# A Novel Voltage-to-Voltage Logarithmic Converter with High Accuracy 

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#### Abstract

A novel BiCMOS voltage-to-voltage converter with logarithmic characteristics and very high accuracy is presented. The relationship between the emitter current and the base-emitter voltage in bipolar transistors is used to realize the logarithmic function. With 1.8 supply voltage, the total power consumption is less than 15.75 mW and an error of $<-\mathbf{3 6 d B}$ is shown in the ADS simulations. Compared to the other method in the literature, very better accuracy in logarithm calculation is achieved. The proposed method can be used in arithmetical operation circuits like analog processors.


Index Terms- Arithmetical circuits, Logarithmic Amplifier, Logarithmic converter.

## I. Introduction

Logarithmic functions are widely used in instrumentation telecommunications, medical equipments, radar receivers and arithmetical operation circuits [1-3]. Logarithmic circuits need to have high input dynamic range to compress the large amplitude of the signals in the radar receivers input, high accuracy for arithmetical operation functions and low power consumption in order to be useful in communication circuits [1]. A square-law characteristic in strong inversion of MOS transistors cannot lead to logarithmic function while the bipolar transistors behavior can be used to generate it easily. On the other hand, good performance bipolar transistors are not available in CMOS-based technologies [4]. Moreover, utilizing MOS transistors in the weak inversion region which has the exponential behavior will reduce the input dynamic range significantly.

Several approaches of generating logarithmic functions for different applications have been proposed in the literature which are discussed here. Fig. 1 shows the progressivecompression structure which was used in [5], [6]. In this approach several auxiliary voltages are created using series of linear-limit amplifiers. A current proportional to the voltage of

[^0]each stage is generated by taking advantage of a transconductance element. The summation of all these currents with proper transconductance ratio can approximate the logarithmic function piecewise. With a little systematic difference with the previous method, the parallel amplification type circuit was used in [7],[8]. Fig. 2 presents the system diagram of this approach. High symmetry in different path which is lead to the good phase and group delay matching is the strength of this method, while its input dynamic range is lower than the previous on [1].


Fig.1. Progressive-compression topology


Fig.2. Parallel amplification topology
By taking advantages of the above approaches combination, which are the subdivisions of parallel-summation technique, [1] got better properties in approximating logarithmic function piecewise. All reviewed approaches
which are based on piecewise approximation, can be employed where high input dynamic range compression is needed but are not useful in basic arithmetic function circuits as they are complicated while suffer from poor accuracy.
Motivated by the need for good accuracy, some other techniques like Taylor series [9],[10] and current conveyors [11], were utilized to attain logarithmic and exponential behaviors, but none of them could be realized with a simple structure. However, utilizing a single MOS transistor with gate-to-substrate biasing technique in [2] can solve the complicated circuit and accuracy problems simultaneously, but a very poor input dynamic range of about 1.5 uA makes it impractical.

In this paper, a simple circuit based on intrinsic exponential characteristic of the bipolar devices is proposed. At first a MOS transistor is used in order to convert voltage to current and then logarithmic characteristic is obtained by injecting the current to a bipolar transistor. Simulation results in ADS software using TSMC 0.18 um BiCMOS process models confirm the well acceptable accuracy for arithmetic functions applications. In section II the basis of logarithmic behaviors will be considered and completed with the circuit design procedure and the simulation results in section III. Concluding remarks are provided in section IV.

## II. The Basis of Logarithmic Behavior

Exponential function can be obtained via the relationship between emitter current and base-emitter voltage in a bipolar transistor.

$$
\begin{equation*}
I_{E}=I_{S}\left(e^{\frac{\eta V_{B E}}{V_{t}}}-1\right) \tag{1}
\end{equation*}
$$

Therefore, logarithmic characteristics can be achieved by a little change in (1).
$V_{B E}=\frac{V_{t}}{\eta} \ln \left(\frac{I_{E}}{I_{S}}\right) \quad$ while $\quad I_{E} \gg I_{S}$
(2) can be written as below, too.
$V_{B E}=\frac{V_{t}}{\eta} \ln I_{E}-\frac{V_{t}}{\eta} \ln I_{S}=a \ln I_{E}-b$
So a linear relationship between $\mathrm{V}_{\mathrm{BE}}$ and $\ln (\mathrm{IE})$ is available. Thus far logarithmic current to voltage converter is available by a single bipolar transistor; however, voltage to voltage converter is the final goal. In this case, a voltage to current converter is also needed. This can be done by means of a single MOS transistor. Because of the logarithm function characteristics, a square-law behavior in strong inversion region of the MOS elements cannot destroy logarithmic relationship.
$I_{D}=k\left(V_{g s}-V_{T}\right)^{2}$
Using (4) instead of $I_{E}$ in (3):

