant attention [23-26]. While the channel can be so nonstationary that it cannot be estimated in any useful sense [27], in this article we assume the channel is quasistatic. MIMO systems provide a number of advantages over single antenna to single antenna communication. Sensitivity to fading is reduced by the spatial diversity provided by multiple spatial paths. Under certain environmental conditions, the power requirements associated with high spectral efficiency communication can be significantly reduced by avoiding the compressive region of the information-theoretic capacity bound. Here, spectral efficiency is defined as the total number of information bits per second per Hertz transmitted from one array to the other.

Appropriately exploiting partial channel knowledge at the transmitter can always increase the information throughput of a wireless multiple-antenna system with either a multi-input multi-output (MIMO) [21], [22] or a multi-input single-output (MISO) [18]–[20] configuration. [19], [29], [30] have discussed how to utilize the channel knowledge of MIMO system in correlated Rayleigh fading to obtain the optimal input covariance matrix for which the channel capacity is maximized. They find that the optimal input covariance matrix should have the same eigenvectors as the transmit covariance matrix, which means that independent Gaussian signals be transmitted along the direction defined by the eigenvectors of the transmit covariance matrix.

Manuscript received on July 7, 2011. The author is with the Department of Operation Research and Information Engineering, Cornell University, Ithaca, NY, USA; e-mail: fl263@cornell.edu

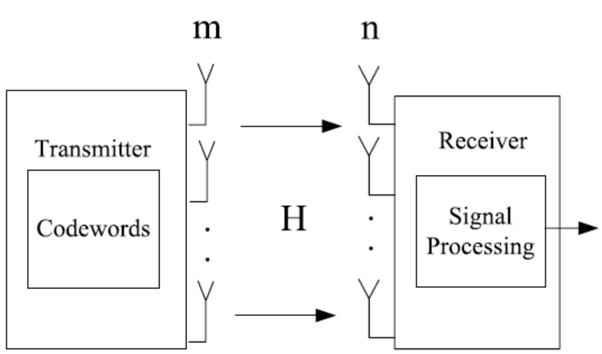

Fig. 1. $m \times n$ MIMO systems

Till now, however, the determination of the eigenvalues for the optimal input covariance matrix in MIMO correlated Rayleigh fading channels still relies on the exhaustive search over all the whole valid domain, directly based on the original objective function for optimization except the insertion of the optimal eigenvectors. Although accurate, this method is extremely complicated and time consuming, thereby calling for more feasible theoretic results to be used in the practical system design. Note that we have already found a method to find the optimal optimal input covariance matrix for multiple input single output(MISO) system [31]. Unfortunately, the proposed method cannot work for MIMO system. In this paper, we derive a new algorithm that works for MIMO system.

The rest of this paper is organized as follows. In Section II, we present our system model. In Section III, we derive a upper bound of the capacity used to determine the optimal power allocation. In Section IV, we present the detailed procedure of our proposed algorithm to determine the structure of the optimum input covariance matrix. In Section V, some numerical results are given illustrating the efficiency of our algorithm. Finally, Section VI contains some concluding remarks.

As a convention in this paper, we will use superscript $\dagger$ to signify conjugate transposition, and use $E[\cdot]$, diag$\{\cdots\}$ and tr$(\cdot)$ to denote expectation, the diagonal matrix and the trace of a matrix, respectively. The notation $\mathbf{x} \sim CN_m(\mu, \mathbf{R})$ implies that the m-by-1 vector $\mathbf{x}$ is complex Gaussian distributed with mean μ and covariance matrix $\mathbf{R}$. For the case of $m = 1$, the subscript m will be dropped for simplicity.

II. FORMUNATION

We assume that a wireless MIMO system has m transmit antennas and n receive antenna as shown in Figure 1. $\mathbf{x}$ and $\mathbf{H}$ denote the $m \times 1$ transmitted signal vector and the $n \times m$ channel gains linking the transmit antennas to the receiver, respectively, The received signal $\mathbf{y}$ can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}. \tag{1}$$

where $\mathbf{n}$ is an $n \times 1$ vector of additive white Gaussian noise (AWGN), i.e.,

$$n_i \sim CN(0, \sigma_n^2), i = 1, \ldots, n \tag{2}$$

For Rayleigh fading, we assume the channel matrix H has both transmit and receive correlation and it can be modeled as

$$\mathbf{H} = \mathbf{R}_R^{1/2} \cdot \mathbf{W} \cdot \mathbf{R}_T^{1/2}. \tag{3}$$

with receive correlation matrix $\mathbf{R}_R$ and transmit correlation matrix $\mathbf{R}_T$. $\mathbf{W}$ has i.i.d. complex Gaussian distributed entries w_{ij} with zero mean and variance one.

