# System Identification Solution for Developing an AdeptThree Robot Arm Model 

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#### Abstract

An AdeptThree robot arm is a SCARA robot that widest working envelope in its class. The robot should be represented in its mathematical model in order to be able to study the behavior and also to design a controller for the robot. Jacobean matrix of a robot can be used to determine the position and motion of a robot end-effector. In this paper, an AdeptThree robot was selected to develop its represented model and a Jacobean model was also built for the robot. System identification method had been chosen to build four models of each joint of the AdeptThree robot in time domain system. All of the models were accepted after passing both whiteness and independent tests in residual analysis plots.


Index Terms- AdeptThree robot, SCARA, System Identification, Jacobean.

## I. Introduction

Robots are mostly used to replace workers in such dangerous, high precision or in routine and repeated works. Robots often do a better job than human. A robot arm is a type of robot which works similarly to a human arm. A robot arm or also called manipulator is composed of a set of joints separated in space by the arm links and it looks like our own wrist and elbow [1].

Due to imprecision, it is not easy to control the robot motion to do an appropriate job perfectly. The imprecision will happen along the robot motion or operation and it might be caused by its structure or control algorithm. Dynamic parameters of the robot also will not able to be brought together into the robot model because of the imprecision. On the other hand, knowledge about the robot parameter values should match to the robot system in detail to get a good robot model.

The uncertainty in modeling a robot will make difficulty to design its good model. Therefore, system identification is often needed to include the uncertainty into the robot model. System identification is widely used in engineering and nonengineering areas [2]. The system identification gives possibility to construct a model from experimental data.

An AdeptThree robot arm is a selectively compliant assembly robot arm (SCARA) manufactured by the Adept Company. In general, traditional SCARA's are 4 -axis robot arms within their work envelope. They have the jointed twolink arm layout similar to our human arms and commonly used

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in pick-and-place, assembly, and packaging applications. As a SCARA robot, an AdeptThree robot has 4 joints which mean that it has 4 degree of freedom (DOF). The Adept robot has been designed with complete components including operating system and programming language namely $\mathrm{V}+[1]$.

Some studies and researches were done in the field of robot arm i.e. Bulent [3] studied about possibility to used two cooperating SCARA manipulator while doing a job, Mustafa [4] implemented fuzzy and neural network for control 3-DOF robot manipulator, Rasit [5] presented a neural network for solving 3-joint robot inverse kinematics, Toshio [6] introduced a neural network in case of robotic arm on-line learning,. The work presented in this paper is aimed to investigate and to develop the mathematical model of an AdeptThree robot arm and its Jacobean model.

## II. SUPPORTING THEORY

## A. System identification

System or process identification is the field of mathematical modeling of systems (processes) from test or experimental data. The input-output data are usually collected from an identification test or experiment that is designed to make the measured data maximally informative about the system properties that are of interest to the user [7]. A system identification procedure can be exploited in which experimental data is used directly [8], whereas identification produces usually input-output models [9].

A general procedure for process estimation includes the following steps [9]:

- Determination of the model structure. This often makes use of empirical experience about the process, or information from some basic experiments
- Parameter estimation. The procedure for parameter estimation depends on the type and characteristics of the process input, as well as the desired model structure.
- Model verification. A suitable model should agree with the experimental data, it should describe the process accurately, and it should meet the purpose in which it was obtained for. Further, it can be verified whether the parameters obtained are within physical limits. It is also possible to reduce the model and to compare it with the original model to see if a simpler model suffices.
Three ways to define a mathematical model in system
identification are:
- White box modeling. A model will be built from a clear system which is mean that all of its parameters can be found.
- Grey box modeling. In this case, many parameters are known but not all of these parameters. A measurement can be used to construct unknown parameter(s). This is a collaborated model between white box and black box modeling.
- Black box modeling. In this kind of system identification, no parameter is known. The way to get the model is by using data measurement. Data input and output will be compared to implement the relationship of both data.
In order to get a good model, a model should be validated. MATLAB System Identification Toolbox has provided two kinds of model validation by using residual analysis plots or Akaike's Final Prediction Error (FPE) criterion.


