

Precoded Differential OFDM for Relay Networks

Homa Eghbali and Sami Muhaidat

Abstract — We study the performance of differential space-time codes with linear constellation precoding (LCP) for orthogonal frequency division multiplexing (OFDM) cooperative networks over frequency selective channels. Through exploiting the unitary structure of the orthogonal STBCs, we design a low complexity differential STBC-LCP-OFDM receiver for cooperative networks. We assume the amplify-and-forward protocol and consider both single relay and multi-relay scenarios. Under the assumption of perfect power control for the relay terminal and high signal-to-noise ratio for the underlying links, our performance analysis demonstrates that the considered scheme is able to exploit fully the spatial diversity.

Index Terms — Linear constellation precoding, orthogonal frequency division multiplexing, differential detection, cooperative communications.

I. INTRODUCTION

There has been a growing demand for high data rate services for wireless multimedia and internet services. Spatial diversity offers significant improvement in link reliability and spectral efficiency through the use of multiple antennas at the transmitter and/or receiver side [1]-[4]. Recently, cooperative communications have gained much attention due to the ability to explore the inherent spatial diversity in relay channels [5]-[9]. The idea behind cooperative diversity is that in a wireless environment, the signal transmitted by the source nodes is overheard by other nodes, which are also known as partners. The source and its partners jointly process and transmit their information, creating a "virtual antenna array" although each of them is equipped with only one antenna.

Most the current works on cooperative diversity consider coherent detection and assume the availability of perfect channel state information (CSI) at the receiver. In fading channels where the coherence time is large enough, the channel estimation can be carried out through the use of pilot symbols [10]. For fast fading channels where the phase carrier recovery is more difficult, differential detection provides a more practical solution. In [10-14], differential detection has been investigated for cooperative transmission scenarios. The works in [10-14] assume an idealized transmission environment with an underlying frequency-flat fading channel. This assumption can be justified for narrowband cooperative scenarios with fixed infrastructure; however, it is impractical if wideband cooperative networks are considered. In [26], the applicability of differential STBC to broadband cooperative transmission over frequency-selective channels was

investigated for OFDM systems. However, the proposed system could not fully exploit the underlying multipath diversity. Motivated by this practical concern, we extend the work in [26] to linearly coded and grouped precoded OFDM systems for cooperative communications.

LCP-OFDM was developed in [25] for multicarrier wireless transmissions over frequency selective fading channels. Not only LCP improves the uncoded OFDM performance, but also doesn't reduce the transmission rates of uncoded OFDM, and guarantees symbol detectability [24]. LCP-OFDM was extended to GLCP-OFDM to exploit the correlation structure of subchannels, such that, the set of correlated subchannels are split into subsets of less correlated subchannels. Within each subset of subcarriers, a linear constellation precoder (LCP) is designed to maximize diversity and coding gains. The LCPs are in general complex and could possibly be nonunitary.

While greatly reducing the system complexity, subcarrier grouping maintains the maximum possible diversity and coding gains [25]. GLCP-OFDM is a considerably flexible system that offers maximum multipath diversity, as well as large coding gains, and guaranteed symbol detectability with low decoding complexity.

Related work and contributions: Although there have been considerable research efforts on differential STBC (conventional and distributed) for frequency flat fading channels (see for example [10]-[17]), only a few isolated results have been reported on conventional differential STBC for frequency-selective channels [18]-[20]. Distributed differential STBC multi-carrier transmission for broadband cooperative networks was investigated in [26], yet was suboptimal. Our contributions in this work are summarized as follows:

1. We propose a distributed differential linear constellation precoded OFDM (DD-LCP-OFDM) STBC scheme for broadband cooperative systems with amplify-and-forward (AF) relaying. The proposed scheme can be considered as an extension of the DD-OFDM-STBC scheme proposed in [26]. Carefully exploiting the underlying orthogonality of distributed STBC, the proposed scheme is able to fully exploit the available underlying diversity.
2. To further reduce the decoding complexity while preserving the performance, we extend the DD-LCP-OFDM STBC scheme to grouped linear constellation precoded (GLCP)-OFDM system. We observe that the optimal performance of the DD-GLCP-OFDM relies on the design of the GLCP precoder.
3. We present a comprehensive Monte Carlo simulation study to confirm the analytical observations and give insight into system performance. We extend our work

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on single-relay scenarios to multi-relay scenarios and analyze system's performance via simulation results.

The rest of the paper is organized as follows: In Section II, the transmission model is introduced. The differential scheme under consideration for distributed LCP-OFDM STBC is described in Sections III, and is extended to DD-GLCP-OFDM STBC in section IV. Section V extends our analysis on single relay scenarios to multiple relay scenarios. Numerical results are presented in Section VI and the paper is concluded in Section VII.

Notation: $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$ denote conjugate, transpose, and Hermitian transpose operations, respectively. \otimes denotes Kronecker product, $|\cdot|$ denotes the absolute value, $\|\cdot\|$ denotes the Euclidean norm of a vector, $[\cdot]_{k,l}$ denotes the $(k,l)^{th}$ entry of a matrix, $[\cdot]_k$ denotes the k^{th} entry of a vector, and $tr(\cdot)$ denotes the matrix trace. \mathbf{I}_N denotes the identity matrix of size N , and $\mathbf{0}_{M \times M}$ denotes all-zero matrix of size $M \times M$. For a vector $\mathbf{a} = [a_0 \dots a_{N-1}]^T$, $[\mathbf{P}_N^q \mathbf{a}]_s = \mathbf{a}((N-s+q) \bmod N)$ where \mathbf{P}_N is a $N \times N$ permutation matrix. \mathbf{Q} represents the $N \times N$ FFT matrix whose $(l,k)^{th}$ element is given by $\mathbf{Q}(l,k) = 1/\sqrt{N} \exp(-j2\pi lk/N)$ where $0 \leq l, k \leq N-1$. A circularly symmetric complex Gaussian random variable is a random variable $Z = X + jY \sim CN(0, \sigma^2)$, where \mathbf{X} and \mathbf{Y} are i.i.d. $N(0, \frac{\sigma^2}{2})$. Bold upper-case letters denote matrices and bold lower-case letters denote vectors.

