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Performance Improvement of Persistent Relay CSMA Using Random Relaying of Partitioned MDS Codeword Block on Error-Prone Channels

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***Abstract*—We propose incorporation of Random relaying of Partitioned Maximum Distance Separable codeword blocks (RP-MDS), which has been proposed for multi-hop cooperative relay networks, to Persistent Relay Carrier Sense Multiple Access (PRCSMA) on error-prone channels. The proposed protocol elaborately employs the powerful error-correcting capability of MDS codes into cooperative communication systems and introduces the incremental redundancy concept to PRCSMA. A destination node can reinforce an error-correcting capability when it receives a new frame. The performance of the proposed protocol is analyzed with a Markov model in terms of the average duration of a cooperation phase and the energy efficiency. Numerical results indicate that the proposed protocol can significantly improve the performance, compared to the original PRCSMA.**

***Index Terms*— Cooperative communications, Error-prone channels, Markov model, MDS codes, Persistent relay CSMA**

I. INTRODUCTION

COOPERATIVE communications with relay nodes have been recognized as one of effective and promising techniques in wireless/mobile communication systems. Relay standards are on the way to successful implementation in Long Term Evolution (LTE)-Advanced by the Third Generation Partnership Project (3GPP) and 802.16m by IEEE [1],[2]. Relay techniques have been enthusiastically investigated from the viewpoint of not only the physical (PHY) layer but also the data-link layer [2],[3]. In the PHY layer perspective, Multiple-Input and Multiple-Output (MIMO) and diversity techniques are attractive. In the data-link layer perspective, a number of Cooperative Automatic Repeat reQuest (C-ARQ) protocols have been proposed and analyzed. Particularly, the design of Medium Access Control (MAC) protocols employed between relay nodes and the destination node influences the performance, when two or more relay nodes collaborate on an identical channel.

Not a few MAC protocols for C-ARQ systems have been proposed recently. Dianati et al. [4] proposed a Node-Cooperation Stop-and-Wait (NCSW) ARQ protocol. The performance of NCSW with a single relay node was analyzed over two-state Markovian channels. Morillo and Garcia-Vidal [5] proposed a C-ARQ scheme with an integrated frame combiner. They analyzed the performance with round-robin cooperation among relay nodes and with Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA). Alonso-Zarate et al. [6],[7] proposed Persistent Relay CSMA (PRCSMA), which elaborately incorporates well-known IEEE 802.11 Distributed Coordination Function (DCF) [8]. In [6], the performance of PRCSMA was analyzed based on a steady-state two-dimensional Markov model proposed by Bianchi [7]. In the above literature [4]–[7], it is basically assumed that a node can correctly receive a transmitted frame if no frame collisions occur. Thus, when we consider a scenario where a channel adds errors to a non-colliding frame, it is expected that the use of error-correcting codes can improve the performance.

In this paper, we propose incorporation of Random relaying of Partitioned Maximum Distance Separable codeword block (RP-MDS) [8] to PRCSMA on error-prone channels. The proposed protocol elaborately takes advantage of the powerful error-correcting capability of MDS codes. Incorporating RP-MDS into PRCSMA may introduce effective performance improvement in accordance with the concept of incremental redundancy [10]. A destination node can take an opportunity to reinforce an error-correcting capability when it receives a new frame, even if it includes channel errors. The performance of the proposed protocol is analyzed with the aid of a Markov model. The accuracy of the model is verified by means of computer simulation.

The rest of the present paper is organized as follows: Section II presents a system model with two or more relay nodes. PRCSMA is briefly reviewed in Section III. In Section IV, after a short reminder of useful properties of MDS codes, the proposed protocol is described. Performance of the proposed protocol is analyzed in Section V, based on the analysis in [6].

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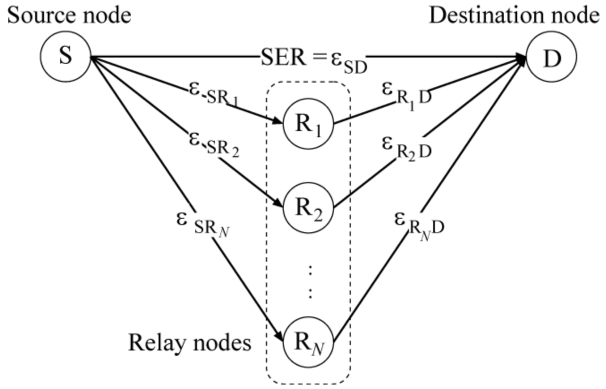

Fig. 1. System model with N relay nodes.

Numerical results are presented in Section VI in comparison with results obtained from computer simulation. Finally, Section VII concludes the present paper.

II. SYSTEM MODEL

Consider a wireless network consisting of a pair of source node S and destination node D with N relay nodes; R_1, R_2, ..., R_N, as shown in Fig. 1. All channels are half-duplex, so that a node cannot transmit and receive simultaneously. All nodes are located within their transmission range. Hence, each node can overhear ongoing transmission originating from other nodes. Let $\varepsilon_{\mathrm{SD}}$, $\varepsilon_{\mathrm{SR}_n}$, and $\varepsilon_{\mathrm{R}_n\mathrm{D}}$ be the symbol error probabilities on channels between source node S and destination node D, between source node S and relay node R_n, and between relay node R_n and destination node D, respectively, for $n = 1, 2, \ldots, N$.[1] If frame transmission from source node S resulted in erroneous reception at destination node D and if one or more relay nodes succeeded in error-free reception of the frame, then such relay nodes can collaboratively serve as supporters for frame retransmission. For effective use of cooperative communications, we generally assume that $\varepsilon_{\mathrm{SD}} > \varepsilon_{\mathrm{R}_n D}$. The duration in which relay nodes collaborate frame retransmissions is referred to as a "cooperation phase" [6]. Note that every frame is assumed to include an appropriate header and an ideal Frame Check Sequence (FCS) for error/collision detection, in addition to the payload.[2]

III. PERSISTENT RELAY CSMA (PRCSMA)

PRCSMA [6],[7] is a MAC protocol which sophisticatedly resolves frame collisions among transmission from relay nodes, based on IEEE 802.11 DCF [8]. Similarly to IEEE 802.11 DCF, each relay node in PRCSMA inserts random backoff delay before every frame transmission in a distributed manner according to its own contention window (CW). Let $\boldsymbol{m}$ denote a message block, which is generated at source node S. A DATA frame consists of a header, payload $\boldsymbol{m}$, and FCS. Note that the terms "message block $\boldsymbol{m}$" and "DATA frame" are used interchangeably hereafter, unless ambiguity arises.