## Residual Analysis Plots

Residual analysis plots can be used for either time domain or frequency domain input-output data. In time domain validation data, the plots will show both autocorrelation function of residuals for each model and cross-correlation between the input and the residuals for each input-output pairs. Conversely, for frequency domain validation data, the plots will give both estimated power spectrum of residuals for each model and transfer-function amplitude from the input to the residuals for each input-output pairs [10].

Residual analysis consists of whiteness test and independent test. In the whiteness test criteria, a good model has autocorrelation function inside the confidence interval of the corresponding estimates, indicating that the residuals are uncorrelated. The confidence interval corresponds to the range of residuals value with specific probability of being statistically significant for the system. On the other hand, in independent test, a good model has residuals uncorrelated with past inputs. The evidence indicates that the model does not describe how part of the output relates to the corresponding input [10].

## Akaike's Criterion

Akaike's criterion provides a measure of model quality by simulating the situation where the model is tested on different data set. The FPE of Akaike's criterion is formulated as follows [10].

$$
\begin{align*}
& \text { FPE }=V\left(\frac{1+\frac{d}{N}}{1-\frac{d}{N}}\right)  \tag{1}\\
& V=\operatorname{det}\left(\frac{1}{N} \sum_{1}^{N} \varepsilon\left(t, \theta_{N}\right)\left(\varepsilon\left(t, \theta_{N}\right)\right)^{T}\right) \tag{2}
\end{align*}
$$

Where $V$ is the loss function, $d$ is the number of estimated parameters, $N$ is the values number in estimation data set and $\theta_{N}$ represents the estimated parameters. By assuming that FPE is asymptotic for $\mathrm{d} \ll \mathrm{N}$, the eq. 1 can be simplified as:

$$
\begin{equation*}
F P E=V(1+2 d / N) \tag{3}
\end{equation*}
$$

The FPE can be computed for either linear or non linear model and according to Akaike's theory, a model that has smallest FPE is the most accurate model [10].

## B. Kinematics

The aim of kinematics is to define relative position of a frame to its original coordinates. Basic vector algebra can be used to solve the problem in kinematics. The sequences to get the relative position are finding the A matrixes, building an arm T matrix and calculating it with the coordinate position which is desired.

## A matrix

The A matrix is a homogenous $4 \times 4$ transformation matrix which describe the position of a point on an object and the orientation of the object in a three dimensional space. The A matrixes can be built by using the Denavid-Hartenberg (D-H) convention with the relation of frame $i-1$ and base frame $i$ given in following equation[11].

$$
A_{i}^{i-1}=\left[\begin{array}{cccc}
\cos \theta_{i} & -\cos \alpha_{i} \sin \theta_{i} & \sin \alpha_{i} \sin \theta_{i} & a_{i} \cos \theta_{i}  \tag{4}\\
\sin \theta_{i} & \cos \alpha_{i} \cos \theta_{i} & -\sin \alpha_{i} \cos \theta_{i} & a_{i} \sin \theta_{i} \\
0 & \sin \alpha_{i} & \cos \alpha_{i} & d_{i} \\
0 & 0 & 0 & 1
\end{array}\right]
$$

The matrix contains link parameters and joint parameters. The link parameters are $\alpha_{i}$ which is the twist of the link $i$ and $\alpha_{i}$ which is the length of link $i$. The joint parameters are joint angle $\theta_{i}$ and the joint offset $d_{i}$. The matrix can be simplified as follows [11].

$$
A_{i}^{i-1}=\left[\begin{array}{cc}
R_{i} & P_{i}  \tag{5}\\
0 & 1
\end{array}\right]
$$

Where $R_{i}$ is a $3 \times 3$ rotation matrix and $P_{i}$ is a $3 \times 1$ translation matrix.

## Arm $T$ Matrix

The arm T matrix is a kinematics chain of transformation. It can be built from 2 or more A matrixes which is shown by following equation [11].

$$
\begin{equation*}
T \equiv T_{n}^{0}=A_{1}^{0} A_{2}^{1}, \ldots, A_{n}^{n-1} \tag{6}
\end{equation*}
$$

The arm T matrix is usable to obtain coordinates of a point in terms of the base link.