II. SYSTEM MODEL

A single-relay assisted cooperative communication scenario is considered. All terminals are equipped with single transmit and receive antennas. We assume AF relaying and adopt the user cooperation protocol proposed by Nabar *et al.* [9]. Specifically, the source terminal communicates with the relay terminal during the first signaling interval. There is no transmission from source-to-destination within this period. In the second signaling interval, both the relay and source terminals communicate with the destination terminal.

The CIRs for $S \rightarrow R$, $S \rightarrow D$ and $R \rightarrow D$ links for the j^{th} transmission block are by $\mathbf{h}_{SR}^j = [h_{SR}^j[0], \dots, h_{SR}^j[L_{SR}]]^T$, $\mathbf{h}_{SD}^j = [h_{SD}^j[0], \dots, h_{SD}^j[L_{SD}]]^T$, and $\mathbf{h}_{RD}^j = [h_{RD}^j[0], \dots, h_{RD}^j[L_{RD}]]^T$, respectively, where \mathbf{L}_{SR} , \mathbf{L}_{SD} , \mathbf{L}_{RD} denote the corresponding channel memory lengths. All $S \rightarrow R$, $S \rightarrow D$, and $R \rightarrow D$ links are assumed to experience frequency selective Rayleigh fading. The random

vectors \mathbf{h}_{SR}^j , \mathbf{h}_{RD}^j , and \mathbf{h}_{SD}^j , are assumed to be independent zero-mean complex Gaussian with power delay profile vectors denoted by $\mathbf{v}_{SR} = [\sigma_{SR}^2(0), \dots, \sigma_{SR}^2(L_{SR})]$, $\mathbf{v}_{RD} = [\sigma_{RD}^2(0), \dots, \sigma_{RD}^2(L_{RD})]$, and $\mathbf{v}_{SD} = [\sigma_{SD}^2(0), \dots, \sigma_{SD}^2(L_{SD})]$ that are normalized such that $\sum_{l_{SR}=0}^{L_{SR}} \sigma_{SR}^2(l_{SR}) = 1$, $\sum_{l_{RD}=0}^{L_{RD}} \sigma_{RD}^2(l_{RD}) = 1$, and $\sum_{l_{SD}=0}^{L_{SD}} \sigma_{SD}^2(l_{SD}) = 1$. The CIRs are assumed to be constant over four consecutive blocks and vary independently every four blocks.

We consider LCP-OFDM transformation [25] where the LCP is defined by the $N \times N$ matrix Φ that has entries over the complex field. Φ should satisfy the transmit-power constraint $tr(\Phi\Phi^H) = N$. Information symbols are parsed into frames, where each frame consists of two information blocks. Let the $N \times 1$ vector \mathbf{y}_{SRD} represent the data vector that is transmitted to the relay terminal during the first block of each frame, whose entries are complex MPSK symbols that are generated through differential space-time (ST) encoding.

After linear constellation precoding, the $N \times 1$ precoded block $\tilde{\mathbf{y}}_{SRD} = \Phi \mathbf{y}_{SRD}$ is IFFT processed by the inverse FFT matrix \mathbf{Q}^H to yield the discrete time signal $\mathbf{Q}^H \Phi \mathbf{y}_{SRD}$. To further remove the IBI, a cyclic prefix of length $l \geq \max(L_{SR}, L_{RD}, L_{SD})$ is inserted per transmitted OFDM block and is removed from the corresponding received block. In the following, without loss of generality, we drop the index j for brevity.

The signal received at the relay terminal during the first signalling interval (broadcasting phase) of each frame after the CP removal is

$$\mathbf{r}_R = \sqrt{E_{SR}} \mathbf{H}_{SR} \mathbf{Q}^H \Phi \mathbf{y}_{SRD} + \mathbf{n}_R, \quad (1)$$

Where \mathbf{E}_{SR} is the average signal energy over one symbol period received at relay terminal, \mathbf{H}_{SR} is the $N \times N$ circulant matrix with entries $[\mathbf{H}_{SR}]_{m,n} = h_{SR}((m-n) \bmod N)$, and \mathbf{n}_R is the additive white Gaussian noise vector with each entry having zero-mean and variance of $N_0/2$ per dimension. To ensure that the power budget is not violated, the relay terminals normalizes each entry of the respective received signal $[\mathbf{r}_R]_n$, $\mathbf{n} = 1, 2, \dots, N$, by a factor of $\mathbf{E}(|[\mathbf{r}_R]_n|^2) = E_{SR} + N_0$ to ensure unit average energy and retransmits the signal during the second signalling interval (relaying phase) of each frame. After some mathematical manipulations, the received signal at the destination terminal during the relaying phase is given by

$$\mathbf{r}_D = \sqrt{\frac{E_{RD} E_{SR}}{E_{SR} + N_0}} \mathbf{H}_{RD} \mathbf{H}_{SR} \mathbf{Q}^H \Phi \mathbf{y}_{SRD} + \sqrt{E_{SD}} \mathbf{H}_{SD} \mathbf{Q}^H \Phi \mathbf{y}_{SD} + \mathbf{n}_D, \quad (2)$$

where \mathbf{E}_{RD} and \mathbf{E}_{SD} are the average signal energies over one

symbol period received at destination terminal, \mathbf{H}_{RD} and \mathbf{H}_{SD} are the $N \times N$ circulant matrices with entries $[\mathbf{H}_{RD}]_{m,n} = \mathbf{h}_{RD}((m-n) \bmod N)$ and $[\mathbf{H}_{SD}]_{m,n} = \mathbf{h}_{SD}((m-n) \bmod N)$, respectively, and \mathbf{n}_D is conditionally (conditioned on \mathbf{h}_{RD}) complex Gaussian with zero mean and variance $\sigma_{\mathbf{n}_D}^2 = N_0 \left(1 + \frac{E_{RD}}{E_{SR} + N_0} \sum_{m=0}^{L_{RD}} |\mathbf{h}_{RD}(m)|^2 \right)$.