We assume that partial channel information is fed back to the receiver. So the receiver knows the Gaussianity of $\mathbf{H}$ with zero mean and transmit covariance matrix $\mathbf{R}_T$ and receive covariance matrix $\mathbf{R}_R$. According to Shannon, the optimal distribution of $\mathbf{x}$ that maximizes the channel capacity is the joint Gaussian distribution taking the form of

$$\mathbf{x} \sim CN_m(\mathbf{0}, \mathbf{Q}) \tag{4}$$

The question is for given partial channel information $\mathbf{R}_T$ and $\mathbf{R}_R$ at the receiver, how to determine the optimal covariance structure $\mathbf{Q}$ subject to the constraint of a constant transmitted power, i.e.,

$$\text{tr}\{\mathbf{Q}\} = P \tag{5}$$

such that

$$C = E[\log\det(\mathbf{I} + \frac{1}{\sigma_n^2}\mathbf{H}\mathbf{Q}\mathbf{H}^\dagger)]. \tag{6}$$

is maximized. The authors of [19] have found that the optimal $\mathbf{Q}$ should have the same unitary matrix of eigenvectors as that of $\mathbf{R}_T$. Efficient techniques, however, are not available for determining the eigenvalues of the optimal $\mathbf{Q}$ except for time consuming exhaustive search.

In this paper, we investigate how to optimize the optimal eigenvalues such that the best throughput can be achieved. Our main idea is that we first derive the upper bound of the channel capacity. Then by maximizing this bound instead of the real capacity, a group of eigenvalues of input covariance matrix is obtained and serves as the optimal power allocation. So the complete structure of the optimal input covariance matrix can be determined

III. UPPER BOUND OF THE CAPACITY

Since we have known the optimal eigenvector of the input covariance matrix, we remove its effect and remain the effect of eigenvalues of input covariance matrix only. First, we assume the eigenvalue decomposition of the input covariance, transmit correlation matrix, receive correlation matrix be

$$\mathbf{Q} = \mathbf{U}_Q\mathbf{\Lambda_Q}\mathbf{U}_\mathbf{Q}^\dagger = \mathbf{U_Q}\mathbf{diag}(\beta_\mathbf{1}, \ldots, \beta_\mathbf{m})\mathbf{U}_\mathbf{Q}^\dagger. \tag{7}$$

$$\mathbf{R}_T = \mathbf{U}_T\mathbf{\Lambda_T}\mathbf{U}_\mathbf{T}^\dagger = \mathbf{U_T}\mathbf{diag}(\alpha_\mathbf{1}^\mathbf{T}, \ldots, \alpha_\mathbf{m}^\mathbf{T})\mathbf{U}_\mathbf{T}^\dagger. \tag{8}$$

$$\mathbf{R}_R = \mathbf{U}_R\mathbf{\Lambda_R}\mathbf{U}_\mathbf{R}^\dagger = \mathbf{U_R}\mathbf{diag}(\alpha_\mathbf{1}^\mathbf{R}, \ldots, \alpha_\mathbf{n}^\mathbf{R})\mathbf{U}_\mathbf{R}^\dagger. \tag{9}$$

where $\mathbf{U}_Q$, $\mathbf{U}_T$, $\mathbf{U}_R$ are all unitary matrices. Substitute (3), (7), (8), (9) into (6), we have

$$\begin{aligned} C &= E[\log\det(\mathbf{I} + \frac{1}{\sigma_n^2}\mathbf{H}\mathbf{Q}\mathbf{H}^\dagger)] \\ &= E[\log\det(\mathbf{I} + \frac{1}{\sigma_n^2}\mathbf{U}_R\mathbf{\Lambda}_\mathbf{R}^{1/2}\mathbf{U}_\mathbf{R}^\dagger\mathbf{W}\mathbf{U_T}\mathbf{\Lambda}_\mathbf{T}^{1/2}\mathbf{U}_\mathbf{T}^\dagger \\ &\qquad \mathbf{U}_Q\mathbf{\Lambda_Q}\mathbf{U}_\mathbf{Q}^\dagger\mathbf{U_T}\mathbf{\Lambda}_\mathbf{T}^{1/2}\mathbf{U}_\mathbf{T}^\dagger\mathbf{W}^\dagger\mathbf{U_R}\mathbf{\Lambda}_\mathbf{R}^{1/2}\mathbf{U}_\mathbf{R}^\dagger)] \end{aligned} \tag{10}$$