## Direct Kinematics

To get the position of the frame which is relative to the base frame, the arm T matrix should be multiplied with the coordinate matrix $r_{n}$ given by position of the end effector. A parameter, which is a scaling factor, should be added to the matrix $r_{n}$ to become a $4 \times 1$ matrix and be able to be multiplied. The final matrix of the robot kinematics is [11],[12],[13]:

$$
r_{o}=\left[\begin{array}{cc}
R_{i} & P_{i}  \tag{7}\\
0 & 1
\end{array}\right]\left[\begin{array}{c}
r_{n} \\
1
\end{array}\right]
$$

The direct kinematics can be found in the matrix $P_{i}$. The $X$, $Y$ and $Z$ positions are $P_{1}, P_{2}$ and $P_{3}$ respectively.

## Velocity Transformation

Given joint variable coordinate of the end effectors $q=\left[q_{1}\right.$, $\left.q_{2} \ldots q_{n}\right]^{\mathrm{T}}$. On the matrix $q, q=\theta$ for a rotary joint and $q=d$ for a prismatic joint. Then a generally nonlinear transformation from joint variable $q(t)$ to $y(t)$ is $y=h(q)$ and the velocities of joint axes is given as[11][12]:

$$
\begin{equation*}
\dot{y}=\frac{\partial h}{\partial q} \dot{q}=J \dot{q} \tag{8}
\end{equation*}
$$

Where $J$ is the Jacobean manipulator and inverse of the Jacobean $J^{l}$ relates the change in the end-effector to the change in axis displacements.

The Jacobean is an important component in many robot control algorithms. Many ways to design a Jacobean matrix of a robot arm were presented. Zomaya et al. (1999) had presented three kinds of algorithms to perform a Jacobean matrix. The other algorithm which uses tool configuration vector $\omega$ was provided by Manjunath and Ardil (2007) and Frank et al.(2006) as follows:

$$
\begin{equation*}
w(q)=\left[\frac{P_{i}}{R_{i 3} e^{\left(q_{n} / \pi\right)}}\right] \tag{9}
\end{equation*}
$$

Therefore the Jacobean matrix can be found using following equation [11][12].

$$
\begin{equation*}
J(q)=\frac{\partial w}{\partial q_{i}} \tag{10}
\end{equation*}
$$

## III. Results

## A. Physical System

An AdeptThree robot consists of 4 joints and the joint motions are revolution, revolution, prismatic and revolution (RRPR) respectively from 1 st until 4th joint [1]. Fig. 1 shows the schematic figure and the joints of AdeptThree robot.


Fig. 1 Physic of AdeptThree Robot

The AdeptThree robot is controlled by a central processing unit (CPU). The CPU has input/output devices and communication devices. The input/output devices are keyboard, mouse, monitor, printer and robot arm itself. The CPU also supports the communication using some devices, such as area network (LAN), digital input/output (DIO), and serial port. With almost two meters in diameter of working area, the AdeptThree robot has widest working envelope in its class [1].

## B. Mathematical Model

Many parameters of the AdeptThree robot are not provided by its manufacturing company. Therefore, the appropriate model will be constructed by assuming the robot as black box models and gaining the models by using MATLAB System Identification Toolbox based on the data input-output measured from the robot. The input to the black box is voltage and the output is either joint angle or link offset.

A number of experimental joint tracking has been made to collect the joint data. From the data, transfer functions of each joint were designed in time domain system identification. Each joint model is built using a $2^{\text {nd }}$ order system with a gain (K), a zero $\left(\mathrm{T}_{\mathrm{z}}\right)$, an integrator and a pole $\left(\mathrm{T}_{\mathrm{p}}\right)$. The estimated model of first joint was fixed at $3^{\text {rd }}$ iteration and the parameters of the model are $\mathrm{K}_{1}=0.22773, \mathrm{~T}_{\mathrm{z} 1}=0.004892$ and $\mathrm{T}_{\mathrm{p} 1}=0.0015202$. Second joint model was found at $5^{\text {th }}$ iteration and its model parameters are $K_{2}=0.24557, \mathrm{~T}_{\mathrm{z} 2}=0.004892$ and $\mathrm{T}_{\mathrm{p} 2}=$ 0.0014101. The model parameters of third joint and fourth joint are $K_{3}=1.218, K_{4}=2.1202, \quad \mathrm{~T}_{23}=0.001106$, $\mathrm{T}_{\mathrm{z} 4}=0.001517, \mathrm{~T}_{\mathrm{p} 3}$ and $\mathrm{T}_{\mathrm{p} 4}=0.001$. The third and fourth joints were defined in $7^{\text {th }}$ iteration and $1^{\text {st }}$ iteration for third and fourth joint respectively. Henceforward the transfer functions of first to fourth joints are given in the following equations.