The destination terminal further normalizes the received signal by a factor of $\sqrt{\rho}$,

where $\rho = 1 + \alpha = 1 + \frac{E_{RD}}{E_{SR} + N_0} \sum_{m=0}^{L_{RD}} |\mathbf{h}_{RD}(m)|^2$. Note that this

does not affect the SNR, but simplifies the ensuing presentation [21]. After normalization, we obtain

$$\mathbf{r} = \sqrt{\gamma_1} \mathbf{H}_{RD} \mathbf{H}_{SR} \mathbf{Q}^H \Phi \mathbf{y}_{SRD} + \sqrt{\gamma_2} \mathbf{H}_{SD} \mathbf{Q}^H \Phi \mathbf{y}_{SD} + \mathbf{n}, \quad (3)$$

where \mathbf{n} is $CN(0, N_0)$ and the scaling coefficients γ_1 and γ_2 are defined as $\gamma_1 = \frac{a}{b}$, and $\gamma_2 = \frac{c}{b}$ respectively, where

$$\mathbf{a} = \left(\frac{E_{SR}}{N_0} \right) E_{RD}, \quad b = 1 + E_{SR} / N_0 + \sum_{m=0}^{L_{RD}} |\mathbf{h}_{RD}(m)|^2 E_{RD} / N_0,$$

and $\mathbf{c} = \left(1 + \frac{E_{SR}}{N_0} \right) E_{SD}$. Note that at the end of each frame, the receiver is provided with the time-domain observation \mathbf{r} in (3).

Inspired by Alamouti code [4], we can further extend (3) to a distributed (D)STBC-LCP-OFDM scenario, by using the transmit diversity scheme $\mathbf{y}_{SRD}(t+1) = -\mathbf{y}_{SD}^*(t)$ and $\mathbf{y}_{SD}(t+1) = \mathbf{y}_{SRD}^*(t)$, leading to

$$\begin{aligned} \mathbf{r}(t) &= \sqrt{\gamma_1} \mathbf{H}_{RD} \mathbf{H}_{SR} \mathbf{Q}^H \mathbf{y}_{SRD}(t) \\ &+ \sqrt{\gamma_2} \mathbf{H}_{SD} \mathbf{Q}^H \Phi \mathbf{y}_{SD}(t) + \mathbf{n}(t), \end{aligned} \quad (4)$$

$$\begin{aligned} \mathbf{r}(t+1) &= -\sqrt{\gamma_1} \mathbf{H}_{RD} \mathbf{H}_{SR} \mathbf{Q}^H \Phi \mathbf{y}_{SD}^*(t) \\ &+ \sqrt{\gamma_2} \mathbf{H}_{SD} \mathbf{Q}^H \Phi \mathbf{y}_{SRD}^*(t) + \mathbf{n}(t+1), \end{aligned}$$

Exploiting the circulant structure of the channel matrices \mathbf{H}_{RD} , \mathbf{H}_{SR} , and \mathbf{H}_{SD} , we have $\mathbf{H}_i = \mathbf{Q}^H \Lambda_i \mathbf{Q}$, where Λ_i , \mathbf{i} denotes SR, RD, SD, is a diagonal matrix whose $(n, n)^{th}$ element is equal to the n^{th} DFT coefficient of \mathbf{h}_i . Thus, transforming the received signal $\mathbf{r}(2t)$ to the frequency domain by multiplying it with \mathbf{Q} matrix and further writing the result in matrix form we have

$$\begin{aligned} \overbrace{\begin{bmatrix} \mathbf{Q}\mathbf{r}(t) \\ \mathbf{Q}\mathbf{r}^*(t+1) \end{bmatrix}}^{\mathbf{R}} &= \begin{bmatrix} \mathbf{Q}\mathbf{n}(t) \\ \mathbf{Q}\mathbf{n}^*(t+1) \end{bmatrix} + \end{aligned} \quad (5)$$

$$\begin{bmatrix} \sqrt{\gamma_1} \Lambda_{RD} \Lambda_{SR} \Phi & \sqrt{\gamma_2} \Lambda_{SD} \Phi \\ \sqrt{\gamma_2} \Lambda_{SD}^* \Phi & -\sqrt{\gamma_1} \Lambda_{RD}^* \Lambda_{SR}^* \Phi \end{bmatrix} \begin{bmatrix} \mathbf{y}_{SRD}(t) \\ \mathbf{y}_{SD}(t) \end{bmatrix}.$$

Assuming without loss of generality the symbols to have variance $\sigma_{\mathbf{y}_{SRD}}^2 = \sigma_{\mathbf{y}_{SD}}^2 = 1$, applying the minimum mean square error (MMSE) equalization we have

$$\begin{aligned} \begin{bmatrix} \hat{\mathbf{y}}_{SRD}(t) \\ \hat{\mathbf{y}}_{SD}(t) \end{bmatrix} &= \begin{bmatrix} \sqrt{\gamma_1} \Phi^H \Lambda_{RD}^* \Lambda_{SR}^* \mathbf{A} & \sqrt{\gamma_2} \Phi^H \Lambda_{SD} \mathbf{A} \\ \sqrt{\gamma_2} \Phi^H \Lambda_{SD}^* \mathbf{A} & -\sqrt{\gamma_1} \Phi^H \Lambda_{RD} \Lambda_{SR} \mathbf{A} \end{bmatrix} \mathbf{R}, \end{aligned} \quad (6)$$

where $\mathbf{A} = (\gamma_1 \Lambda_{RD} \Lambda_{SR} \Phi \Phi^H \Lambda_{SR}^* \Lambda_{RD}^* + \gamma_2 \Lambda_{SD} \Phi \Phi^H \Lambda_{SD}^*)^{-1}$. In the following, we will discuss the distributed differential (DD)-LCP-OFDM system and the ST encoding and decoding procedures employed by it.