Since unitary matrices $\mathbf{U}_T$, $\mathbf{U}_R$ do not change the distribution of $\mathbf{W}$, (10) can be rewritten as

$$C = E[\log\det(\mathbf{I} + \frac{1}{\sigma_n^2}\mathbf{U}_R\mathbf{\Lambda}_\mathbf{R}^{1/2}\mathbf{W}\mathbf{\Lambda}_\mathbf{T}^{1/2}\mathbf{U}_\mathbf{T}^\dagger \mathbf{U}_Q\mathbf{\Lambda}_\mathbf{Q}\mathbf{U}_\mathbf{Q}^\dagger\mathbf{U}_\mathbf{T}\mathbf{\Lambda}_\mathbf{T}^{1/2}\mathbf{W}^\dagger\mathbf{\Lambda}_\mathbf{R}^{1/2}\mathbf{U}_\mathbf{R}^\dagger)]. \quad (11)$$

By [19],

$$\mathbf{U}_Q = \mathbf{U}_T \quad (12)$$

so (11) becomes

$$E[\log\det(\mathbf{I} + \frac{1}{\sigma_n^2}\mathbf{U}_R\mathbf{\Lambda}_\mathbf{R}^{1/2}\mathbf{W}\mathbf{\Lambda}_\mathbf{T}\mathbf{\Lambda}_\mathbf{Q}\mathbf{W}^\dagger\mathbf{\Lambda}_\mathbf{R}^{1/2}\mathbf{U}_\mathbf{R}^\dagger)]. \quad (13)$$

For arbitrary compatible matrices $\mathbf{A}$ and $\mathbf{B}$, it can be shown that

$$\det(\mathbf{I} + \mathbf{AB}) = \det(\mathbf{I} + \mathbf{BA}). \quad (14)$$

By (14), (13) can be rewritten as

$$C = E[\log\det(\mathbf{I} + \frac{1}{\sigma_n^2}\mathbf{W}^\dagger\mathbf{\Lambda}_\mathbf{R}\mathbf{W}\mathbf{\Lambda}_\mathbf{T}\mathbf{\Lambda}_\mathbf{Q})]. \quad (15)$$

In (15), the effect of eigenvector of input covariance matrix has been eliminated and the capacity is only the function of eigenvalues of input covariance matrix.

Directly optimizing (15) to get the optimal eigenvalues of input covariance matrix is so complicated that no close form solution exists or the solution is too lengthy to be used in practice. So we propose to optimize an upper bound of the capacity to get the optimal eigenvalues.

Note that the *log* function is concave in its definition domain, so

$$E[\log\det(\mathbf{I} + \frac{1}{\sigma_n^2}\mathbf{W}^\dagger\mathbf{\Lambda}_\mathbf{R}\mathbf{W}\mathbf{\Lambda}_\mathbf{T}\mathbf{\Lambda}_\mathbf{Q})] \leq \log E[\det(\mathbf{I} + \frac{1}{\sigma_n^2}\mathbf{W}^\dagger\mathbf{\Lambda}_\mathbf{R}\mathbf{W}\mathbf{\Lambda}_\mathbf{T}\mathbf{\Lambda}_\mathbf{Q})] \quad (16)$$

And for any matrix $\mathbf{A}$, $\mathbf{A} \rightarrow \det(\mathbf{A})$ is a concave function. So we have

$$E[\det(\mathbf{I} + \frac{1}{\sigma_n^2}\mathbf{W}^\dagger\mathbf{\Lambda}_\mathbf{R}\mathbf{W}\mathbf{\Lambda}_\mathbf{T}\mathbf{\Lambda}_\mathbf{Q})] \leq \det E[\mathbf{I} + \frac{1}{\sigma_n^2}\mathbf{W}^\dagger\mathbf{\Lambda}_\mathbf{R}\mathbf{W}\mathbf{\Lambda}_\mathbf{T}\mathbf{\Lambda}_\mathbf{Q}]. \quad (17)$$

In the right part of the inequality (17)

$$E[\mathbf{I} + \frac{1}{\sigma_n^2}\mathbf{W}^\dagger\mathbf{\Lambda}_\mathbf{R}\mathbf{W}\mathbf{\Lambda}_\mathbf{T}\mathbf{\Lambda}_\mathbf{Q}] = \mathbf{I} + \frac{\mathbf{1}}{\sigma_\mathbf{n}^2}\mathbf{E}[\mathbf{W}^\dagger\mathbf{\Lambda}_\mathbf{R}\mathbf{W}]\mathbf{\Lambda}_\mathbf{T}\mathbf{\Lambda}_\mathbf{Q}. \quad (18)$$