$$
\begin{align*}
& H_{1}(s)=\frac{0.004892 s+0.22773}{0.0015202 s^{2}+s}  \tag{12}\\
& H_{2}(s)=\frac{0.004892 s+0.24557}{0.0014101 s^{2}+s}  \tag{13}\\
& H_{3}(s)=\frac{0.001106 s+1.218}{0.001 s^{2}+s}  \tag{14}\\
& H_{4}(s)=\frac{0.001517 \mathrm{~s}+2.1202}{0.001 \mathrm{~s}^{2}+\mathrm{s}} \tag{15}
\end{align*}
$$

System Identification toolbox in MATLAB will also generate model validation automatically after a model had been built. The default method for validating model in this MATLAB toolbox is residual analysis plots. In this case the models for first, second, third and fourth joints are match at $99.81 \%, 99.81 \%, 100 \%$ and $100 \%$ respectively.

Using residual analysis plots for time domain input-output data, autocorrelations and cross correlations of each joint model were illustrated in fig. 2 downward from $1^{\text {st }}$ to $4^{\text {th }}$ joint. The figure shows that the model residuals for both correlations of each joint are laid between the confidential lines (dotted lines). This is the evidence that all models are acceptable.


Fig. 2 a) Autocorrelations and b) cross correlations

## C. The Jacobean

The steps to get the relative position are calculating the A matrices and multiplying the A matrix of each joint to build an arm T matrix. To build the A matrices, first step that must be done is to define the DH parameters of the robot as shown in table 1 .

TABLE I

| TABLE I |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| DH PARAMETERS OF AN ADEPTTHREE ROBOT |  |  |  |  |
| $l^{\text {st }}$ | $\theta_{i}$ | $d_{i}$ | $a_{i}$ | $\alpha_{i}$ |
| $2^{\text {nd }}$ | $\theta_{1}$ | $h$ | $l_{1}$ | 0 |
| $3^{\text {rd }}$ | $\theta_{2}$ | 0 | $l_{2}$ | 0 |
| $4^{\text {th }}$ | 0 | $-d_{3}$ | 0 | $\pi$ |

Joints and links parameters are shown in fig.1. The joints are where the motion in the arm occurs while the links are a fixed construction. Thus the link has a fixed relationship between the joints [11].

The arm T matrix of the AdeptThree robot which is found using eq. 2 and eq. 3 is as follows.

$$
T=\left[\begin{array}{cccc}
-s_{4} s_{1+2}+c_{4} c_{1+2} & -s_{4} c_{1+2}-c_{4} s_{1+2} & 0 & l_{2} c_{1+2}+l_{1} c_{1}  \tag{16}\\
s_{4} c_{1+2}+c_{4} s_{1+2} & -s_{4} s_{1+2}+c_{4} c_{1+2} & 0 & l_{2} s_{1+2}+l_{1} s_{1} \\
0 & 0 & 1 & -d_{4}-d_{3}+h \\
0 & 0 & 0 & 1
\end{array}\right]
$$

Where $c_{1}, c_{2}, c_{4}$ are the cosines of $\theta_{1}, \theta_{2}$, and $\theta_{4} ; s_{1}, s_{2}, s_{4}$ are the sinus of $\theta_{l}, \theta_{2}$, and $\theta_{4} ; l_{1}$ and $l_{2}$ are the length of link 1 , and link $2 ; d_{3}$ and $d_{4}$ are the length of link 3 and $4 ; h$ is the length of $1^{\text {st }}$ joint column.