III. DD-LCP-OFDM

The data vectors $\mathbf{d}_i(t) = [d_i^0(t), \dots, d_i^{N-1}(t)]^T$ $\mathbf{i} = \mathbf{1}, \mathbf{2}$, represent the OFDM-STBC symbols, where \mathbf{t} is the time index and complex symbols $d_i^n(t)$ $n = 1, \dots, N$, are drawn from an unit-energy MPSK constellation. We are encoding the LCP-OFDM-STBC data vectors $\mathbf{d}_1(t) = \Phi \mathbf{d}_1(t)$ and $\mathbf{d}_2(t) = \Phi \mathbf{d}_2(t)$ into their linear constellation precoded differentially encoded frequency domain counterparts $\mathbf{y}_{SRD}(m) = [\mathfrak{y}_{SRD}^0(m), \dots, \mathfrak{y}_{SRD}^{N-1}(m)]^T$ and $\mathbf{y}_{SD}(m) = [\mathfrak{y}_{SD}^0(m), \dots, \mathfrak{y}_{SD}^{N-1}(m)]^T$ $\mathbf{m} = \mathbf{2t}, \mathbf{2t} + \mathbf{1}$ as following $\mathbf{Y}^n(t) = \mathbf{D}^n(t) \mathbf{Y}^n(t-1)$ (7)

where

$$\mathbf{D}^n(t) = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathfrak{d}_1^n(t) & \mathfrak{d}_2^n(t) \\ -(\mathfrak{d}_2^n(t))^* & (\mathfrak{d}_1^n(t))^* \end{bmatrix}, \quad (8)$$

and

$$\mathbf{Y}^n(t) = \begin{bmatrix} \mathfrak{y}_{SRD}^n(2t) & \mathfrak{y}_{SD}^n(2t) \\ \mathfrak{y}_{SRD}^n(2t+1) & \mathfrak{y}_{SD}^n(2t+1) \end{bmatrix}, \quad (9)$$

having

$$\begin{aligned} \mathfrak{y}_{SRD}^n(2t+1) &= -(\mathfrak{y}_{SD}^n(2t))^*, \\ \mathfrak{y}_{SD}^n(2t+1) &= (\mathfrak{y}_{SRD}^n(2t))^*. \end{aligned} \quad (10)$$

Note that (7) is relating 4 frames of information in a differential manner. Applying (10) at the sequence level to the OFDM blocks $(2t)$ and $(2t+1)$ in (9) and substituting them into (4), we obtain

$$\begin{aligned} \mathbf{r}(2t) &= \sqrt{\gamma_1} \mathbf{H}_{RD} \mathbf{H}_{SR} \mathbf{Q}^H \mathbf{y}_{SRD}^n(2t) \\ &+ \sqrt{\gamma_2} \mathbf{H}_{SD} \mathbf{Q}^H \mathbf{y}_{SD}^n(2t) + \mathbf{n}(2t), \end{aligned} \quad (11)$$

$$\begin{aligned} \mathbf{r}(2t+1) &= -\sqrt{\gamma_1} \mathbf{H}_{RD} \mathbf{H}_{SR} \mathbf{Q}^H (\mathbf{y}_{SD}^n(2t))^* \\ &+ \sqrt{\gamma_2} \mathbf{H}_{SD} \mathbf{Q}^H (\mathbf{y}_{SRD}^n(2t))^* + \mathbf{n}(2t+1), \end{aligned}$$

which represents the two consecutive received OFDM frames (2t) and (2t+1) at the destination terminal.

To recover the OFDM data vectors $\mathbf{d}_1(t)$ and $\mathbf{d}_2(t)$ from the differentially encoded received signals in (12), we exploit the circulant structure of the channel matrices \mathbf{H}_{RD} , \mathbf{H}_{SR} , and \mathbf{H}_{SD} , similar to (7) as following

$$\begin{aligned} \mathbf{Q}\mathbf{r}(2t) &= \tilde{\mathbf{r}}(2t) = \sqrt{\gamma_1} \Lambda_{RD} \Lambda_{SR} \mathbf{y}_{SRD}^n(2t) \\ &+ \sqrt{\gamma_2} \Lambda_{SD} \mathbf{y}_{SD}^n(2t) + \underbrace{\mathbf{Q}\mathbf{n}(2t)}_{\tilde{\mathbf{n}}(2t)}, \end{aligned} \quad (12)$$

$$\begin{aligned} \mathbf{Q}\mathbf{r}(2t+1) &= \tilde{\mathbf{r}}(2t+1) = -\sqrt{\gamma_1} \Lambda_{RD} \Lambda_{SR} (\mathbf{y}_{SD}^n(2t))^* \\ &+ \sqrt{\gamma_2} \Lambda_{SD} (\mathbf{y}_{SRD}^n(2t))^* + \underbrace{\mathbf{Q}\mathbf{n}(2t+1)}_{\tilde{\mathbf{n}}(2t+1)}, \end{aligned}$$

Note that in here, we are performing the differential decoding upon the n^{th} subchannel and n^{th} subcarrier to recover $d_1^n(t)$ and $d_2^n(t)$. Considering the n^{th} subchannel, writing (12) in the matrix form we have

$$\begin{aligned} \underbrace{\begin{bmatrix} \tilde{\mathbf{r}}^n(2t) \\ \tilde{\mathbf{r}}^n(2t+1) \end{bmatrix}}_{\mathbf{R}^n(t)} &= \underbrace{\begin{bmatrix} \tilde{\mathbf{n}}^n(2t) \\ \tilde{\mathbf{n}}^n(2t+1) \end{bmatrix}}_{\tilde{\mathbf{n}}^n(t)} + \\ &\underbrace{\begin{bmatrix} \mathbf{y}_{SRD}^n(2t) & \mathbf{y}_{SD}^n(2t) \\ -(\mathbf{y}_{SD}^n(2t))^* & (\mathbf{y}_{SRD}^n(2t))^* \end{bmatrix}}_{\mathbf{Y}^n(t)} \underbrace{\begin{bmatrix} \sqrt{\gamma_1} \Lambda_{RD}^n \Lambda_{SR}^n \\ \sqrt{\gamma_2} \Lambda_{SD}^n \end{bmatrix}}_{\Lambda^n}, \end{aligned} \quad (13)$$

where Λ_{SR}^n , Λ_{RD}^n , and Λ_{SD}^n , stand for the n^{th} diagonal elements of matrices Λ_{SR} , Λ_{RD} , and Λ_{SD} . Thus, substituting (7) into (13), the current input to the distributed STBC differential detector, $\mathbf{R}^n(t)$, for each sub-channel is related to the previous input, $\mathbf{R}^n(t-1)$, according to

$$\begin{aligned} \mathbf{R}^n(t) &= \mathbf{D}^n(t) \mathbf{Y}^n(t-1) \Lambda^n + \tilde{\mathbf{n}}^n(t) \\ &= \mathbf{D}^n(t) \mathbf{R}^n(t-1) + \underbrace{\tilde{\mathbf{n}}^n(t) - \mathbf{D}^n(t) \tilde{\mathbf{n}}^n(t-1)}_{\mathbf{U}^n(t)}. \end{aligned} \quad (14)$$

