Chaotic Multiple Access System Based on Orthogonal Chaotic Vector of Rossler Sequence

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***Abstract*—Chaos phenomenon is capable of generating chaotic sequences with low cross-correlation, which can be useful for spreading the direct sequence code division multiple access (DS-CDMA) system. In this paper, a chaotic multiple access communication system based on Orthogonal Chaotic Vector (OCV) generated from Rössler system has been proposed. The simulation results showed that the performance of the proposed system is almost the same regardless of the number of users in Additive White Gaussian noise AWGN channel. The results also showed that at bit-error-rate of 10^{-3}, the proposed system has achieved a gain of 6 dB in signal-to-noise ratio over traditional DS-CDMA based on Walsh-Hadamard sequence for single user case in both AWGN and Rayleigh fading channels.**

***Index Terms*—Chaos theory, DS- CDMA, orthogonal chaotic vector, Rössler system**

I. Introduction

Recently, many works [1-6] have shown the effectiveness of applying chaotic maps' signals/sequences in multiple access digital communication systems to replace the quasi-orthogonal binary code sequences. For CDMA systems, sequences with low cross-correlation properties are desired. Such sequences can be produced by chaotic systems due to its sensitivity to initial conditions. A concrete proposal for the chaotic CDMA system could already be found as early as 1992 [7], followed by other similar proposals [8-11]. Mazzini et al. [12] presented the performance analysis of the chaotic CDMA system. They showed that chaos-based sequences can outperform m-sequences & Gold codes.

Tse [13] warned that annotating engineering solutions with chaos terminologies may divert reviewers' attention away from comparing with conventional methods. Also researchers tend to present themselves very positively in talking about chaos. Thus potential advantages of chaotic solutions are emphasized, but difficult problems are often not discussed adequately. In case of CDMA systems, the current conventional spreading codes used in the 3G systems are orthogonal Walsh codes. For such systems, it has been recognized (although generally not published) that the chaos codes cannot beat the orthogonal codes in AWGN environment. However, because of the rather bad cross-correlation properties of the orthogonal codes, there should be cases in multipath channel environment, in which chaos codes outperform orthogonal codes.

Unfortunately, publications on the performance comparison of chaotic CDMA based on Orthogonal Chaotic Vector (OCV) with traditional CDMA based on orthogonal spreading codes are rare. Probably because it is commonly recognized that orthogonal spreading codes will generally outperform chaos map spreading codes. Furthermore, the little existing works considered the use of chaotic maps for CDMA although the chaotic flows like Rössler [14] are easier to be generated practically from physical electronic circuit. In this work, direct sequence DS-CDMA digital communication system based on the OCV Rössler flow sequences (x, y, and z) has been proposed and simulated. A random time-invariant multipath channel is used for BER simulations. The receiver is a RAKE receiver with enough fingers to capture all the energy from the multipath. Perfect channel estimation is assumed, and only downlink multiuser cases are simulated. The performance of the proposed system has been compared with traditional CDMA system.

The rest of the paper is organized as follows: Section II describes the architecture of the proposed system. Section III describes the Rössler system and orthogonal chaotic vector generation. The Performance of the proposed system over AWGN and fading channels are given in section IV and section V respectively. Finally, the simulation results and conclusions are presented in sections VI and VII respectively.

II. Architecture of the Proposed DS-CDMA Based on Chaotic Rössler System

The block diagram of the proposed CDMA based on chaotic Rössler system (CDS-CDMA) is shown in Fig.1. In the transmitter side, for each user (say jth user) out of N_u users, the data $d^{(j)}$ are generated and modulated using baseband BPSK modulator (QPSK or other digital modulation schemes are also applicable). The modulated data $d_m^{(j)}$ of each user are spread using a unique Rössler chaotic sequence. The Rössler