By using the arm T matrix, it is possible to calculate the values of $\left(P_{x}, P_{y}, P_{z}\right)$ with respect to the fixed coordinate system. Length of link $4\left(d_{4}\right)$ was removed from the equations due to no hand gripper. Then the end-effector position obtained with direct kinematics is equations which are listed in
following equations.

$$
\begin{align*}
& P_{X}=l_{2} c_{1+2}+l_{1} c_{1} \\
& P_{Y}=l_{2} s_{1+2}+l_{1} s_{1}  \tag{17}\\
& P_{Z}=h-d_{3}-d_{4}
\end{align*}
$$

Where constant parameters $l_{1}=559 \mathrm{~mm}, l_{2}=508 \mathrm{~mm}$, and $h=876.3 \mathrm{~mm}$. The direct kinematics can be used to find the end-effector coordinate of the robot movement by substituting the constant parameter values to the above equation.

The manipulator's speeds are calculated by means of the Jacobean matrix. By knowing the speeds of the articulations we obtain the speed with which the SCARA describes a trajectory. With the equations of the kinematics models, the following matrix is obtained.

$$
w(q)=\left[\begin{array}{c}
l_{2} c_{1+2}+l_{1} c_{1}  \tag{18}\\
l_{2} s_{1+2}+l_{1} s_{1} \\
-d_{4}-d_{3}+h \\
0 \\
0 \\
e^{\left(\frac{\left.\theta_{4}\right)}{\pi}\right)}
\end{array}\right]
$$

From the formulation above, 4 tool configuration vectors can be built and a Jacobean $6 \times 4$ matrix is found from the vectors. The vectors are constructed using eq. 7 and as the result four sub matrices forming the Jacobean matrix are obtained as follows.

$$
\begin{gather*}
J_{1}\left(q_{1}\right)=\left[\begin{array}{c}
-l_{2} s_{1+2}-l_{1} s_{1} \\
l_{2} c_{1+2}+l_{1} c_{1} \\
0 \\
0 \\
0 \\
0
\end{array}\right] \quad J_{2}\left(q_{2}\right)=\left[\begin{array}{c}
-l_{2} s_{1+2} \\
l_{2} c_{1+2} \\
0 \\
0 \\
0 \\
0
\end{array}\right] \\
J_{3}\left(q_{3}\right)=\left[\begin{array}{c}
0 \\
0 \\
\frac{-1}{0} \\
0 \\
0
\end{array}\right] \quad J_{4}\left(q_{4}\right)=\left[\begin{array}{c}
0 \\
0 \\
0 \\
\hline 0 \\
0 \\
e^{\left(\frac{\theta_{4}}{\pi}\right)} / \pi
\end{array}\right] \tag{19}
\end{gather*}
$$

The Jacobean is formed from the $4^{\text {th }}$ vectors as $J(q)=\left[J_{1}\left(q_{1}\right), J_{2}\left(q_{2}\right), J_{3}\left(q_{3}\right), J_{4}\left(q_{4}\right)\right]$. The similar Jacobean was also build by Manjunath and Ardil (2007) for a developed SCARA as follows.

$$
J(q)=\left[\begin{array}{cccc}
-l_{2} s_{1+2}-l_{1} s_{1} & -l_{2} s_{1+2} & 0 & 0  \tag{20}\\
l_{2} c_{1+2}+l_{1} c_{1} & l_{2} c_{1+2} & 0 & 0 \\
0 & 0 & \frac{-1}{0} & 0 \\
\hline 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & e^{\left(\frac{\theta_{4}}{\pi}\right)} / \pi
\end{array}\right]
$$

To be specific for an AdeptThree robot, the constant parameters of the robot should be substituted into the eq. 20 . Finally, the Jacobean matrix for the AdeptThree robot is found
in the following equation.

$$
J(q)=\left[\begin{array}{cccc}
-508 s_{1+2}-559 s_{1} & 508 s_{1+2} & 0 & 0  \tag{21}\\
508 c_{1+2}+559 c_{1} & -508 c_{1+2} & 0 & 0 \\
0 & 0 & \frac{-1}{0} & 0 \\
\hline 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & e^{\left(\frac{\theta_{4}}{\pi}\right)} / \pi
\end{array}\right]
$$

The above Jacobean matrix is not a square matrix because it has $6 \times 4$ components. Because the Jacobean is not a square matrix, the matrix can not be inverted. Therefore, it can only be used to calculate the robot joint velocities. The Jacobean matrix of the AdeptThree robot which is built with using tool configuration vectors indicates that about 24 of 32 or about $3 / 4$ of the matrix components are equal to zero.