The expectation

$$\begin{aligned} E[\mathbf{W}^\dagger\mathbf{\Lambda}_\mathbf{R}\mathbf{W}] &= E[\sum_{k=1}^{n}\alpha_k^R\mathbf{w}_k\mathbf{w}_k^\dagger] \\ &= \frac{1}{n}\sum_{k=1}^{n}\alpha_k^R \cdot \mathbf{I}. \end{aligned} \quad (19)$$

where $\mathbf{w}_k$ is the kth column of matrix $\mathbf{W}$. Substitute (19) to (18), we have

$$E[\mathbf{I} + \frac{1}{\sigma_n^2}\mathbf{W}^\dagger\mathbf{\Lambda}_\mathbf{R}\mathbf{W}\mathbf{\Lambda}_\mathbf{T}\mathbf{\Lambda}_\mathbf{Q}] = \mathbf{I} + \frac{\mathbf{1}}{\mathbf{n}\sigma_\mathbf{n}^2}\sum_{\mathbf{k=1}}^{\mathbf{n}}\alpha_\mathbf{k}^\mathbf{R}\mathbf{\Lambda}_\mathbf{T}\mathbf{\Lambda}_\mathbf{Q}. \quad (20)$$

Combining (16), (17), (20) leads to

$$E[\log\det(\mathbf{I} + \frac{1}{\sigma_n^2}\mathbf{W}^\dagger\mathbf{\Lambda}_\mathbf{R}\mathbf{W}\mathbf{\Lambda}_\mathbf{T}\mathbf{\Lambda}_\mathbf{Q})] \leq \log\det(\mathbf{I} + \frac{1}{n\sigma_n^2}\sum_{k=1}^{n}\alpha_k^R\mathbf{\Lambda}_\mathbf{T}\mathbf{\Lambda}_\mathbf{Q}). \quad (21)$$

So we get the upper bound of channel capacity

$$C_{\text{upper}} = \log\det(\mathbf{I} + \frac{1}{n\sigma_n^2}\sum_{k=1}^{n}\alpha_k^R\mathbf{\Lambda}_\mathbf{T}\mathbf{\Lambda}_\mathbf{Q}). \quad (22)$$

We will optimize (22) to get the optimal eigenvalues of input covariance matrix.

IV. Optimal Power Allocation

We use the Lagrange multipliers to form a new objective function, as shown by

$$\begin{aligned} J(\mathbf{\Lambda}_\mathbf{Q}) &= \log|\mathbf{I} + \frac{1}{n\sigma_n^2}\sum_{k=1}^{n}\alpha_k^R\mathbf{\Lambda}_\mathbf{T}\mathbf{\Lambda}_\mathbf{Q}| + \lambda(\text{tr}\{\mathbf{\Lambda}_\mathbf{Q}\} - \mathbf{P}) \\ &= C_{\text{upper}} + \lambda(\text{tr}\{\mathbf{\Lambda}_\mathbf{Q}\} - \mathbf{P}). \end{aligned} \quad (23)$$

where λ is a constant. The task is to find $\mathbf{\Lambda}_\mathbf{Q}$ that maximizes this objective function.

To maximize $J(\mathbf{\Lambda}_\mathbf{Q})$, we take its derivative with respect to $\mathbf{\Lambda}_\mathbf{Q}$ yielding

$$\frac{dJ(\mathbf{\Lambda}_\mathbf{Q})}{d\mathbf{\Lambda}_\mathbf{Q}} = \frac{dC_{\text{upper}}}{d\mathbf{\Lambda}_\mathbf{Q}} + \lambda\frac{\text{tr}\{\mathbf{\Lambda}_\mathbf{Q}\}}{d\mathbf{\Lambda}_\mathbf{Q}}. \quad (24)$$

The second term on the right is simply equal to the identity matrix $\mathbf{I}$; namely,

$$\frac{d\,\text{tr}\{\mathbf{\Lambda}_\mathbf{Q}\}}{d\mathbf{\Lambda}_\mathbf{Q}} = \mathbf{I} \quad (25)$$

We therefore focus on the first term. Using the rule (10.17) for matrix differentiation [24], we obtain

$$\frac{dC_{\text{upper}}}{d\mathbf{\Lambda}_\mathbf{Q}} = \det(\mathbf{I} + \frac{1}{n\sigma_n^2}\sum_{k=1}^{n}\alpha_k^R\mathbf{\Lambda}_\mathbf{T}\mathbf{\Lambda}_\mathbf{Q})^{-1} \cdot \frac{d\det(\mathbf{I} + \frac{1}{n\sigma_n^2}\sum_{k=1}^{n}\alpha_k^R\mathbf{\Lambda}_\mathbf{T}\mathbf{\Lambda}_\mathbf{Q})}{d\mathbf{\Lambda}_\mathbf{Q}}. \quad (26)$$