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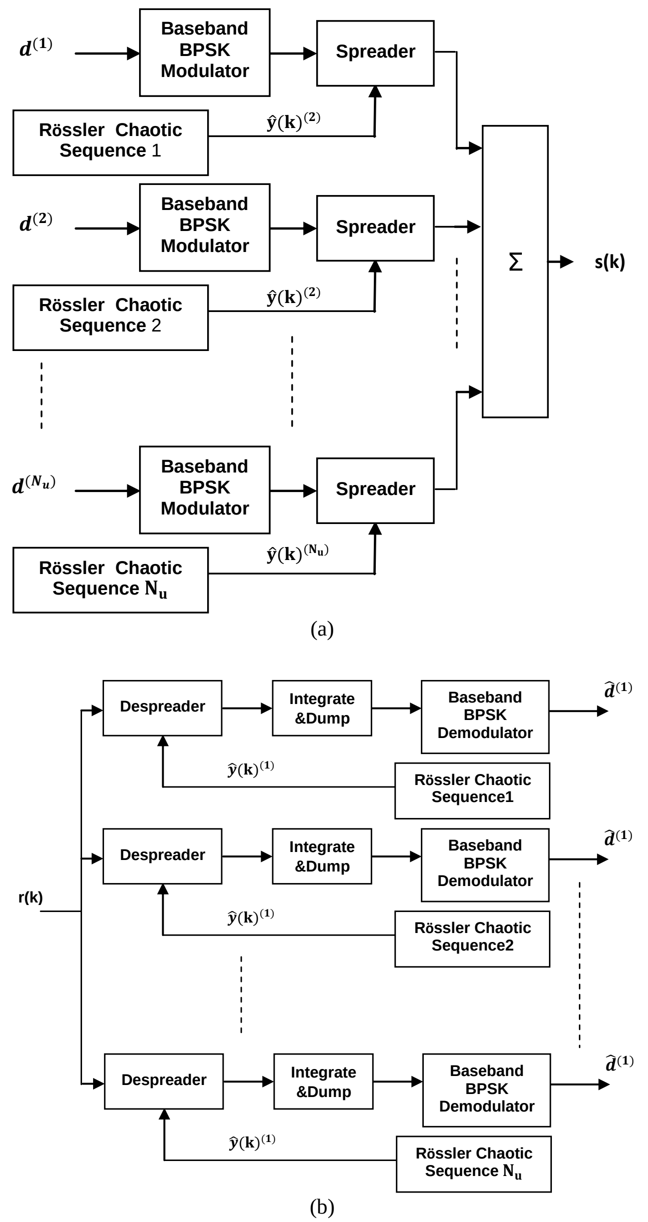

Fig.1. The proposed chaotic multiple access system based on Rössler model. (a) the transmitter, (b) the receiver.

sequences are generated by taking one of the outputs of Rössler system (i.e. x-sequence, y-sequence, or z-sequence) and applying it to an OCV circuitry that generates N_u Rössler sequences orthogonal to each other for N_u different users. The spreading operation is simply a multiplication process between the chaotic and the BPSK modulated signals. The spread signals are added to each other and sent though the transmission channel. At the receiver, each of the N_u received signals is despread by multiplying it by an exactly similar replica of chaotic signal at transmitter of the corresponding user. An integrate and dump operation is performed to detect the energy produced by the despread signal and finally the recovered signal is demodulated to obtain the transmitted data. The difference between the proposed system and the traditional DS-CDMA used for wireless applications is that the digital Walsh-Hadamard sequence is replaced by Rössler OCV analogue sequence to improve the system performance and capacity.

III. Rössler Chaotic System and Orthogonal Chaotic Vector

The Rössler system has only one quadratic nonlinearity xy numerical integration shows that this system has a strange attractor for a = b= 0.2, c = 5.7 [14].

$$\dot{x} = -y - z \quad (1)$$

$$\dot{y} = x + ay \quad (2)$$

$$\dot{z} = b + z(x - c) \quad (3)$$

where [x y z] is the output state vector while a, b and c are three real valued parameters. As a three-dimensional system with three parameters, the Rössler system can lead to very complicated behavior on changing the parameter values. Choosing the standard parameter values a =b=0.2, and c = 5.7, the well-known "buttery" attractor is obtained, as shown in Fig.2.

The orthogonal chaotic vector algorithm:

The non-zero values of cross-correlation between mutual chaotic sequences (x, y, and z) of Rössler system in chaotic CDMA system result in Multiple Access Interference (MAI) between users. The effect of MAI increases as the number of users increases. In order to eliminate the effect of problem (MAI), we apply rule of Gram-Schmidt's ortho-normalization process [15] on flow chaotic sequence vectors to obtain flow orthogonal chaotic vectors OCV codes (x, y and z) respectively. Gram-Schmidt ortho-normalization process for N_u orthogonal chaotic signal generators is given by:

$$\hat{y}(k)^{(p)} = \frac{y(k)^{(p)} - \sum_{q=1}^{p-1}\left[\sum_{k=1}^{\beta} y(k)^{(p)}\hat{y}(k)^{(q)}\right]\hat{y}(k)^{(q)}}{\sqrt{\sum_{k=1}^{\beta}\left[y(k)^{(p)} - \sum_{q=1}^{\beta}\left[\sum_{k=1}^{\beta} y(k)^{(p)}\hat{y}(k)^{(q)}\right]\hat{y}(k)^{(q)}\right]^2}} \quad (4)$$

where p = 2, 3. . . N_u. For p = 1(i.e. single user)

$$\hat{y}(k)^{(1)} = \frac{y(k)^{(1)}}{\sqrt{\sum_{k=1}^{\beta}[y(k)^{(1)}]^2}} \quad (5)$$

where $\hat{y}(k)^{(j)}$ is the chaotic carrier for jth user and β is the number of chaotic samples used to transmit single binary bit (i.e. spreading factor). These OCVs are used as spreader to spread message bits, and to increase the number of active users. The orthogonal chaotic sequences can be generated from the different outputs of Rössler chaotic system (x, y, and z) with similar or different initial conditions.

Fig.3. shows the orthogonal chaotic sequences generated using the output y of Rössler chaotic system. The mean value of the chaotic carrier is made equal to zero in order to avoid

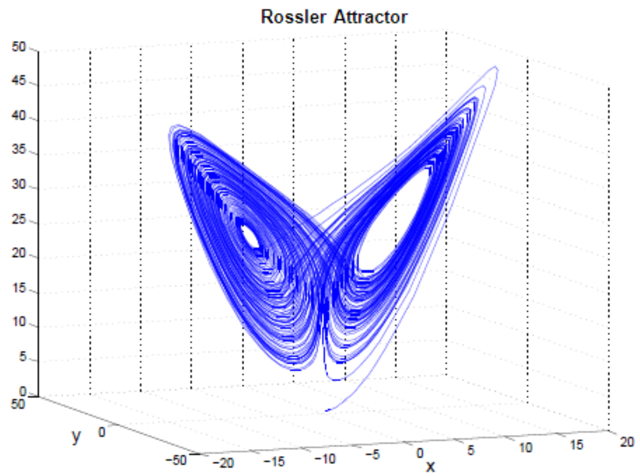

Fig.2. Rössler attractor with parameters values: a =0.2, b= 0.2, and c= 5.7, initial conditions [x_0 y_0 z_0] = [0.1 0.1 0.1].

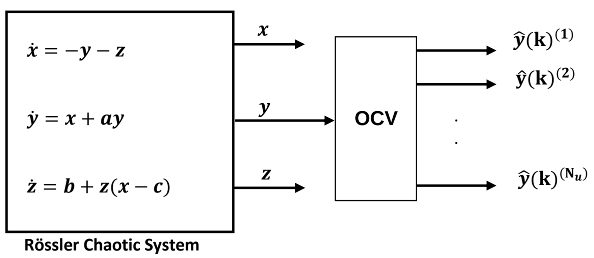

Fig.3. Generation of orthogonal chaotic sequences from Rössler system.

unwanted dc power transmission. As it was mentioned in section II, the chaotic sequences are multiplied by baseband modulated BPSK data sequence d_1{-1, +1} to obtain the spread vector $v(k)^{(j)}$.The transmitted signal $s(k)$ is the sum of modulated orthogonal chaotic vectors of each user and can be represented as:

$$s(k) = \sum_{i=1}^{N_u} v(k)^{(j)} \tag{6}$$

IV. Performance Analysis Over AWGN Channel

Assuming that the signal is corrupted only due to AWGN, the received signal $r(k)$ can be represented as:

$$r(k) = \sum_{i=1}^{N_u} V(k)^{(i)} + \xi(k) \tag{7}$$

where $\xi(k)$ is the additive white Gaussian noise with zero mean and $N_o/2$ variance. At the receiver, it is assumed that a similar replica of spreading sequence is available and it is exactly synchronized with the transmitted one. The mth decoded symbol for the jth user, denoted by $\tilde{d}_m^{(j)}$, is determined according to the rule:

$$\tilde{d}_m^{(j)} = \begin{cases} +1, if\ O^{(j)} = \sum_{k=1}^{\beta} r(k)\hat{y}(k)^{(j)} > 0 \\ -1, if\ O^{(j)} = \sum_{k=1}^{\beta} r(k)\hat{y}(k)^{(j)} \le 0 \end{cases} \tag{8}$$