We set $\mathbf{Y}^n(0)$; $n=0, \dots, N-1$ to \mathbf{I}_2 and linearly detect $\mathbf{D}^n(t)$ from $\mathbf{Y}^n(t)$ using the orthogonal structure in (13). This can be done by rewriting (14) as follows

$$\begin{aligned} \begin{bmatrix} \tilde{\mathbf{r}}^n(2t) \\ (\tilde{\mathbf{r}}^n(2t+1))^* \end{bmatrix} &= \begin{bmatrix} \mathbf{U}_1^n(t) \\ (\mathbf{U}_2^n(t))^* \end{bmatrix} + \\ &\frac{1}{\sqrt{2}} \begin{bmatrix} \tilde{\mathbf{r}}^n(2t-2) & \tilde{\mathbf{r}}^n(2t-1) \\ (\tilde{\mathbf{r}}^n(2t-1))^* & -(\tilde{\mathbf{r}}^n(2t-2))^* \end{bmatrix} \begin{bmatrix} \mathbf{d}_1^n(t) \\ \mathbf{d}_2^n(t) \end{bmatrix}. \end{aligned} \quad (15)$$

To perform MMSE equalization, we can further cascade every n^{th} elements to use the vector forms in (15) as follows

$$\begin{aligned} \begin{bmatrix} \tilde{\mathbf{r}}(2t) \\ \tilde{\mathbf{r}}^*(2t+1) \end{bmatrix} &= \begin{bmatrix} \mathbf{U}_1(t) \\ \mathbf{U}_2^*(t) \end{bmatrix} + \\ &\frac{1}{\sqrt{2}} \begin{bmatrix} \tilde{\mathbf{r}}(2t-2) & \tilde{\mathbf{r}}(2t-1) \\ \tilde{\mathbf{r}}^*(2t-1) & -\tilde{\mathbf{r}}^*(2t-2) \end{bmatrix} \begin{bmatrix} \mathbf{d}_1(t) \\ \mathbf{d}_2(t) \end{bmatrix}. \end{aligned} \quad (16)$$

Thus, performing blind MMSE equalization with no access to CSI, we have

$$\begin{aligned} \begin{bmatrix} \mathbf{d}_1(t) \\ \mathbf{d}_2(t) \end{bmatrix} &= \\ &\begin{bmatrix} \sqrt{\gamma_1} \Phi^H \tilde{\mathbf{r}}^H(2t-2) \mathbf{B} & \sqrt{\gamma_2} \Phi^H \tilde{\mathbf{r}}(2t-1) \mathbf{B} \\ \sqrt{\gamma_2} \Phi^H \tilde{\mathbf{r}}^H(2t-1) \mathbf{B} & -\sqrt{\gamma_1} \Phi^H \tilde{\mathbf{r}}(2t-2) \mathbf{B} \end{bmatrix} \tilde{\mathbf{R}}, \end{aligned} \quad (17)$$

where $\tilde{\mathbf{R}} = \begin{bmatrix} \tilde{\mathbf{r}}(2t) & \tilde{\mathbf{r}}^*(2t+1) \end{bmatrix}^T$ and $\mathbf{B} = (\gamma_1 \tilde{\mathbf{r}}(2t-2) \Phi \Phi^H \tilde{\mathbf{r}}^H(2t-2) + \gamma_2 \tilde{\mathbf{r}}(2t-1) \Phi \Phi^H \tilde{\mathbf{r}}^H(2t-1))^{-1}$.

IV. DD-GLCP-OFDM

Due to high decoding complexity of LCP-OFDM, an optimal subcarrier grouping technique was proposed in [25] in which the decoder's complexity is reduced by dividing the set of all subcarriers into nonintersecting subsets of subcarriers, called subcarrier groups. In this approach, every information symbol is transmitted over subcarriers within only one of these subsets. Note that if the subcarrier grouping is properly done, not only the decoding complexity is reduced, but also systems performance is preserved [25].

The GLCP matrix θ is designed such that the decoding complexity is reduced, while preserving the maximum diversity and coding gains. Assuming that $\mathbf{N} = \mathbf{K}\mathbf{M}$, in DD-GLCP-OFDM STBC system, the information symbols $\mathbf{d}_i(t)$, $i=1,2$, are divided into \mathbf{M} blocks, $\mathbf{d}_i^m(t) = \psi_m \mathbf{d}_i(t)$, where ψ_m is a $K \times N$ permutation matrix built from the rows $(m-1)K+1 \rightarrow mK$ of \mathbf{I}_N ; and then precoded by the GLCP matrix θ . Thus, as an example, (12) can be rewritten as

$$\begin{aligned} \tilde{\mathbf{r}}(2t) &= \sqrt{\gamma_1} \Lambda_{RD}^m \Lambda_{SR}^m \mathbf{g}_{SRD}^m(2t) \\ &+ \sqrt{\gamma_2} \Lambda_{SD}^m \mathbf{g}_{SD}^m(2t) + \tilde{\mathbf{n}}(2t), \\ \tilde{\mathbf{r}}(2t+1) &= -\sqrt{\gamma_1} \Lambda_{RD}^m \Lambda_{SR}^m (\mathbf{g}_{SD}^m(2t))^* \\ &+ \sqrt{\gamma_2} \Lambda_{SD}^m (\mathbf{g}_{SRD}^m(2t))^* + \tilde{\mathbf{n}}(2t+1), \end{aligned} \quad (18)$$

where $\Lambda_i^m = \psi_m \Lambda_i \psi_m^T$, $\mathbf{i} = \mathbf{SR, RD, SD}$, and $\mathbf{g}_i^m(2t)$, $\mathbf{i} = \mathbf{SR, SD}$, are differentially encoded from the GLCP-OFDM symbols $\mathbf{g}_i^m(t) = \theta \mathbf{d}_i^m(t)$ following similar steps as in (7).

Following [24-25], for any \mathbf{K} , QAM, PAM, BPSK, and QPSK constellation, the optimal can be constructed through LCP-A, which can be generally written as a Vandermonde matrix as following

$$\theta = \frac{1}{\beta} \begin{bmatrix} 1 & \alpha_1 & \cdots & \alpha_1^{K-1} \\ 1 & \alpha_2 & \cdots & \alpha_2^{K-1} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \alpha_K & \cdots & \alpha_K^{K-1} \end{bmatrix}, \quad (19)$$

where β is a constant such that $\text{tr}(\theta\theta^H) = K$, and the parameters $\{\alpha_k\}_{k=1}^K$ are selected depending on \mathbf{K} [25].