The operation in PRCSMA is summarized as follows. The detailed description can be found in [6]. After erroneous reception of a DATA frame, destination node D broadcasts a Call For Cooperation (CFC) frame. If one or more relay nodes receive both the DATA frame and the CFC frame, then the cooperation phase is invoked. A relay node which joins in the cooperation phase is referred to as an active relay node. Active relay nodes simultaneously start the DCF operation, after the reception of the CFC frame followed by DIFS (Distributed Inter-Frame Space). When destination node D correctly receives a frame, it broadcasts an ACK frame to announce not only the correct reception of the DATA frame to source node S but also the completion of the cooperation phase to all the nodes.

An illustrative operational example with two active relay nodes, R_1 and R_2, is shown in Fig. 2. Both active relay nodes independently set their backoff counter to seven and a cooperation phase is invoked. The first DATA frame transmission from these relay nodes results in collision. The second transmission from relay node R_1 suffers from channel errors. Finally, an ACK frame is returned by destination node D corresponding to error-free reception of the second transmission from R_2. It completes the cooperation phase. Notice that source node S does not participate in a cooperation phase [6].

IV. PRCSMA WITH RANDOM RELAYING OF PARTITIONED MDS CODEWORD BLOCK

In a cooperation phase in PRCSMA on error-prone channels, destination node D may successively receive erroneous frames one by one in between backoff intervals. It suggests possibility to effectively utilize the concept of incremental redundancy [10], where the error-correcting capability at a receiving node is reinforced upon frame reception. In this context, we propose incorporating RP-MDS into PRCSMA. RP-MDS has been proposed for multi-hop cooperative relay networks on noisy channels [9]. The proposed protocol, designated as PRCSMA+RP-MDS, is described after some properties of MDS codes are reviewed.

A. MDS Codes

Denote a linear block code of length n and dimension k over a certain finite field by an $[n, k]$ code. An $[n, k]$ code is

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[1] Using the symbol error rate ε, we can evaluate the bit error rate as $1-\sqrt[b]{1-\varepsilon}$ when a symbol consists of b bits.

[2] The term "ideal" implies that the probability of undetected errors can be neglected.

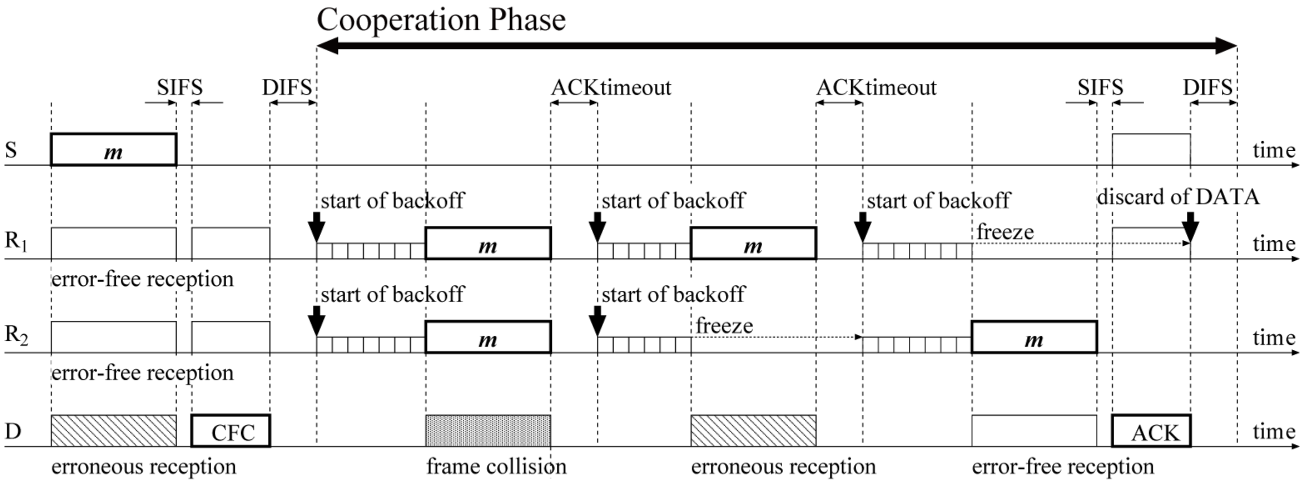

Fig. 2. Illustrative example of PRCSMA.

MDS if its minimum distance is $n-k+1$ [12]. A class of MDS codes, including Reed-Solomon codes, is known to be fruitful in advantageous properties [13]. Among them, the following two theorems; Theorems 8-4 and 8-6 in [13], respectively, are used afterward:

Theorem 1. For an $[n, k]$ MDS code, a receiver can recover the encoded message of length k, if it receives at least k code symbols with no errors. □

Theorem 2. Punctured MDS codes are also MDS, that is, the minimum distance of an $[n-p, k]$ punctured MDS code is $n-p-k+1 \geq k$. □

Let k be the length of a message block $\boldsymbol{m}$. Suppose a systematic $[Lk, k]$ MDS code *C*.[3] Let $\boldsymbol{G}$ be a generator matrix of *C*. It is clear that $\boldsymbol{G}$ is a $k \times Lk$ matrix. Let

$$\boldsymbol{G} = [\ \underset{k}{\boldsymbol{I}} \ |\ \underset{k}{\boldsymbol{G}_1} \ |\ \underset{k}{\boldsymbol{G}_2} \ | \cdots |\ \underbrace{\boldsymbol{G}_{L-1}}_{k}\ ] \tag{1}$$

be a partition of $\boldsymbol{G}$ into L blocks of identical size, where $\boldsymbol{I}$ and $\boldsymbol{G}_\ell$ are an identity matrix and a square matrix of order k for $\ell = 1, 2, \cdots, L-1$, respectively. Then, for a message block $\boldsymbol{m}$ of length k to be encoded, a codeword of *C* can be also partitioned into L codeword blocks $\boldsymbol{c}_\ell$ of length k;

$$\boldsymbol{c} = \boldsymbol{mG} = [\ \underset{k}{\boldsymbol{c}_0} \ |\ \underset{k}{\boldsymbol{c}_1} \ |\ \underset{k}{\boldsymbol{c}_2} \ | \cdots |\ \underbrace{\boldsymbol{c}_{L-1}}_{k}\ ], \tag{2}$$

where $\boldsymbol{c}_0 = \boldsymbol{m}$ and $\boldsymbol{c}_\ell = \boldsymbol{mG}$ for $\ell = 1, 2, \cdots, L-1$. From Theorem 1 and Theorem 2, the following corollary holds at a receiver when one or more codeword blocks $\boldsymbol{c}_\ell$ are received:

Corollary 1. Assume that u distinct codeword blocks, $\boldsymbol{c}_{\ell_1}$, $\boldsymbol{c}_{\ell_2}$, $\cdots, \boldsymbol{c}_{\ell_u}$, are received and that a receiver can identify the received codeword block number, $\ell_1, \ell_2, \ldots, \ell_u$, for $u \leq L$ and for $0 \leq \ell_1 < \ell_2 < \cdots < \ell_u < L$. Then, a k-symbol message $\boldsymbol{m}$ can be recovered, if either of the following conditions is satisfied: (i) at least one codeword block $\boldsymbol{c}_\ell$ is error-free; and (ii) the total number of errors occurred in the u codeword blocks is less than or equal to

$$t_u = \left\lfloor \frac{(u-1)k}{2} \right\rfloor, \tag{3}$$

where $\lfloor x \rfloor$ is the maximum integer not greater than x. □

Proof. Since every codeword block $\boldsymbol{c}_\ell$ consists of k symbols, it is straightforward from Theorem 1 that a receiver can recover the message $\boldsymbol{m}$ from one or more error-free codeword blocks. This leads to the first condition.

Next, aggregation of the u distinct received codeword blocks results in a codeword of a $[uk, k]$ punctured MDS code of *C*. Thus, t_u or less errors can be corrected according to Theorem 2, which provides the second condition. (Q.E.D.)

B. *Proposed Protocol (PRCSMA+RP-MDS)*

In PRCSMA, as described in Section III, what a relay node transmits is a replica of the message block $\boldsymbol{m}$. Therefore, it is required for destination node D to receive a frame with no errors in order to complete the cooperation phase. Therefore, erroneously received block is discarded. By contrast, in the proposed protocol, an active relay node randomly transmits one out of $L-1$ redundant MDS codeword blocks; $\boldsymbol{c}_1, \boldsymbol{c}_2, \ldots, \boldsymbol{c}_{L-1}$, after encoding the received message block $\boldsymbol{m}$ by *C*, as in (2). In addition, destination node D should store erroneously received frames in the buffer rather than discard in order to make provision for forthcoming decoding procedures.

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[3] Using a systematic code, an encoded message appears explicitly in the corresponding codeword vector. It implies that its generator matrix includes an identity matrix, as its submatrix. In the case that a given generator matrix is nonsystematic, we can convert it into a systematic form with the aid of appropriate elementary row operations [12].

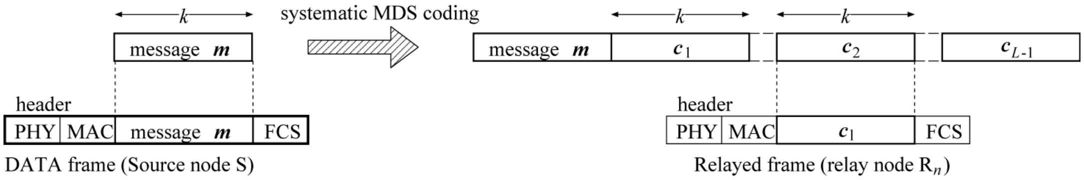


Fig. 3. Systematic encoding by MDS code and frame format of the proposed protocol.

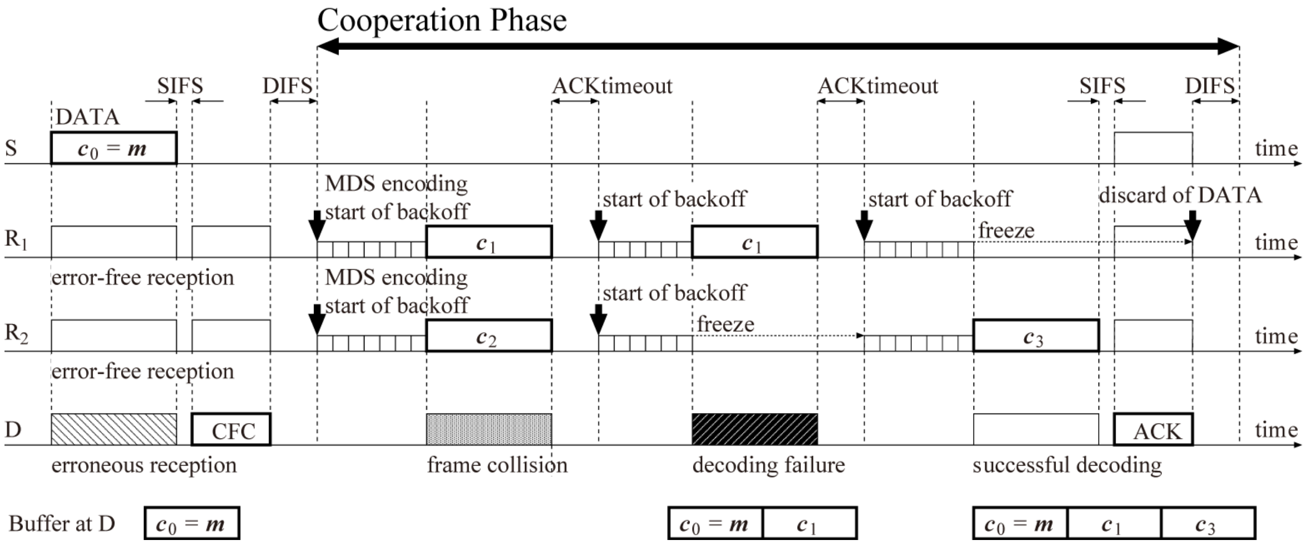


Fig. 4. Illustrative example of the proposed protocol (PRCSMA+RP-MDS).

A frame format used in the proposed protocol is depicted in Fig. 3. The codeword block number ℓ should be appropriately embedded in a header part, which can be digitized by $\lceil \log_2 L \rceil$ bits, where $\lceil x \rceil$ is the minimum integer not less than x. For small L, it can be negligible.