Without the loss of generality, we consider the probability of error for the first symbol. Omitting the subscripts of the variables $\tilde{d}_m^{(j)}$ and $O^{(j)}$ for the sake of brevity, the decision parameter of the jth user is given by:

$$O^{(j)} = d^{(j)} \sum_{k=1}^{\beta} \left[\hat{y}(k)^{(j)}\right]^2 + d^{(j)} \sum_{i=1, i\neq j}^{N_u} \sum_{k=1}^{\beta} \left(\hat{y}(k)^{(i)}\hat{y}(k)^{(j)}\right)^2 + \sum_{k=1}^{\beta} \xi(k)\hat{y}(k)^{(j)} \tag{9}$$

Since, chaotic vectors used for each user is ortho-normal to each other, the second term in equation 9 causing MAI will be equal to zero. Assuming, that $O^{(j)}$ has a Gaussian distribution, the BER for jth user can be written as [16]:

$$BER^{(j)} = \frac{1}{2} erfc\left(\frac{E\left(O^{(j)} | d^{(j)} = +1\right)}{\sqrt{\left(2\ var\left(O^{(j)} | d^{(j)} = +1\right)\right)}}\right) \tag{10}$$

where, mean value of ($O^{(j)}$ | $d^{(j)}$ = +1) is given by:

$$E\left(O^{(j)} | d^{(j)} = +1\right) = \beta E\left[\left(\hat{y}(k)^{(j)}\right)^2\right] = E_b \tag{11}$$

where E_b is energy per bit. The variance is given by:

$$Var\left(O^{(j)} | d^{(j)} = +1\right) = var\left[\sum_{k=1}^{\beta} \left[\hat{y}(k)^{(j)}\right]^2\right] + \beta \frac{N_O}{2} E\left[\left(\hat{y}(k)^{(j)}\right)^2\right] = \frac{E_b N_O}{2} \tag{12}$$

substituting equations 11 & 12 in equation 10, we get:

$$BER^{(j)} = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_O}}\right) \tag{13}$$

From equation 13 it can be concluded that BER performance of the proposed system is independent on the number of users and spread factor which is a unique and strong feature does not exist in other multiple access systems.

V. Performance Analysis over Rayleigh Fading Channel

Assuming that the channel is a slow Rayleigh fading channel, let α is a Rayleigh distributed random variable denoting fading gain. Then it can be shown that the BER of the jth user in symbol duration is:

$$BER_{\alpha}^{(j)} = \frac{1}{2} erfc(\gamma) \tag{14}$$

where,

$$\gamma = \frac{\alpha E_b}{N_o} \quad (15)$$

Since α is Rayleigh-distributed random variable, γ (the received instantaneous signal to noise ratio per bit) will be chi-square distributed and has the form [14]:

$$f_{Rayleigh} = \frac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}}, \gamma \geq 0 \quad (16)$$

where

$$\bar{\gamma} = \mathrm{E}[\gamma] = E[\alpha].\frac{E_b}{N_o} \quad (17)$$

Therefore, the average BER for jth user is:

$$BER_{Rayleigh} = \int_0^\infty BER_\alpha(\gamma)\, f_{Rayleigh}(\gamma)\, d\gamma \quad (18)$$

The last equation shows that in case of Rayleigh fading channel, there is no clear relation weather the BER performance of the proposed system is independent on the number of users and spread factor or not. Therefore, the simulation results may show us more clear view about this issue.

VI. Simulation Results

A complete simulation model for the proposed OCV Rössler based CDS-CDMA system shown in Fig.1 has implemented using MATLAB package. From other hand, another simulation model for traditional CDMA system based on orthogonal Walsh-Hadamard code with the same simulation parameters has also implemented for the purpose of performance comparison. In all simulation results the spreading factor is chosen to be 64 in both systems. The parameters of the generated Rössler sequence were a = 0.2, b = 0.2, and c= 5.7 while the initial conditions were $[x_0 \;\; y_0 \;\; z_0]$ = [0.155 0.25 0.155]. The number of users has changed from single user to a maximum of 16 users.

A. Simulation Results in AWGN Channel

Fig.4 shows the BER performances of CDMA based on OCVs generated from individual outputs of Rössler system (x, y, and z) for single user transmission in AWGN channel together with traditional CDMA system. It can be noticed from this figure that the performances of OCVs generated using different outputs of Rössler system is not the same. The OCV associated with output y has the best performance. Therefore, it will be selected for all the remaining simulation results. Also it is obvious in this figure that OCVs outperforms the traditional CDMA. For instant, at BER=10^{-3} a gain of 6 dB in E_b/N_0 can be obtained when we use OCV (y-output) as compared with the traditional CDMA.