V. EXTENSION TO MULTIPLE RELAY SCENARIOS

We consider a multiple-relay assisted cooperative wireless communication system with a single source S , R half-duplex relay terminals R_i , $i=1, 2, \dots, R$, and a single destination D . The source, destination, and all relays are equipped with single transmit and receive antennas. We adopt the transmission protocol in [27] and consider non-regenerative relays. Note that unlike [27], we assume that there is no direct transmission between the source and destination terminals due to the presence of shadowing.

The CIRs for $S \rightarrow R_i$ and $R_i \rightarrow D$ links for the i^{th} relay terminal are given by $\mathbf{h}_{SR_i} = [h_{SR_i}[0], \dots, h_{SR_i}[L_{SR_i}]]^T$, $\mathbf{h}_{R_iD} = [h_{R_iD}[0], \dots, h_{R_iD}[L_{R_iD}]]^T$, respectively, where L_{SR_i} and L_{R_iD} denote the corresponding channel memory lengths. All the $S \rightarrow R_i$ and $R_i \rightarrow D$ links are assumed to be frequency selective Rayleigh fading. The random vectors \mathbf{h}_{SR_i} and \mathbf{h}_{R_iD} are assumed to be independent zero-mean complex Gaussian with power delay profile vectors denoted by $\mathbf{v}_{SR_i} = [\sigma_{SR_i}^2(0), \dots, \sigma_{SR_i}^2(L_{SR_i})]$ and $\mathbf{v}_{R_iD} = [\sigma_{R_iD}^2(0), \dots, \sigma_{R_iD}^2(L_{R_iD})]$ that are normalized such that $\sum_{l_{SR_i}=0}^{L_{SR_i}} \sigma_{SR_i}^2(l_{SR_i}) = 1$ and $\sum_{l_{R_iD}=0}^{L_{R_iD}} \sigma_{R_iD}^2(l_{R_iD}) = 1$.

We consider a complex space-time block code C with its entries \mathbf{c}_i [1]. For demonstration purposes, we focus on the G_3 code [3] with three relays in a cooperative DD-OFDM-STBC scenario with AF relaying. The extension to other orthogonal space-time block codes and DD-LCP/GLCP-OFDM STBC systems is straightforward. During every four time slots, three time slots are devoted to signal transmission from source to relays, one at a time, and during the last time slot, all R relays retransmit the normalized received signals to the destination terminal. During the last signaling interval of every four time slots, all the three relays retransmit the

normalized received signal to the destination terminal, and the destination receives $\mathbf{r}_D^4, \mathbf{r}_D^8, \mathbf{r}_D^{12}, \mathbf{r}_D^{16}, \mathbf{r}_D^{20}, \mathbf{r}_D^{24}, \mathbf{r}_D^{28}, \mathbf{r}_D^{32}$, where \mathbf{r}_D^t denotes the received signal at the destination terminal at time slot t . Assuming that the Information symbols are first parsed as four streams of $N \times 1$ blocks x_i , $\mathbf{i} = \mathbf{1, 2, 3, 4}$ and encoded to \mathbf{c}_i using the orthogonal space-time block code design C , (7) holds true with

$$\mathbf{D}^n(t) = \frac{1}{\sqrt{3}} \begin{bmatrix} x_1^n & x_2^n & x_3^n \\ -x_2^n & x_1^n & -x_4^n \\ -x_3^n & x_4^n & x_1^n \\ -x_4^n & -x_3^n & x_{12}^n \\ (x_1^n)^* & (x_2^n)^* & (x_3^n)^* \\ -(x_2^n)^* & (x_1^n)^* & -(x_4^n)^* \\ -(x_3^n)^* & (x_4^n)^* & (x_1^n)^* \\ -(x_4^n)^* & -(x_3^n)^* & (x_{12}^n)^* \end{bmatrix}, \quad (20)$$

and $\mathbf{Y}^n(t)$ changes accordingly. Following this, $\mathbf{R}^n(t)$ in (13) changes to

$$\mathbf{R}^n(t) = [\mathbf{r}_D^4 \quad \mathbf{r}_D^8 \quad \mathbf{r}_D^{12} \quad \mathbf{r}_D^{16} \quad \mathbf{r}_D^{20} \quad \mathbf{r}_D^{24} \quad \mathbf{r}_D^{28} \quad \mathbf{r}_D^{32}]^T. \quad (21)$$

VI. NUMERICAL RESULTS

In this section, we present Monte-Carlo simulation results for the proposed receiver.

Fig.1. depicts the SER performance of the DD-LCP-OFDM STBC scheme assuming for the following three different scenarios:

- 1) $L_{SR} = L_{RD} = L_{SD} = 0$,
- 2) $L_{SR} = L_{RD} = L_{SD} = 1$,
- 3) $L_{SR} = 5, L_{RD} = 2, L_{SD} = 1$.

For LCP-OFDM to achieve maximum diversity order, maximum diversity encoders should be used. Two classes of maximum achievable diversity order (MADO) enabling LCP encoders are introduced in [24], namely: Vandermonde encoders and cosine encoders. In here, we are using Vandermonde encoders and we assume 4-PSK modulation. To further minimize the receiver complexity, we are applying the low-cost minimum mean-square error (MMSE) equalizer. Our simulation results indicate that with the optimal design of LCP encoder matrix, the DD-LCP-OFDM STBC system is able to achieve full spatial and multipath diversity, $\min(L_{SR}, L_{RD}) + L_{SD} + 1$ and it has been confirmed.

Following [25], we suggest an alternative low complexity implementation of the DD-LCP-OFDM STBC system, namely, DD grouped linear constellation precoded GLCP-OFDM STBC subsystems. The aim is to reduce system complexity while preserving maximum possible diversity and coding gains. The proposed system's optimal performance relies on the design of the GLCP matrix [25].