$$
\begin{align*}
V_{B E} & =a \ln \left[k\left(V_{g s}-V_{T}\right)^{2}\right]-b \\
& =2 a \ln \left[\sqrt{k}\left(V_{g s}-V_{T}\right)\right]-b \\
& =2 a \ln (\sqrt{k})+2 a \ln \left(V_{g s}-V_{T}\right)-b \\
& =p \ln \left(V_{g s}-V_{T}\right)+q \tag{5}
\end{align*}
$$

Using proper dimensions (w/l) for MOS element may cause the q to become zero while it can be made zero by taking advantage of DC level shifting in the output stage, too. In this section, it is demonstrated that logarithmic converter can be realized using a MOS element for voltage to current converting and a bipolar transistor for logarithmic behaving.

## III. Circuit Design And Simulation Results

Circuit-level implementation of the proposed method is presented here in Fig. 3.


Fig.3. Simple logarithmic converter
Logarithmic behavior in the output voltage can be achieved through feeding the current which is in proportion to the input voltage, into a bipolar transistor. The simple topology has two issues discussed below. The first one is about the linear voltage to current converting which is not accessible in this case and can be obtained if the input voltage is equaled to $\left(\mathrm{V}_{\mathrm{gs}}-\mathrm{V}_{\mathrm{T}}\right)$. Furthermore in this technology, the P-channel transistor characteristics are not as well as the N-channel. As the accurate square-law behavior is needed for the accurate logarithmic function according to mathematical equations, The NMOS transistor is a better choice for the circuit input actually.

The second issue is the low output dynamic range. Big changes in the collector current value will cause low alteration in the base-emitter voltage due to the exponential relationship. The lower output dynamic range requires higher gain for the
next stage to achieve logarithmic converter, while the higher gain can lead to the higher power dissipation obviously. The problems are solved in the new topology which is depicted in Fig. 4. By taking advantage of a current mirror, An NMOS transistor is selected for the circuit input. Also, two baseemitter voltage variations will affect the output voltage and it can improve the output dynamic range. Another stage should be added for amplifying and level shifting. Fig. 5 shows the final circuit. As the input of M6 is small signal, the final stage (M6 \& M7) does not have destroyer effect on logarithmic behavior of it. Anyway, in order to decrease the short channel effect on the final result, large dimension transistors are used in the final stage while it will increase the power dissipation, undoubtedly. But, they can control DC level of the output signal and adjust the amplifying coefficient to reach logarithmic function.


Fig.4. Repaired Logarithmic converter


Fig.5. Final Circuit for logarithmic converter
To enclose the operation of the proposed circuit it should be expressed that the logarithm of all positive numbers can be calculated using the logarithm of numbers between 1 to e. (6) shows how all positive numbers can be mapped in to the [1,e] zone.
$x=y \cdot e^{k} \Rightarrow \ln (x)=\ln (y)+k$
If $x$ is a real positive number and $k$ is an integer, $y$ will be a real number between 1 and e. So logarithm of $y$ is sufficient to calculate the logarithm of $x$. For this reason the input dynamic range of the proposed circuit is determined between 1 and e . The equaled resistors are used to divide the input voltage by two, because the positive supply voltage of 1.3 volt is not enough to support the dynamic range of [1,e]. Also, as the threshold voltage of the transistors in the used technology is about 0.5 volt, the minus supply voltage is fixed to -0.5 volt to recoup the input voltage. It is manifestly shown in (7).

$$
\begin{gather*}
I_{d} \alpha\left(v_{g s}-v_{T}\right) \rightarrow I_{d} \alpha\left(v_{A}+v_{E E}-v_{T}\right) \\
\xrightarrow{v_{s s}=v_{T}}+I_{d} \propto v_{A} \rightarrow I_{d} \alpha v_{i n} \tag{7}
\end{gather*}
$$

Fig. 6 shows a comparison of the proposed circuit and an ideal logarithmic converter in 100 MHz .


Fig.6. Simulation result of proposed circuit and ideal logarithmic converter in 100 MHz

Time domain simulation is depicted in Fig. 7 to verify the proper operation of the designed circuit.