After invoking the formula for matrix differentiation [24]

$$\frac{d\det(\mathbf{A}^n)}{d\mathbf{A}} = n\det(\mathbf{A}^n)(\mathbf{A}^{-1})^\dagger. \quad (27)$$

and simplifying, produces

$$\frac{dC_{\text{upper}}}{d\mathbf{\Lambda}_\mathbf{Q}} = \frac{1}{n\sigma_n^2}\sum_{k=1}^{n}\alpha_k^R\mathbf{\Lambda}_\mathbf{T} \cdot (\mathbf{I} + \frac{\mathbf{1}}{\mathbf{n}\sigma_\mathbf{n}^2}\sum_{\mathbf{k=1}}^{\mathbf{n}}\alpha_\mathbf{k}^\mathbf{R}\mathbf{\Lambda}_\mathbf{T}\mathbf{\Lambda}_\mathbf{Q})^{-\mathbf{1}}. \quad (28)$$

By inserting (28) into (24) and setting the derivative to zero, we obtain the simultaneous equations

$$\mathbf{A} + \lambda\mathbf{I} = \mathbf{0}. \quad (29)$$

where $\mathbf{A}$ is a matrix function of $\mathbf{\Lambda}_\mathbf{Q}$ defined by

$$\mathbf{A} = \frac{1}{n\sigma_n^2}\sum_{k=1}^{n}\alpha_k^R\mathbf{\Lambda}_\mathbf{T} \cdot (\mathbf{I} + \frac{\mathbf{1}}{\mathbf{n}\sigma_\mathbf{n}^2}\sum_{\mathbf{k=1}}^{\mathbf{n}}\alpha_\mathbf{k}^\mathbf{R}\mathbf{\Lambda}_\mathbf{T}\mathbf{\Lambda}_\mathbf{Q})^{-\mathbf{1}}. \quad (30)$$

The solution to (29) defines the optimal $\mathbf{\Lambda}_\mathbf{Q}$.

From (29), it follows that the optimal $\mathbf{\Lambda}_\mathbf{Q}$ must be chosen such that $\mathbf{A}$ is, up to a factor, the identity matrix. This requires, in turn, that all the eigenvalues of $\mathbf{A}$ be identical. The optimal $\mathbf{\Lambda}_\mathbf{Q}$ that meets this requirement implies that

$$a_1 = a_2 = \cdots = a_m. \quad (31)$$

where the eigenvalues a_i of $\mathbf{A}$ are given by

$$a_i = \frac{1}{n\sigma_n^2}\sum_{k=1}^{n}\alpha_k^R\alpha_i^T \cdot (1+\frac{1}{n\sigma_n^2}\sum_{k=1}^{n}\alpha_k^R\alpha_i^T\beta_i)^{-1} \quad i=1,\cdots,m. \tag{32}$$

So the relation among $\{a_i\}$, plus the power constraint, defines the following simultaneous equations

$$\begin{aligned} &a_1 - a_2 = 0 \\ &a_1 - a_3 = 0 \\ &\vdots \\ &a_1 - a_m = 0 \\ &\beta_1 + \cdots \beta_m - P = 0. \end{aligned} \tag{33}$$

which is a set of nonlinear equations in the eigenvalues $\{\beta_i\}$ of the optimal $\mathbf{\Lambda_Q}$.

Substitute (32) into (34), after simplifying, (34) becomes

$$\begin{aligned} &\beta_1 - \beta_2 = (\frac{1}{n\sigma_n^2}\sum_{k=1}^{n}\alpha_k^R)^{-1}(\frac{1}{\alpha_2^T} - \frac{1}{\alpha_1^T}) \\ &\beta_1 - \beta_3 = (\frac{1}{n\sigma_n^2}\sum_{k=1}^{n}\alpha_k^R)^{-1}(\frac{1}{\alpha_3^T} - \frac{1}{\alpha_1^T}) \\ &\vdots \\ &\beta_1 - \beta_m = (\frac{1}{n\sigma_n^2}\sum_{k=1}^{n}\alpha_k^R)^{-1}(\frac{1}{\alpha_m^T} - \frac{1}{\alpha_1^T}) \\ &\beta_1 + \cdots \beta_m - P = 0. \end{aligned} \tag{34}$$

We use Cramer rule to solve equations (32). Define D and $\mathbf{G}$ as the coefficient determinant and constant matrix, i.e.,

$$D = \det \begin{pmatrix} 1 & -1 & 0 & \cdots & 0 \\ 1 & 0 & -1 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}. \tag{35}$$

$$\mathbf{G} = \begin{pmatrix} (\frac{1}{n\sigma_n^2}\sum_{k=1}^{n}\alpha_k^R)^{-1}(\frac{1}{\alpha_2^T} - \frac{1}{\alpha_1^T}) \\ (\frac{1}{n\sigma_n^2}\sum_{k=1}^{n}\alpha_k^R)^{-1}(\frac{1}{\alpha_3^T} - \frac{1}{\alpha_1^T}) \\ \vdots \\ (\frac{1}{n\sigma_n^2}\sum_{k=1}^{n}\alpha_k^R)^{-1}(\frac{1}{\alpha_m^T} - \frac{1}{\alpha_1^T}) \\ P \end{pmatrix}. \tag{36}$$