We describe the proposed protocol with the aid of an illustrative operational example, as shown in Fig. 4, with the same scenario as in Fig. 2. Destination node D stores an erroneous message block $\boldsymbol{m}$ into its buffer, when the cooperation phase starts. Two active relay nodes R_1 and R_2 independently encode $\boldsymbol{m}$ and randomly select one codeword block. In Fig. 4, R_1 selects $\boldsymbol{c}_1$ and R_2 selects $\boldsymbol{c}_2$. After frame collision occurs, each relay node re-selects one codeword block; R_1 does $\boldsymbol{c}_1$ again and R_2, $\boldsymbol{c}_3$, and restarts another backoff interval. Upon a reception of $\boldsymbol{c}_1$ from relay node R_1, destination node D aggregates the received $\boldsymbol{c}_1$ and $\boldsymbol{m}$ in the buffer, and then, decodes $[\,\boldsymbol{m} \mid \boldsymbol{c}_1\,]$ by a $[2k, k]$ punctured MDS code of C. According to Corollary 1, the message block $\boldsymbol{m}$ can be retrieved if $\boldsymbol{c}_1$ is received with no errors or if the total number of symbol errors in $[\,\boldsymbol{m} \mid \boldsymbol{c}_1\,]$ is not greater than $\lfloor k/2 \rfloor$. However, it fails in Fig. 4. At this time, destination node D stores two erroneous blocks, $\boldsymbol{m}$ and $\boldsymbol{c}_1$. Subsequently to reception of $\boldsymbol{c}_3$ from R_2, the message block $\boldsymbol{m}$ is successfully recovered by decoding $[\,\boldsymbol{m} \mid \boldsymbol{c}_1 \mid \boldsymbol{c}_3\,]$ with a $[3k, k]$ MDS code, which can correct up to k errors. Finally, an ACK frame is returned from destination node D. It completes the cooperation phase.

Notice that source node S does not take part in a cooperation phase similarly to the original PRCSMA [6]. Furthermore, for $L = 1$ the proposed protocol is reduced to the original PRCSMA, since no error-correcting capability is available at destination node D.

V. PERFORMANCE ANALYSIS

A. Assumptions and Markov model

In this section, we analyze the performance in the cooperation phase, based on the Markov model in [6]. We impose identical assumptions with [6]. Since we focus on the cooperation phase, it is presumed that destination node D has stored an erroneous message block $\boldsymbol{m}$. We assume that a cooperation phase start with N active relay nodes, that is, N relay nodes correctly receive both the message block $\boldsymbol{m}$ from source node S and the CFC frame from destination node D. We ignore erroneous reception of control frames; ACK frames, and

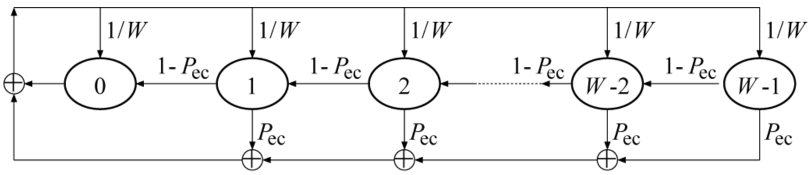


Fig. 5. Markov model with respect to the residual backoff counter at a relay node [6]

of a header part in each frame. The CW value at each relay node remains constant W all the time, that is, no doubling procedure is carried out even if frame transmission failure occurs, as opposed to the legacy DCF [8]. All frames involved in collision are to be retransmitted, until the cooperation phase is completed. We assume symmetric channels between relay node R_n and destination node D, that is, the symbol error rates between each relay node and destination node D are identical and independent; $\varepsilon_{\text{R}_1\text{D}} = \varepsilon_{\text{R}_2\text{D}} = \cdots = \varepsilon_{\text{R}_N\text{D}} = \varepsilon_{\text{RD}}$.

Then, a Markov model with respect to the value of backoff counter at a relay node is quoted in Fig. 5 from [6]. In Fig. 5, P_{ec} represents the probability that the cooperation phase ends in a slot. Each relay node transits states in the Markov model in a slot-by-slot basis. Note that a sojourn time varies depending on frame transmissions in the slot as in [6],[11].

B. Equations in Equilibrium

In equilibrium, an in-flow and an out-flow are balanced for every state in Fig. 5. Letting π_w be the steady-state probability of state w for $w = 0, 1, \ldots, W-1$. we obtain

$$\pi_w = \begin{cases} \pi_{w+1} + \dfrac{1}{W}\left(\pi_0 + \displaystyle\sum_{i=0}^{W-1} P_{\text{ec}}\pi_i\right), & \text{for } w = 0, 1, \ldots, W-2; \\ \dfrac{1}{W}\left(\pi_0 + \displaystyle\sum_{i=0}^{W-1} P_{\text{ec}}\pi_i\right), & \text{for } w = W-1, \end{cases} \tag{4}$$

as equilibrium equations. Solving the recursive expression and the boundary condition in (4) under the normalizing condition $\pi_0 + \pi_1 + \cdots + \pi_{W-1} = 1$, we have

$$\pi_w = \frac{P_{\text{ec}}\{1-(1-P_{\text{ec}})^{W-w}\}}{WP_{\text{ec}} - (1-P_{\text{ec}})\{1-(1-P_{\text{ec}})^{W-w}\}} \tag{5}$$

for $w = 0, 1, \ldots, W-1$. Since frame transmission occurs only when the backoff counter reaches to zero, the probability of i-frame collision can be given by

$$q_i = \Pr[i\text{-frame collision}] = \binom{N}{i}\pi_0^i(1-\pi_0)^{N-i} \tag{6}$$

for $i = 0, 1, \ldots, N$. Then, a slot is idle with probability q_0, one frame is transmitted in a slot with probability q_1, and frame collision takes place with probability $1 - q_0 - q_1$.