Fig.7. Input and output of the proposed logarithmic converter

The behavior of the proposed logarithmic converter over different frequencies will be changed. It can have a different rise and fall shape and of course it is not unexpected because of the accumulated charges in the base of bipolar devices. Fig. 8 demonstrates the output in 200 MHz and 400 MHz .


Fig.8. Proposed circuit output in two different frequencies

Moreover, Fig. 8 shows the error increases as the frequency goes up. Thus, Fig. 9 is provided to report the details. Very low error especially before 100 MHz shows that the presented approach is very promising for arithmetical applications. Table I shows a comparison of this work with some other logarithmic amplifiers.

## Frequency (MHz)



Fig.9. Maximum error of proposed circuit

## IV. CONCLUSION

In this paper, a novel voltage to voltage logarithmic converter for arithmetical circuits was proposed. The idea was originated from the intrinsic characteristics of bipolar transistors. Very low error in logarithm calculation which is so important for arithmetical circuits, show the strength of the proposed. Additionally, a method of mapping the whole positive real numbers in to the $(1, \mathrm{e})$ zone was used to show that the large input dynamic range is not necessary.

TABLE I
THIS WORK AND SOME OTHER LOGARITHMIC AMPLIFIERS CHARACTERISTICS

|  | $[1]$ | $[9]$ | $[12]$ | This work |
| :---: | :--- | :--- | :--- | :--- |
| Technology | 35 GHz Silicon Bipolar | 0.25 um CMOS | 0.35 um CMOS | 0.18 um BiCMOS |
| Technique | Piecewise approximation | Taylor series | Taylor series | Bipolar intrinsic Behavior |
| Supply voltage | -5 V | 1.5 V | 1.5 V | $-0.5 \mathrm{v}, 1.3 \mathrm{~V}$ |
| power | 0.75 W | 0.8 mW | 0.8 mW | 15.75 mW |
| Error @ low Freq | 2 dB | 0.5 dB | 0.5 dB | -36 dB |
| Applications | Radar Input Stage | Arithmetical circuit, AGC | Arithmetical circuit, AGC | Arithmetical circuit |

## REFERENCES

[1] Chris D. Holdenried, James W. Haslett, John G. McRory, R. Douglas Beards, and A. J. BergsmaJ. A DC-4-GHz True Logarithmic Amplifier: Theory and Implementation, IEEE Journal of Solid-State Circuits, Vol. 37, No. 10, October 2002.
[2] Sanchi Harnsoongnoen, Chiranut Sa-ngiamsak, Poonsak Intarakul and Rardchawadee Silapunt, "Logarithmic and Antilogarithmic Circuit with Gate-to-Substrate Biasing Technique", ITC-CSCC, 2008.
[3] R. F. Wolffenbuttel, Digitally Programmable Accurate Current Source for Logarithmic Control of The Amplification or Attenuation in again Cell, IEEE Journal of Solid-State Circuit, Vol.23, pp. 767-773, 1988.
[4] A. B. Grebene, Bipolar and MOS integrated Circuit Design, John Wiley \& Sons, New York, 1984.
[5] B. Gilbert, "Synchronous logarithmic amplifier," U.S. Patent 5298 811, Mar. 29, 1994.
[6] A. Garskamp, "Logarithmic amplifier with sequentially limiting amplifier stages," U.S. Patent 5049 829, Sept. 17, 1991.
[7] K. Kimura, "Logarithmic amplifying circuit based on the bias-offset technique," U.S. Patent 5506 537, Apr. 9, 1996.
[8] F. W. Olsen and E. J. Tasillo, "Psuedo logarithmic step adder," U.S. Patent 5221 907, June 22, 1993.
[9] Quoc-hoang duong, T.kien nguyen and Sang-gug lee, CMOS Exponential Current-to-Voltage Circuit Based on Newly Proposed Approximation Method, ISCAS, 2004.
[10] Carlos A. De La Cruz-Blas and Antonio Lopez-Martin, Compact PowerEfficient CMOS Exponential Voltage-to-Voltage Converter, ISCAS 2006.
[11] F. Bergouignan, N. Abouchi, R. Grisel, G. Caille, J. Caranana, Designs Of A Logarithmic and Exponential Amplifiers Using Current Conveyors, ICECS 1996.
[12] Wci hsing Liii, Clieng-Chicli Clieilg and Shen-I tian TA, Realisation of exponential V-l converter using composite NMOS transistors, Electronics Letters 6th January 2000, Val. 36 No. I.


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[^0]:    Manuscript received January 5, 2011.
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