Since $D \neq 0$, equations (34) have sole solution, which can be expressed by

$$\beta_i = \frac{D_i}{D}, \quad i = 1, 2, \ldots, m. \tag{37}$$

where D_i is the determinant when the i th column of D is replaced by constant matrix G. To this end, the eigenvalues of input covariance matrix maximizing the upper bound (22) of the capacity are derived. Numerical results shows that the optimal eigenvalues we derived are very close to the Monte Carlo results.

Let us summarize the procedure for the determination of $\mathbf{\Lambda_Q}$.

a) Eigen decompose $\mathbf{R}_T$ to obtain its matrix of eigenvector $\mathbf{U}_T$ and eigenvalues $\{\alpha_T^1 \cdots, \alpha_T^m\}$.

b) Eigen decompose $\mathbf{R}_R$ to obtain its eigenvalues

c) Calculate D and $\mathbf{G}$ using the values of $\{\alpha_T^i\}$ and $\{\alpha_R^i\}$

d) Substitute D and $\mathbf{G}$ into (37).

When the channel matrix $\mathbf{H}$ has only transmit correlation $\mathbf{R}_T$, D remains (35), but $\mathbf{G}$ becomes and the procedure is simplified to a), c), d).

Once we obtain the optimal eigenvalues, we can use them, along with the optimal eigenvectors to determine $\mathbf{Q}$ and maximum average capacity using (6).

V. Numerical results

We examine the optimization technique developed above through some numerical examples. We assume the channel matrix $\mathbf{H}$ has only transmit correlation and the correlation matrix is normalized with respect to tr($\mathbf{R}_T$) = m. Set $\sigma_n^2 = 1$ and $P = 10$, so that $SNR = 10\log P/\sigma_n^2 = 10$dB.

Let us first consider the case with 2 × 2 MIMO systems. To see the relative error between the capacity derived by our algorithm and Monte Carlo simulation, we assign different values to the correlation factor ρ_{ij} and use the optimization technique to determine the optimal $\{\beta_i\}$ and corresponding capacity, ending up with results tabulated in Table I.

We next apply the optimization procedure to the case of 3×3 MIMO systems with different correlation. Here we assume the correlation factors between different antennas are the same. The results are summarized in Table II.

TABLE I

Capacity derived by our algorithm for 2 × 2 MIMO systems

correlation	our capacity	Monte Carlo capacity	relative error
0.1	2.6583	2.7138	0.0204
0.3	2.5998	2.6900	0.0335
0.5	2.5651	2.6143	0.0188
0.7	2.4963	2.5845	0.0341
0.9	2.4753	2.5689	0.0364

TABLE II

Capacity derived by our algorithm for 3 × 3 MIMO systems

correlation	our capacity	Monte Carlo capacity	relative error
0.1	3.3201	3.3387	0.0056
0.3	3.2451	3.2718	0.0082
0.5	3.0295	3.1371	0.0343
0.7	2.9634	3.0532	0.0294
0.9	2.8756	2.9060	0.0105

In Table I and II, our capacity is the theoretical channel capacity derived by the proposed optimization algorithm. The Monte Carlo capacity is the simulation capacity obtained through the Monte Carlo method, based on (6). In simulation, we assume that all β_i are unknown, and thus, for each SNR, we tried each β_i over the range from 0 to P with step increment equal to 0.01P. The value of $E[C]$ is obtained by averaging over 1000 independent computer trials. The maximum $E[C]$ over all possible $\{\beta_i\}$ is the simulation capacity we need. From the tables, you can see the relative capacity error with our algorithm is very small, which illustrates the efficiency and accuracy of our proposed algorithm for the optimal eigenvalues.

To further demonstrate the efficiency of our algorithm, we plot the curve of channel capacity with respect to different SNR. Here we set the correlation is 0.1. The results are shown in

Figure 2 and 3 where excellent agreement between the theoretical capacity and its simulation counterpart is observed verifying the validity of our algorithm.