Next, we evaluate the probability P_{ec} of completing the cooperation phase. Destination node D stores an erroneous DATA frame $\boldsymbol{c}_0 = \boldsymbol{m}$, when the cooperation phase starts. The initial probability that the stored message block $\boldsymbol{m}$ includes e symbol errors is

$$\alpha(e) = \frac{1}{1-(1-\varepsilon_{\text{SD}})^k}\binom{k}{e}\varepsilon_{\text{SD}}^e(1-\varepsilon_{\text{SD}})^{k-e} \tag{7}$$

for $e = 1, 2, \ldots, k$. Then, when destination node D receives a non-colliding frame; say $\boldsymbol{c}_\ell$, $\ell > 0$ if $L > 1$, aggregating two blocks results in $[\ \boldsymbol{c}_0 \mid \boldsymbol{c}_\ell\ ]$. The cooperation phase ends, if either of two conditions in Corollary 1 is satisfied. The probability of error-free reception of a block of length k is $(1-\varepsilon_{\text{RD}})^k$. Taking into account the fact that up to $\lfloor k/2 \rfloor$ errors in $[\ \boldsymbol{c}_0 \mid \boldsymbol{c}_\ell\ ]$ can be corrected, we have the probability of successful decoding at destination node D as

$$P_{\text{succ}} = \begin{cases} (1-\varepsilon_{\text{RD}})^k, & \text{for } L = 1; \\ (1-\varepsilon_{\text{RD}})^k + \displaystyle\sum_{j=1}^{\lfloor k/2 \rfloor - 1}\sum_{e=1}^{\lfloor k/2 \rfloor - j}\binom{k}{j}\varepsilon_{\text{RD}}^j(1-\varepsilon_{\text{RD}})^{k-j}\alpha(e), & \text{for } L \geq 2. \end{cases} \tag{8}$$

In the case of $L > 2$, further gain on P_{succ} can be available when other code word blocks are received. However, we omit it in (8). Finally, we obtain

$$P_{\text{ec}} = q_1 P_{\text{succ}} = N\pi_0(1-\pi_0)^{N-1}P_{\text{succ}}. \tag{9}$$

C. Average Duration of Cooperation Phase

Once P_{ec} is provided, it implies that a cooperation phase consists of $1/P_{\text{ec}}$ slots in average, in which the last slot is the only successful one. Hence, the average numbers of idle slots, of slots with 1-frame transmission, and of slots with frame collision can be evaluated by

$$\#[\text{idle}] = \left(\frac{1}{P_{\text{ec}}} - 1\right)\frac{q_0}{1-q_1P_{\text{succ}}}, \tag{10}$$

$$\#[\text{1-frame transmission}] = 1 + \left(\frac{1}{P_{\text{ec}}} - 1\right)\frac{q_1(1-P_{\text{succ}})}{1-q_1P_{\text{succ}}}, \tag{11}$$

$$\#[\text{frame collison}] = \left(\frac{1}{P_{\text{ec}}} - 1\right)\frac{1-q_0-q_1}{1-q_1P_{\text{succ}}}, \tag{12}$$

respectively. Then, the average duration of a cooperation phase, given that N active relay nodes collaborate, is given by

Table I. Parameters for numerical results.

(a) Frame Format

PHY preamble	96	[μsec]
MAC header (incl. FCS)	34	[byte]
message length	512	[byte]
ACK length	14	[byte]
CFC length	14	[byte]
Block length: k	64	[symbol]

(b) DCF parameters

slot duration	10	[μsec]
DIFS: T_{DIFS}	50	[μsec]
SIFS: T_{SIFS}	20	[μsec]
ACK timeout: $T_{\text{ACKtimeout}}$	50	[μsec]
CW: W	16	

(c) Power [7]

Transmission: P_{T}	1900	[mW]
Reception: P_{R}	1340	[mW]
Channel sensing: P_{S}	1340	[mW]

(d) Channel

channel rate (DATA)	54	[Mbps]
channel rate (control)	6	[Mbps]
symbol error rate: (ε_{SD}, ε_{RD})	(0.1, 0.01)	
	(0.01, 0.001)	

$$\begin{aligned} \mathrm{E}[\text{duration} \mid N] &= T_{\text{succ}} + T_{\text{slot}}\,\#[\text{idle}] \\ &\quad + T_{\text{fail}}(\#[\text{1-frame transmission}] - 1 + \#[\text{frame collision}]) \\ &= T_{\text{succ}} + \left(\frac{1}{P_{\text{ec}}} - 1\right)\frac{T_{\text{slot}} q_0 + T_{\text{fail}}(1 - q_0 - q_1 P_{\text{succ}})}{1 - q_1 P_{\text{succ}}}, \end{aligned} \tag{13}$$

where T_{slot}, T_{succ}, and T_{fail} are the idle slot duration, the duration of successful message transmission consisting of the DATA and the ACK frames, SIFS and DIFT, and the duration of erroneous reception or frame collision consisting of the DATA frame and ACKtimeout, respectively. They are given as

$$T_{\text{succ}} = T_{\text{DATA}} + T_{\text{SIFS}} + T_{\text{ACK}} + T_{\text{DIFS}}, \tag{14}$$

$$T_{\text{fail}} = T_{\text{DATA}} + T_{\text{ACKtimeout}}, \tag{15}$$

where T_{DATA} and T_{ACK} are DATA frame duration and ACK frame duration, respectively, and other T_x's are the duration of element x.

D. Energy Efficiency in Cooperation Phase

Similarly to (13), the average of total energy consumed in a cooperation phase starting with N active relay nodes can be evaluated;

$$\begin{aligned} &\mathrm{E}[\text{energy consumption} \mid N] \\ &= E_{\text{succ}} + E_{\text{slot}}\,\#[\text{idle}] \\ &\quad + E_{\text{fail}}(1)(\#[\text{1-frame transmission}] - 1) \\ &\quad + \sum_{i=2}^{N} E_{\text{fail}}(i)\,\#[i\text{-frame collision}] \\ &= E_{\text{succ}} + \left(\frac{1}{P_{\text{ec}}} - 1\right)\frac{1}{1 - q_1 P_{\text{succ}}} \\ &\quad \times \left\{ E_{\text{idle}} q_0 + E_{\text{fail}}(1) q_1 (1 - P_{\text{succ}}) + \sum_{i=2}^{N} E_{\text{fail}}(i) q_i \right\}, \end{aligned} \tag{16}$$

where E_{succ} is the total energy consumed by N active relay nodes, source node S and destination node D in a successful slot, E_{idle} is that in an idle slot, and $E_{\text{fail}}(i)$ is that in an unsuccessful slot, given that i-frame collision occurs for $i = 1, 2, \ldots, N$, respectively. Let P_{T}, P_{R} and P_{S} be the power consumption at a node when transmitting, receiving, and sensing the channel, respectively. Then, E_{succ}, E_{idle}, and $E_{\text{fail}}(i)$ in (16) of the energy consumption in a slot are given by