## IV. CONCLUSION

MATLAB System Identification Toolbox was used in this project to develop the mathematical models of each joint of the AdeptThree robot. The models were designed with the data measurements, which contain voltages as the input and either joint angles or link offset as the output. Results of model validation show that all of the joint models are accepted.

In different way, Jacobean matrix of an AdeptThree robot also had been developed. The Jacobean consists of $6 \times 4$ components and it can be used for defining the joint speed of the robot. The Jacobean matrix of the AdeptThree robot is simple to be built by using tool configuration vectors.

## REFERENCES

[1] Rehiara Adelhard Beni, Smit Wim, Controller Design of a Modeled AdeptThree Robot Arm, Proceedings of the 2010 International Conference on Modelling, Identification and Control, Japan, July 17-19, 2010, pp 854-858.
[2] Mei Li, Chen Chen, Wenlin Liu, Identification Based on MATLAB, Proceedings of the 2009 International Workshop on Information Security and Application, November 21-22, 2009, China, pp 523-525.
[3] Bulent Ozkan, Kemal Ozgoren, Invalid Joint Arrangements and Actuator Related Singular Configuration of a System of two Cooperating SCARA Manipulator, Journal of Mechatronics, Vol.11, 2001, pp 491-507.
[4] Mustafa Nil, Ugur Yuzgec, Murat Sonmez, Bekir Cakir, Fuzzy Neural Network Based Intelligent Controller for 3-DOF Robot Manipulator, Procedings of Intelligent Manufacturing Systems Symposium, Sakarya University Turkey, May 29-31, 2006.
[5] Rasit Koker, Cemil Oz, Tarik Cakar, Huseyin Ekiz, A Study of Neural Network Based Inverse Kinematics Solution for a Three-Joint Robot, Journal of Robotics and Autonomous System, Vol.49, 2004, pp 227-234
[6] Toshio Tsuji, Yoshiyuki Tanaka, On-line Learning of Robot Arm Impedance using Neural Network, Journal of Robotics and Autonomous System, Vol.52, 2005, pp 257-271.
[7] Yucai Zhu, Multivariable System Identification for Process Control, Elsevier, London, 2001.
[8] Raymond A. de Callafon, Paul M. J. Van den Hof, Multivariable Feedback Relevant System Identification of a Wafer Stepper System, IEEE Transactions on Control Systems Technology, Vol. 9, No. 2, March 2001, pp 381-390.
[9] Ján Mikleš, Miroslav Fikar, Process Modelling, Identification, and Control, Springer-Verlag, Berlin, 2007.
[10] Lennart Ljung, System Identification Toolbox 7 User's Guides, The MathWorks Inc., Natick, 2010. Available at http://www.mathworks.com/help/pdf_doc/ident/ident.pdf
[11] Frank L.Lewis, Darren M.Dawson, Chaouki T.Abdallah, Robot Manipulators Control, Marcel Dekker, Inc., New York, 2006.
[12] Manjunath T.C., Ardil C., Development of a Jacobean Model for 4Axes indigenously developed SCARA System, International Journal of Computer and Information Science and Engineering, Vol. 1 No 3, 2007, pp 152-158.
[13] John Faber Archila Diaz, Max Suell Dutra, Claudia Johana Diaz, Design and Construction of a Manipulator Type Scara, Implementing a Control System, Proceedings of COBEM, 19th International Congress of Mechanical Engineering, November 5-9, 2007, Brasília.
[14] Zomaya A.Y., Smitha H., Olariub S., Computing robot Jacobians on meshes with multiple buses, Microprocessors and Microsystems, no. 23, 1999, pp 309-324.

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