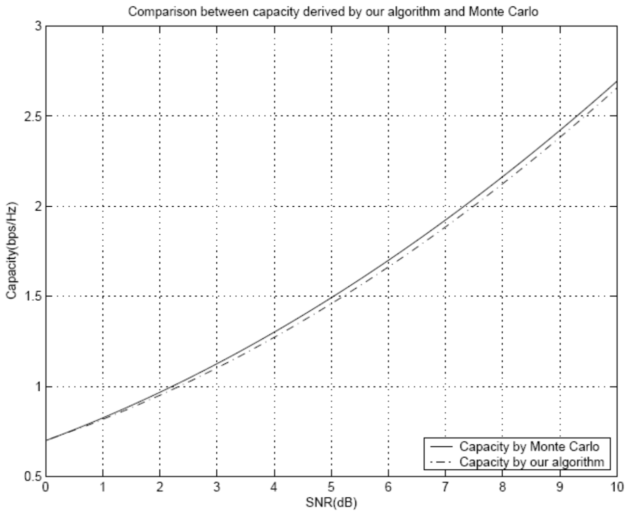

Fig. 2. Channel capacity derived by our algorithm in 2×2 MIMO systems

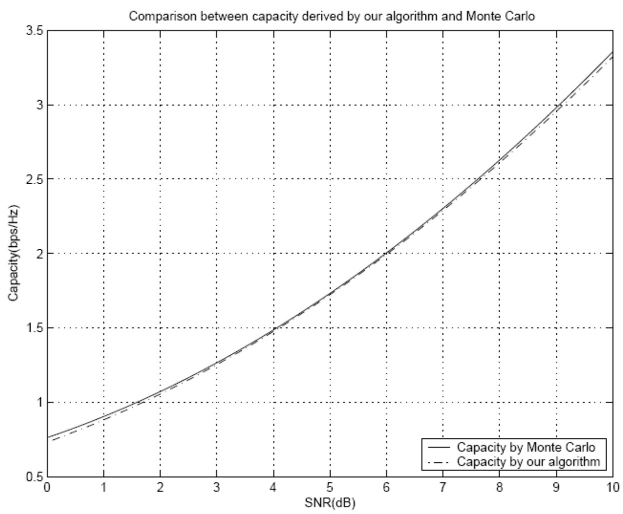

Fig. 3. Channel capacity derived by our algorithm in 3×3 MIMO systems

VI. CONCLUSIONS

In this paper, we investigate how to increase the channel throughput to approach the channel capacity. We propose an efficient algorithm to determine the optimal power allocation for MIMO correlated Rayleigh fading channels, such that the throughput is close to the channel capacity. Instead of the original channel capacity, we first derive an upper bound of the capacity by matrix operation and concavity of the objective function. Then by Lagrange multiplier method and matrix differentiation, we obtain a series of equations which the eigenvalues of optimal input covariance matrix must satisfy. Finally, using Cramer rule, we get the solution to these equations and the general procedure to determine the optimal input covariance matrix is outlined. Numerical results show the channel capacity derived from our proposed algorithm is very close to the capacity from Monte Carlo method, illustrating the validity of our algorithm.

REFERENCES

[1] Y. Hong and E. Viterbo, "Algebraic multiuser space-time block codes for a 2 ×2 MIMO," *IEEE Trans. Veh. Tech.*, vol 58, no. 6, pp. 3062-3066, Jul. 2009.

[2] Syed A. Jafar, Shlomo Shamai, "Degrees of Freedom Region for the MIMO X Channel," *IEEE Transactions on Information Theory*, Vol. 54, No. 1, pp. 151-170, Jan. 2008.

[3] V.S. Annapureddy and V. V. Veeravalli, "Sum Capacity of MIMO Interference Channels in the Low Interference Regime," *IEEE Trans. Inf. Theory*, vol. 57, no. 5, pp. 2565-2581, May 2011.

[4] O. O. Koyluoglu, M. Shahmohammadi, and H. El Gamal, "A New Achievable Rate Region for the Discrete Memoryless X Channel," in *Proc. 2009 IEEE Int. Symp. on Information Theory (ISIT 2009)*, pp. 2427-2431, June 2009.

[5] L. Cottatellucci and M. Debbah, "On the Capacity of MIMO Rice Channels", *Proceedings of the 42nd Allerton Conference*, Illinois 2004.

[6] J. Huang, E. Au, and V. Lau, "Precoding of space-time block codes in multiuser MIMO channels with outdated channel state information," in *Proceedings of the IEEE International Symposium on Information Theory (ISIT '07)*, June 2007.

[7] S.Karmakar, M. K. Varanasi, "The diversity-multiplexing tradeoff of the MIMO Z interference channel," *Proc. IEEE int. Symp. on Inform. Theory*, pp 2188-2192, Jul. 2008.

[8] M. A. Maddah-Ali, A. S. Motahari, and A. K. Khandani, "Communication over MIMO X channels: interference alignment, decomposition, and performance analysis," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3457–3470, Aug., 2008.