$$\begin{aligned} E_{\text{succ}} &= P_{\text{T}} T_{\text{DATA}} + P_{\text{S}} T_{\text{SIFS}} + P_{\text{R}} T_{\text{ACK}} + P_{\text{S}} T_{\text{DIFS}} \\ &\quad + P_{\text{R}} T_{\text{DATA}} + P_{\text{S}} T_{\text{SIFS}} + P_{\text{T}} T_{\text{ACK}} + P_{\text{S}} T_{\text{DIFS}} \\ &\quad + N(P_{\text{R}} T_{\text{DATA}} + P_{\text{S}} T_{\text{SIFS}} + P_{\text{R}} T_{\text{ACK}} + P_{\text{S}} T_{\text{DIFS}}) \end{aligned} \tag{17}$$

$$E_{\text{idle}} = (N+2) P_{\text{S}} T_{\text{slot}} \tag{18}$$

$$\begin{aligned} E_{\text{fail}}(i) &= i(P_{\text{T}} T_{\text{DATA}} + P_{\text{S}} T_{\text{ACKtimeout}}) \\ &\quad + (N + 2 - i)(P_{\text{R}} T_{\text{DATA}} + P_{\text{S}} T_{\text{ACKtimeout}}) \end{aligned} \tag{19}$$

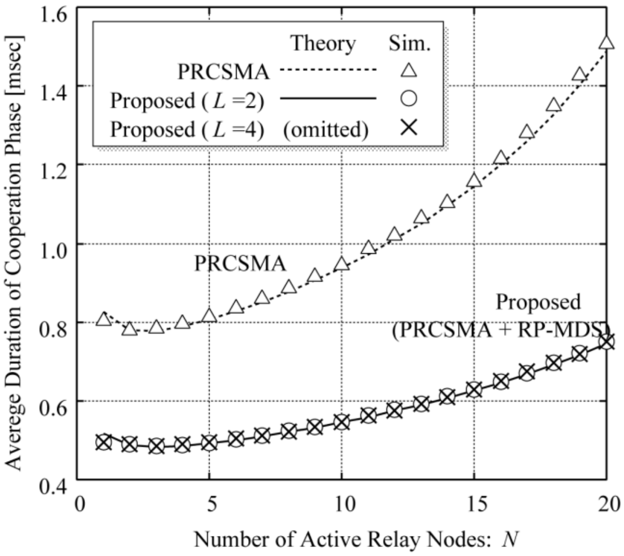
for $i = 1, 2, \ldots, N$, respectively. Finally, we define the energy efficiency η as

$$\eta = \frac{\mathrm{E}[\text{message length in bits}]}{\mathrm{E}[\text{energy consumption} \mid N]} \tag{20}$$

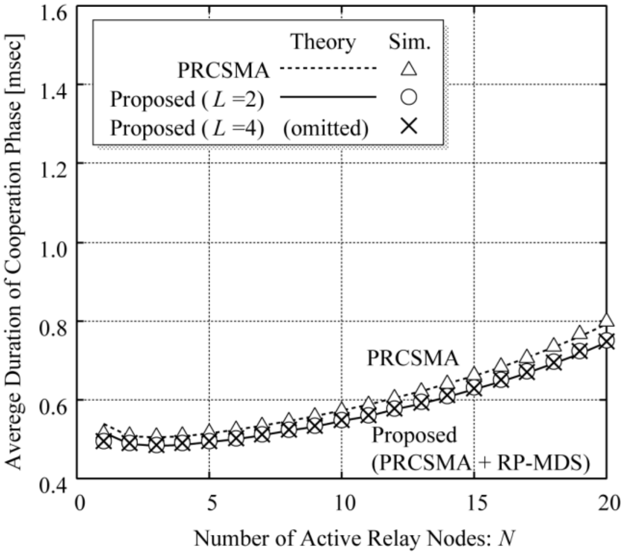
for a cooperation phase starting with N active relay nodes [7].

VI. NUMERICAL RESULTS

We examine the accuracy of the derived expressions with exhaustive computer simulation and compare the performance of the proposed protocol to that of PRCSMA. The values of parameters employed are shown in Table I. The frame format and the DCF parameters are basically extracted from [6], [7] and IEEE 802.11 standard [8]. The power consumption is identical with [7]. Two pairs of the symbol error rates are considered; $(\varepsilon_{\text{SD}}, \varepsilon_{\text{RD}}) = (0.1, 0.01)$ and $(\varepsilon_{\text{SD}}, \varepsilon_{\text{RD}}) = (0.01, 0.001)$. A block length in frame is $k = 64$ symbols and two types of MDS codes C are considered; a half-rate [128,64] MDS code for $L = 2$, a quarter-rate [256,64] MDS code for $L = 4$. Note that for $L = 2$, a relay node always transmits $\boldsymbol{c}_1$, since a codeword consists of two blocks; $\boldsymbol{c} = [\,\boldsymbol{c}_0 = \boldsymbol{m} \mid \boldsymbol{c}_1\,]$. The theoretical results for $L = 4$ are omitted in order to avoid the complexity to derive the