Fig. 2 depicts the SER performance of the DD-GLCP-OFDM STBC scheme for the following combinations of the underlying channel memory lengths:

- 1) $\mathbf{L}_{SR}=\mathbf{L}_{RD}=\mathbf{L}_{SD}=\mathbf{0}$,
- 2) $\mathbf{L}_{SR}=\mathbf{L}_{RD}=\mathbf{L}_{SD}=\mathbf{1}$,
- 3) $\mathbf{L}_{SR}=\mathbf{5}, \mathbf{L}_{RD}=\mathbf{2}, \mathbf{L}_{SD}=\mathbf{1}$.

The MADDO GLCP encoders used for scenarios 1, 2, and 3 are θ_2 , θ_4 , and θ_8 , respectively [25]. We analyze system's performance for $\alpha=1,10$, i.e. $E_{SR}/N_0 = \alpha E_{SD}/N_0$. To minimize the receiver complexity, MMSE equalizer is implemented at the receiver side. In the case of $\alpha=1$, for all three scenarios where the $S \rightarrow R$ and $S \rightarrow D$ links are balanced, the SER performance degrades compared to the corresponding scenario when $\alpha=10$, while preserving the achieved diversity order.

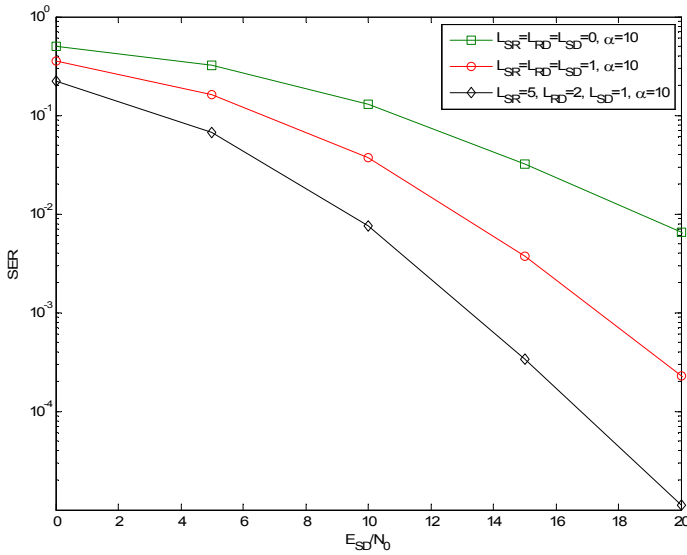


Fig.1. SER performances of DD-LCP-OFDM STBC over frequency-selective $S \rightarrow R$, $R \rightarrow D$, and $S \rightarrow D$ links ($E_{SR}/N_0 = \alpha E_{SD}/N_0, \alpha = 10$)

In Fig.3 the SER performance of DD-OFDM-STBC is compared with that of DD-LCP-OFDM-STBC and DD-GLCP-OFDM-STBC, for $\mathbf{L}_{SR}=\mathbf{L}_{RD}=\mathbf{L}_{SD}=\mathbf{1}$, $\alpha=10$, and θ_4 [25]. As can be observed from Fig. 6, DD-LCP-OFDM-STBC outperforms both DD-GLCP-OFDM-STBC and DD-OFDM-STBC. However, both DD-LCP-OFDM-STBC and DD-GLCP-OFDM-STBC achieve the same diversity gain. As an example, at $\text{BER}=10^{-4}$, the DD-LCP-OFDM-STBC system outperforms DD-GLCP-OFDM-STBC and DD-OFDM-STBC systems by ~ 2 dB and ~ 4 dB, respectively.

In Fig. 4 the SER performance of DD-GLCP-OFDM is provided with optimal and suboptimal subcarrier grouping, assuming $\mathbf{L}_{SR}=\mathbf{L}_{RD}=\mathbf{L}_{SD}=\mathbf{1}$ and 4-PSK modulation. The optimal and suboptimal grouping are specified with $I_{m,opt} = \{m-1, M+m-1, \dots, (K-1)M+m-1\}$ and $I_{m,subopt} = \{(m-1)K+1, (m-1)K+2, \dots, mK\}$ $m \in [1, M]$

subsets, respectively, where $\mathbf{N}=\mathbf{MK}$. As is illustrated in Fig. 7, the optimal subcarrier grouping improves performance, i.e. by ~ 2 dB at $\text{BER}=10^{-3}$.

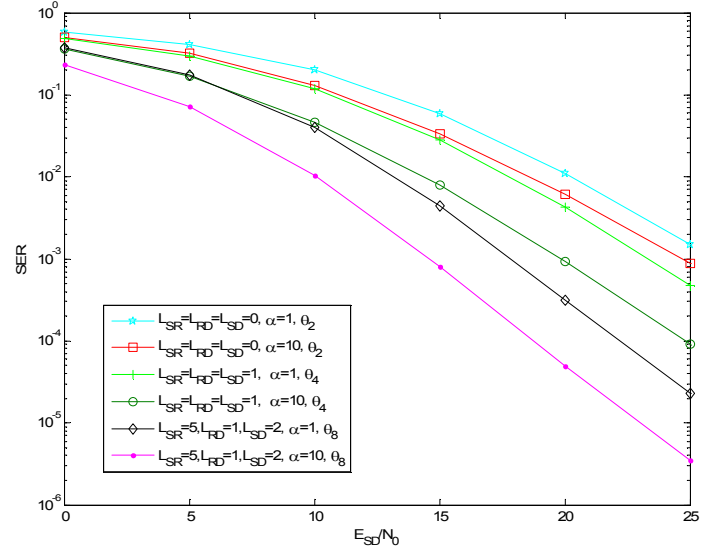


Fig.2. SER performances of DD-GLCP-OFDM STBC over frequency-selective $S \rightarrow R$, $R \rightarrow D$, and $S \rightarrow D$ links ($E_{SR}/N_0 = \alpha E_{SD}/N_0, \alpha = 1,10$)

Next, we extend our analysis to a multiple relay scenario, where we adopt the transmission protocol in [27] and consider non-regenerative relays. Note that unlike [27], we assume that there is no direct transmission between the source and destination terminals due to the presence of shadowing. We assume that there are three relay nodes, where each node is equipped with one antenna. We set $\text{SNR}_{SR_1} = \text{SNR}_{SR_3} = 25$ dB, and $E_{R_1D} = E_{R_2D} = E_{R_3D} = 5$ dB, and the SER curve is plotted against E_{SR_2}/N_0 . 4-PSK modulation and G_3 code [28] are used. The scenarios with different combinations of channel memory lengths are considered for DD-LCP-OFDM-STBC system:

- 1) $L_{SR_1} = L_{R_1D} = L_{SR_2} = L_{R_2D} = L_{SR_3} = L_{R_3D} = 0$,
- 2) $L_{SR_1} = L_{R_1D} = L_{SR_2} = L_{R_2D} = L_{SR_3} = L_{R_3D} = 1$,
- 3) $L_{SR_1} = L_{R_1D} = L_{SR_2} = L_{R_2D} = L_{SR_3} = L_{R_3D} = 2$.