Fig.5 depicts the BER performances when the number of users is 4. Once again, the OCV based CDMA outperforms the traditional CDMA. At BER=10^{-3}, a gain of 6 dB in E_b/N_0 is obtained. The improvement can be attributed to good combination of auto and cross-correlation properties of Rössler OCVs. In other word, OCV sequences has ideal auto-correlation properties and acceptable cross-correlation ones Walsh sequences has ideal cross-correlation properties but worse auto-correlation ones.

Fig.6 shows the performance of the proposed OCV of Rössler based CDMA with different number of users. It is obvious from this figure that the performance is almost the same regardless of the number of users. This result confirms the conclusion mentioned early in section IV upon the derivation of equation 13.

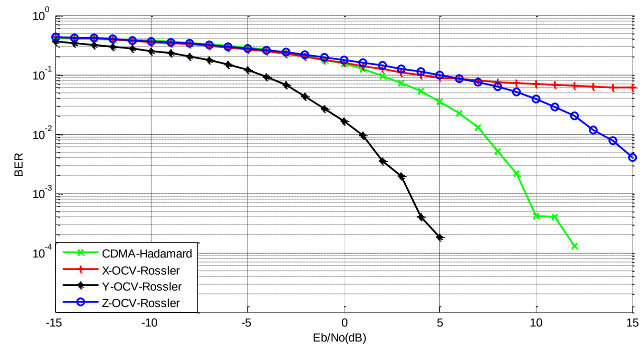

Fig.4. Performance of OCV of Rössler chaotic sequences x, y and z, and the orthogonal Walsh-Hadamard sequence for single user transmission in AWGN channel

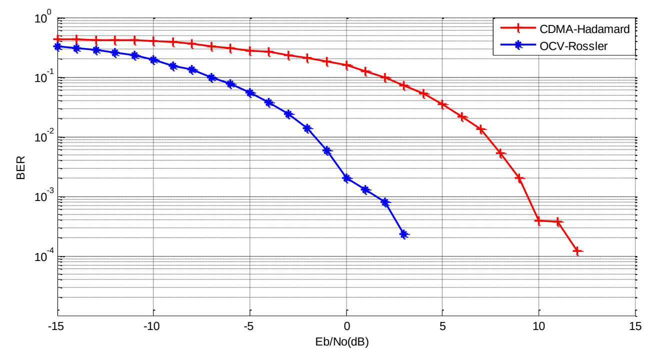

Fig.5. Performance of OCV of Rössler based CDMA (sequence y) and traditional CDMA for 4 users' transmission in AWGN channel.

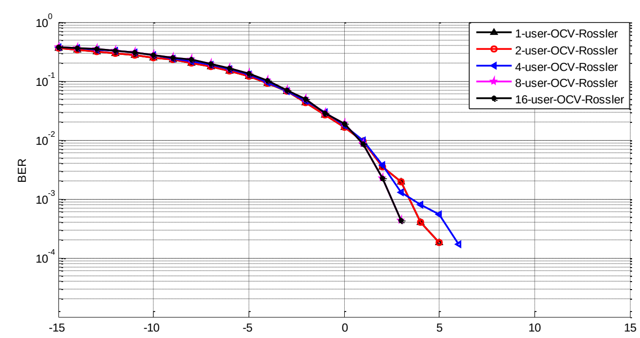

Fig.6. Performance of OCV of Rössler based CDMA for different number of users in AWGN channel.

B. Simulation Results in Rayleigh Fading Channel

Fig.7 shows the BER performances of CDMA based on OCVs generated from individual outputs of Rössler system (x, y, and z) for single user transmission in 3-tap random time-invariant Rayleigh multipath channel [16] together with traditional CDMA system. It can be seen in this figure that all OCVs generated from different Rössler system outputs performs the traditional CDMA. However, the best performance results are obtained in the case of (output y). It is worth noted that for single user case, the improvement in the case Rayleigh fading channel is almost the same as in AWGN case. For instant, at BER=10^{-3}, a gain of 6 dB in SNR is obtained Rayleigh fading channel. Fig.8 shows the BER performance comparison when the number of users is 4. Here, it can be easily noticed that the OCV Rössler based CDMA outperforms the traditional CDMA. The improvement in the case of 4 users is less than that of single user case. For instant, at BER=10^{-3}, a gain of 4 dB in SNR is obtained. Finally, Fig.9 depicts the performance of the proposed OCV of Rössler based CDMA with different number of users. It is obvious from this figure that the performance is improved as the number of users is decreased which is the similar case in traditional CDMA systems. However, this was not the case in AWGN channel discussed in Fig.6 where the performance was the same regardless the number of users. But in all cases the performance of the proposed scheme is better than the traditional CDMA for the similar number of users as we have seen for an example case N_u=4 in the previous figure.