[9] I. Sarris and A. R. Nix, "On the Link-Level Performance of MIMO Systems in Line-Of-Sight," *in Proc. of the IEE 3G and Beyond Conference*, London, Nov. 2005.

[10] K. Yu, M. Bengtsson, B. Ottersten, D. McNamara, P. Karlsson and M. Beach, "Modeling of Wide-Band MIMO Radio Channels Based on NLoS Indoor Measurements," *IEEE Transactions on Vehicular Technology*, vol. 53, no. 3, pp. 655 - 665, May 2004.

[11] Y. Y. Wu and A. R. Calderbank, "Code diversity in multiple antenna wireless communication, in *Proceedings of IEEE International Symposium on Information Theory*, Toronto, Canada, July 2008.

[12] H. Bocskei and A. J. Paulraj, *Multiple-input multiple-output (MIMO) wireless systems*, Cambridge University Press, 2003.

[13] B. Nourani, A.S. Motahari, and A.K. Khandani, "Relay-Aided Interference Alignment for the Quasi-Static X Channel," *Proceedings of IEEE International Symposium on Information Theory 2009*, pp. 1764-1768, June, 2009.

[14] H. Lu, R. Vehkalahti, C. Hollanti, J. Lahtonen, Y. Hong and E. Viterbo, "New space-time code constructions for two-user multiple access channels," *IEEE J. Sel. Top. Sign. Proces.*, vol. 3, no. 6, pp.939-957, Dec. 2009.

[15] W. Zhang and K. B. Letaief, "A systematic design of full diversity multiuser space-frequency codes," *IEEE Trans. Signal Processing*, vol. 58, no. 3, pp.1732-1740, Mar. 2010.

[16] P. D. Karamalis, N. D. Skentos, and A. G. Kanatas, "Adaptive antenna subarray formation for MIMO systems," *IEEE Transactions on Wireless Communications*, vol. 5, no. 11, pp. 2977C 2982, 2006.

[17] R. Monzingo and T. Miller, *Introduction to Adaptive Arrays*, Scitech Publishing, Raleigh, NC, 2004.

[18] E. Visotsky and U. Madhow, "Space-time transmit precoding with imperfect feedback," *IEEE Trans. Inform. Theory*, vol. 47, pp. 2632-2639, Sept. 2001.

[19] S. A. Jafar and A. Goldsmith, "On optimality of beamforming for multiple antenna systems," in *Proc. IEEE Int. Symp. Information Theory*, Washington, DC, June 2001, p. 321.

[20] A. Narula et al., "Eifficient use of side information in multiple-antenna data transmission over fading channels," *IEEE J. Select. Areas Commun.*, vol.16, pp. 1423-1436, Oct. 1998.

[21] A. L. Moustakes et al., "Communication through a diffusive medium: Coherence and capacity," *Science*, vol. 287, pp. 287-290, Jan. 2000.

[22] A. M. Sengupta and P. P. Mitra, "Capacity of multivariate channels with multiplicative noise: I. Random matrix technique and large-n expansions for full tranfer matrices," *LANL arXiv: physics/0010081*, Oct. 2000.

[23] E. Telatar, "Capacity of multi-antenna Gaussian channels," AT&T-Bell Labs, Internal Tech. Memo, June 1995.

[24] W. H. Beyer, *CRC Standard Mathematical Tables and Formulae*, 29th ed. Boca Raton, FL: CRC, 1991.
[25] K.S. Miller, *Complex Stochastic Processes: An Introduction to Theory and Application*, Reading Massachusetts: Addison-Wesley, 1974.
[26] Mariesa Crow, *Computation Methods for Electric Power System*, New York: CRC Press, 2003.
[27] Robb J. Muirhead. *Aspects of Multivariate Statistical Aspects*. New York: Wiley, 1982.
[28] S. Jafar, S. Vishwanath, and A. Goldsmith, "Channel Capacity and beamforming for multiple transmit and receive antennas with covariance feedback", in *Proc. IEEE Int. Conf. Commun.*, vol. 7, June 2001, pp.2266-2270.
[29] S. H. Simon and A. L. Moustakas, "Optimizing MIMO systems with channel covariance feedback," *IEEE J. Select. Areas Commun.*, vol. 21, pp. 406-417, Apr. 2003.
[30] T. Yoo, E. Yoon, and A. Goldsmith, "MIMO capacity with channel uncertainty: does feedback help?," *IEEE Globecom'04*, vol. 1, pp. 96-100, Dec. 2004.
[31] F. Li, Q. T. Zhang, and S. H. Song, "Efficient optimization of input covariance matrix for MISO in correlated Rayleigh fading," *Proceedings of IEEE Wireless Communications and Networking Conference*, pp. 1926-1930, Mar. 2007.