As is illustrated in Fig. 5, as an example, at $\text{BER}=10^{-2}$, the third scenario outperforms the first and second scenarios by ~ 3 dB and ~ 8 dB, respectively. The second scenario outperforms the first scenario by ~ 4 dB at $\text{BER}=10^{-2}$.

VII. CONCLUSION

We have investigated distributed differential LCP-OFDM STBC and GLCP-OFDM for cooperative communications over frequency-selective fading channels. We have carefully exploited the unitary structure of STBCs to design a low complexity distributed differential STBC receiver for broadband cooperative networks. With the optimal design of

the LCP encoder matrix, the DD-LCP-OFDM STBC system is able to achieve full diversity gain. An alternative low-complexity implementation of the DD-LCP-OFDM STBC system, namely DD-GLCP-OFDM-STBC, reduces the complexity while preserving the maximum possible diversity and coding gains by dividing the set of all subcarriers into non-intersecting subcarrier groups. The DD-GLCP-OFDM STBC system's performance relies on the design of the GLCP matrix. Note that with optimal subcarrier grouping, the DD-LCP-OFDM STBC and DD-GLCP-OFDM STBC both achieve the same diversity gain. We have further extended the analysis to multiple relay scenarios. We have presented the comprehensive Monte-Carlo simulations to corroborate the theoretical presentation.

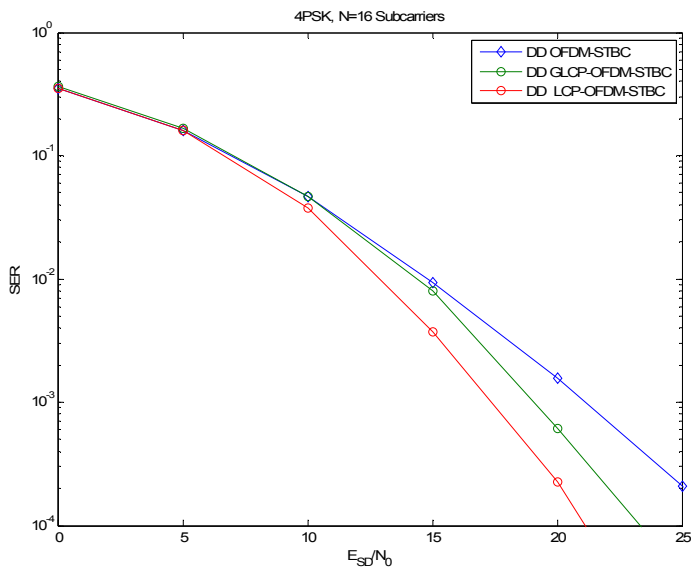


Fig. 3. SER performance comparison between DD-OFDM-STBC, DD-LCP-OFDM STBC, and DD-GLCP-OFDM-STBC systems.

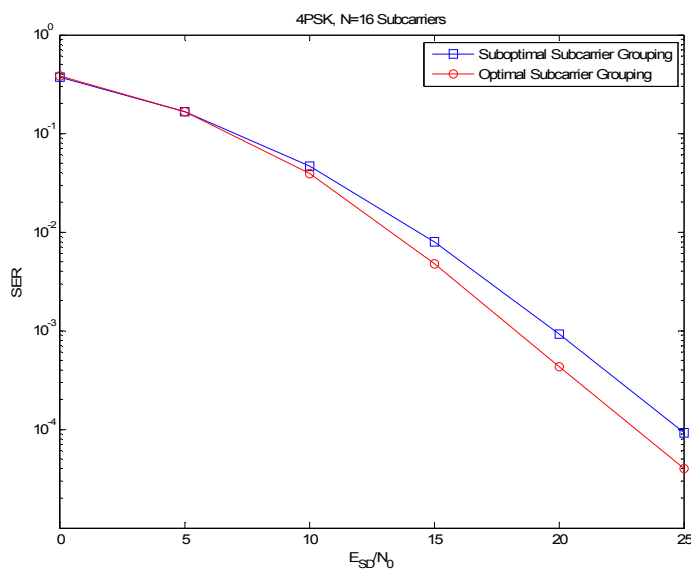


Fig. 4. Optimal versus suboptimal subcarrier grouping for the DD-LCP-OFDM STBC system

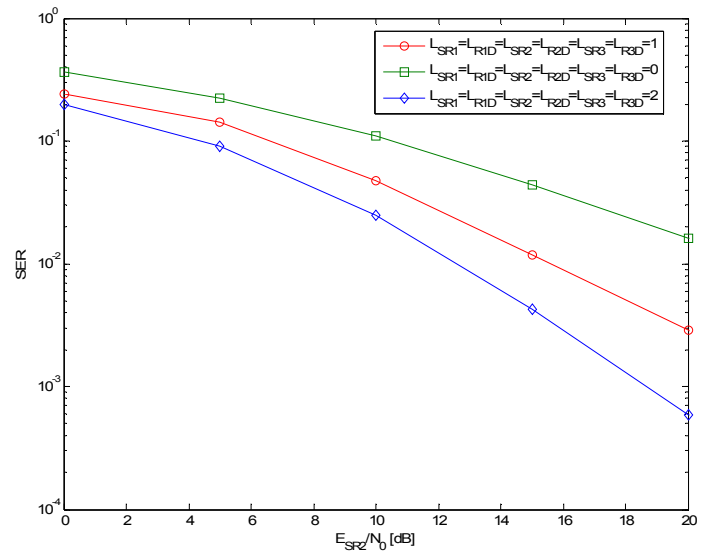


Fig. 5. SER performance of DD-LCP-OFDM STBC system with three relays.

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