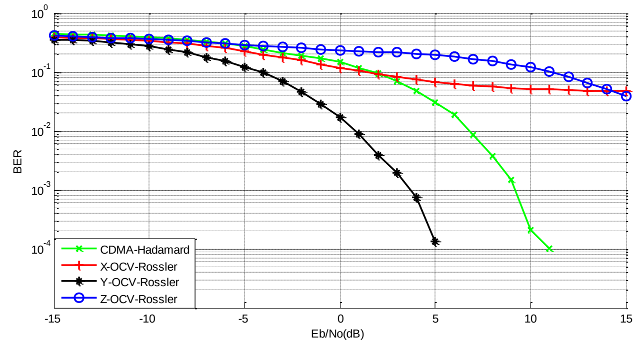

Fig.7. Performance of OCV of Rössler chaotic sequences x, y and z, and the orthogonal Walsh-Hadamard sequence for single user transmission in Rayleigh fading channel.

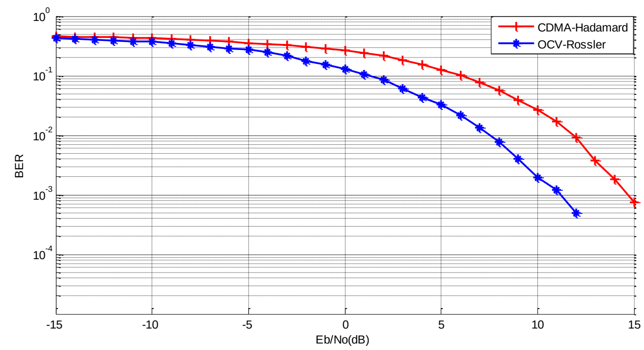

Fig.8. Performance of OCV of Rössler based CDMA (sequence y) and traditional CDMA for 4 users' transmission in Rayleigh fading channel.

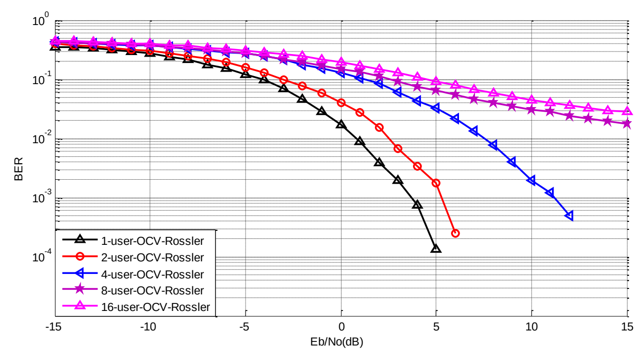

Fig.9. Performance of OCV of Rössler based CDMA for different number of users in Rayleigh fading channel.

VII. Conclusion

In this work, a multiple access communication system based on Rössler chaotic sequences has been proposed. The orthogonally among different users sequences has been improved by the use of orthogonal chaotic vector algorithm which is derived from Gram-Schmidt ortho-normalization process. A mathematical expression for BER performance of the proposed system has been derived for the case of transmission in AWGN channel. This expression showed that the proposed system has the same performance regardless of the number of users or spreading factor value. This result, which is considered superior in multiple access communication systems, has also been approved by computer simulations. By selecting the Rössler chaotic sequence that has the best performance, a 6 dB gain in SNR over traditional CDMA system is obtained. In Rayleigh fading channel, the simulation results showed that the performance of the proposed system is improved as the number of users is decreased. In this case, 6 dB and 4 dB gains in SNR are obtained over traditional CDMA system in single user and 4 users' transmissions respectively. The possible future improvements for the proposed work can be: the inclusion of channel coding prior chaotic modulation to improve BER and the use of differential chaos shift keying to model the users signals to easily achieve the synchronization requirements.

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