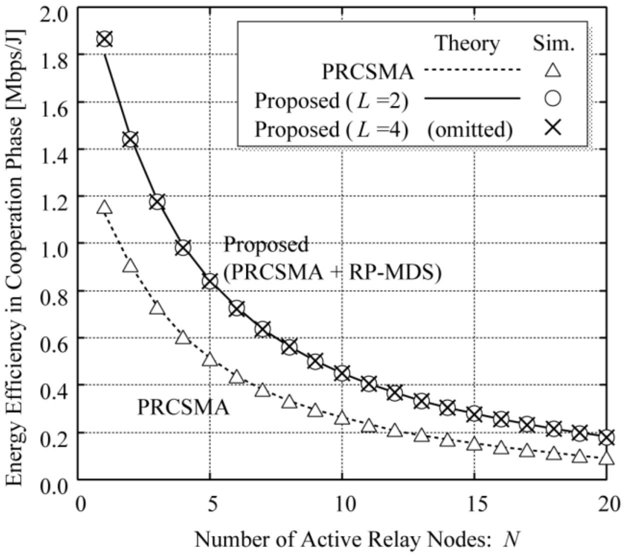


(a) For $(\varepsilon_{SD}, \varepsilon_{RD}) = (0.1, 0.01)$

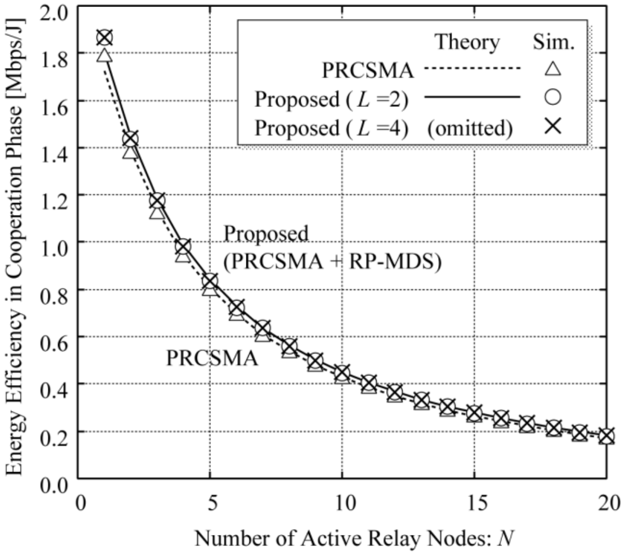


(b) For $(\varepsilon_{SD}, \varepsilon_{RD}) = (0.01, 0.001)$

Fig. 6. Average duration of cooperation phase.



(a) For $(\varepsilon_{SD}, \varepsilon_{RD}) = (0.1, 0.01)$



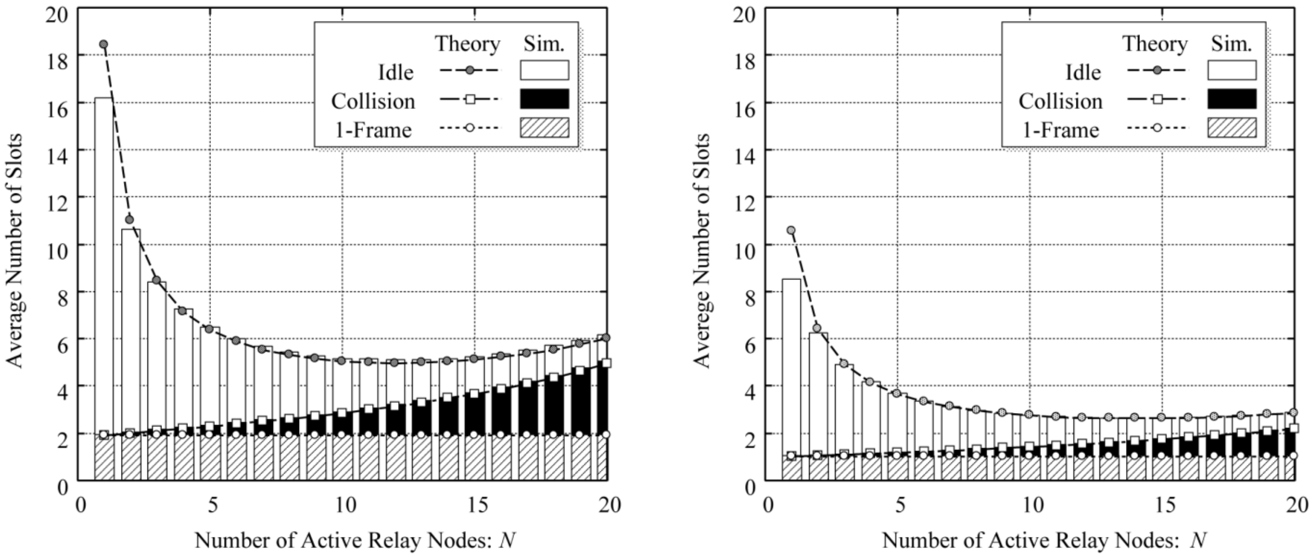
(b) For $(\varepsilon_{SD}, \varepsilon_{RD}) = (0.01, 0.001)$

Fig. 7. Energy efficiency in cooperation phase.

probability of successful decoding at destination node D, (8). The simulation program is written in C language and the results are obtained by averaging 10^5 trials of cooperation phases for given N. Recall that a cooperation phase starts with destination node D which has already held an erroneously received message block $\boldsymbol{m}$ including e errors with probability $\alpha(e)$, (7), for $e = 1, 2, \ldots, k$.

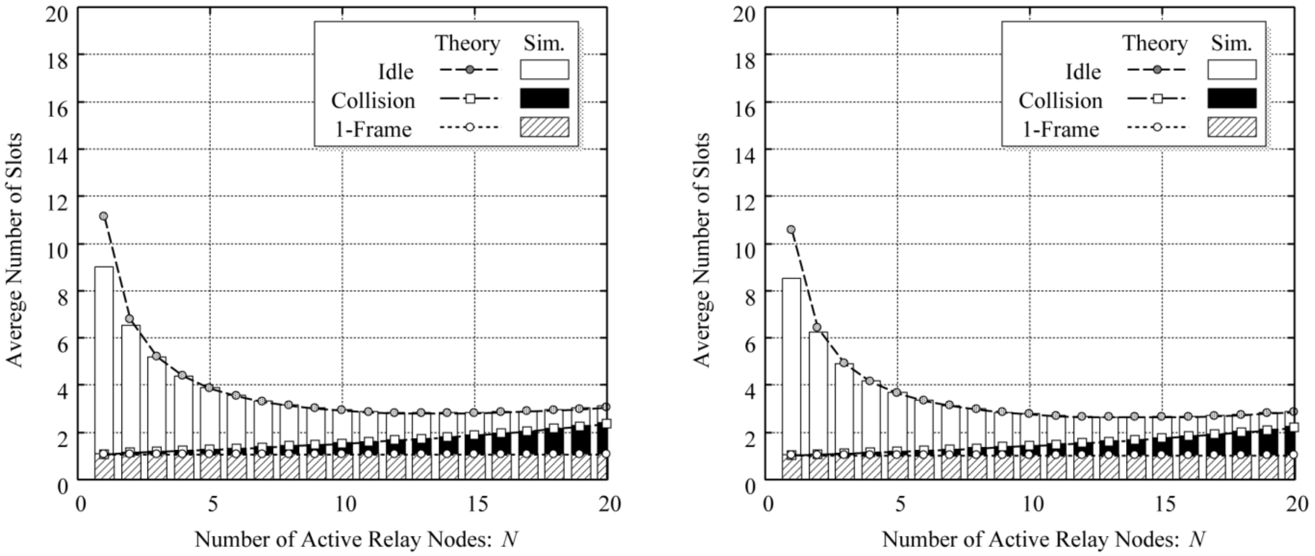
The average duration of a cooperation phase and the energy efficiency in a cooperation phase are presented in Fig. 6 and in Fig. 7, respectively, as a function of the number of active relay nodes N. The agreement between the theoretical and simulation results validates the accuracy of the derived expressions. Evidently, the proposed protocol, PRCSMA+RP-MDS, outperforms the original PRCSMA. In addition, it is revealed from computer simulation that the performance of the proposed protocol, PRCSMA+RP-MDS, for $L = 4$ coincides with that for $L = 2$, so that a half-rate MDS code suffices for PRCSMA+RP-MDS, since the computational complexity for longer MDS codes increases.

From Fig. 6(a) the proposed protocol can achieve approximately 40% reduction in the average duration of a cooperation phase for $(\varepsilon_{\mathrm{SD}}, \varepsilon_{\mathrm{RD}}) = (0.1, 0.01)$. The Energy efficiency is also improved by the proposed protocol, as shown in Fig. 7(a). However, it is clear from Fig. 6(b) and Fig. 7(b) that the degree of performance improvement by the proposed protocol decreases, as the channel quality is enhanced, since the



(a) Original PRCSMA (L=1) for ($\varepsilon_{\mathrm{SD}}$, $\varepsilon_{\mathrm{RD}}$) = (0.1, 0.01)

(b) Proposed protocol (L=2) for ($\varepsilon_{\mathrm{SD}}$, $\varepsilon_{\mathrm{RD}}$) = (0.1, 0.01)



(c) Original PRCSMA (L=1) for ($\varepsilon_{\mathrm{SD}}$, $\varepsilon_{\mathrm{RD}}$) = (0.01, 0.001)

(d) Proposed protocol (L=2) for ($\varepsilon_{\mathrm{SD}}$, $\varepsilon_{\mathrm{RD}}$) = (0.01, 0.001)

Fig. 8. Slot distribution in cooperation phase.

opportunity to take advantage of the error-correcting capability of the MDS code decreases at destination node D. For the values of parameters given in Table I, the probability of error-free reception of a frame is

$$(1-\varepsilon_{\mathrm{RD}})^k \approx \begin{cases} 0.526, & \text{for } \varepsilon_{\mathrm{RD}} = 10^{-2}; \\ 0.938, & \text{for } \varepsilon_{\mathrm{RD}} = 10^{-3}. \end{cases} \tag{21}$$

It implies that destination node D requires to receive a frame approximately 1/0.526 ≈ 1.90 times and 1/0.938 ≈ 1.07 times in average before the message $\boldsymbol{m}$ can be successfully recovered for $\varepsilon_{\mathrm{RD}} = 0.01$ and $\varepsilon_{\mathrm{RD}} = 0.001$, respectively. On the other hand, since destination node D can receive a frame other than $\boldsymbol{m}$ in the cooperation phase in the proposed protocol, the error-correcting decoding for a half-rate $[2k, k]$ MDS code can be carried out. In this case, at most $\lfloor k/2 \rfloor$ symbol errors can be corrected. Then, the probability of decoding failure is given as

$$\begin{aligned} &\sum_{i=\lfloor k/2 \rfloor+1}^{2k} \binom{2k}{i} \varepsilon_{\mathrm{RD}}^{i} (1-\varepsilon_{\mathrm{RD}})^{2k-i} \\ &\approx \begin{cases} 1.70\times 10^{-36}, & \text{for } \varepsilon_{\mathrm{RD}} = 10^{-2}; \\ 3.92\times 10^{-96}, & \text{for } \varepsilon_{\mathrm{RD}} = 10^{-3}, \end{cases} \end{aligned} \tag{22}$$

which is negligibly small, so that one frame reception other than $\boldsymbol{m}$ suffices for destination node D to recover the message block $\boldsymbol{m}$ in most cases. Therefore, the performance of the proposed protocol is independent of the value of $L \geq 2$.

Another observation from Fig. 6 is that the average duration slightly decreases for $N \leq 3$ and then it turns to increase. For $N \leq 3$, frame collisions are rare events. In addition, the more active relay nodes exist, the sooner the first transmission among relay nodes takes place in a cooperation phase. These observations decrease the average duration with or without the use of RP-MDS. However, for $N \geq 4$, the probability of frame collisions cannot be negligible and frame collisions add another backoff interval and frame retransmission. Hence, the average duration of a cooperation phase increase.

Next, as shown in Fig. 2 and Fig. 4, a cooperation phase consists of consecutive and synchronized slots. These slots are classified into three categories; idle slots of duration T_{slot}, slots with 1-frame transmission of duration of T_{succ} or T_{fail}, and slots with frame collisions of duration of T_{fail}. Clearly, one slot in slots with 1-frame transmission is a successful slot of duration of T_{succ} which is the last slot in a cooperation phase. Fig. 8 shows the average number of slots in a cooperation phase, classified into the three categories. The average number of these slots are theoretically evaluated as (10)–(12). Predictably, the average number of slots with frame collision monotonously increases in proportion to increment of the number of active relay nodes. The average number of idle sots decreases on the contrary. The incorporation of RP-MDS successfully facilitates the completion of a cooperation phase. Therefore, the average number of slots can be reduced by the use of the proposed protocol. Particularly, the use of RP-MDS can approximately halve the average number of slots for $(\varepsilon_{\text{SD}}, \varepsilon_{\text{RD}}) = (0.1, 0.01)$, comparing Fig. 8(a) to Fig. 8(b).

VII. CONCLUSIONS

We have proposed incorporation of RP-MDS, which has been proposed for multi-hop cooperative relay networks [9], to PRCSMA on error-prone channels. The proposed protocol elaborately takes advantage of the powerful error-correcting capability of MDS codes into cooperative communication systems and introduces the incremental redundancy concept to PRCSMA. A destination node can reinforce the error-correcting capability when it receives a new frame. Assuming symmetric relay channels, we have analyzed the performance of the proposed protocol in terms of the average duration of a cooperation phase and the energy efficiency in a cooperation phase. The accuracy of theoretical results has been validated by means of computer simulation. Numerical results have indicated that the proposed protocol can improve the performance, compared to the original PRCSMA, particularly over severe noisy channels. It is also revealed that the use of a half-rate MDS code suffices in the proposed protocol.

Further work includes, for example, the consideration of header errors and feedback errors, and the extension to bidirectional communication systems and to the use of network coding.

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