Performance of Random Relaying of MDS Codeword Blocks in Cooperative Multi-Hop Networks over Random Error Channels

Katsumi Sakakibara, Shuji Kobayashi, and Jumpei Taketsugu

***Abstract*—We propose random relaying of MDS codeword blocks in cooperative multi-hop networks and evaluate its performance over random error channels by means of an absorbing Markov chain. The proposed scheme can apply to cooperative multi-hop networks with the arbitrary number of relay nodes at each hop stage and require no feedback channels between nodes. In the proposed scheme, a message block is encoded by a MDS code of coding rate $1/L$, where L is an integer. A relay node partitions a codeword of the MDS code into L blocks and transmits one randomly selected codeword block. When each relay node receives two or more codeword blocks of the MDS code, it aggregates the blocks, corrects channel errors with the aid of the MDS code or its punctured code. Numerical results indicate that significant improvement can be achieved by incorporating random relaying of MDS codeword blocks.**

***Index Terms*—Cooperative multi-hop networks, MDS codes, Non-homogeneous absorbing Markov chain, Outage probability**

I. Introduction

INFORMATIN transmission technologies over multi-hop or relay channels have been extensively investigated for the use of wireless ad-hoc and sensor networks in the recent years [1]–[4]. In multi-hop networks, a source and a destination nodes may be connected through two or more routes. In such a case, it has been reported that collaborative transmission among nodes can significantly improve performance [2]–[6]. In [2], [3], cooperative relaying techniques are discussed in conjunction with space-time coding [7] at the physical level. Miyano et al. presented that the multi-route diversity gain can be obtained in terms of the packet error rate in a two-hop network [2]. Koike et al. demonstrated performance improvement for inter-vehicle networks [3]. In [4], performance of cooperative relay networks is discussed at the link level. However, relayed packets are identical to the original message transmitted at the source node. From the viewpoint of error-correcting codes at the link level, erasure decoding of repetition codes for received blocks has been at the base of performance enhancement of collaborative transmission [4]. By extending the erasure decoding concept of packets, the use of fountain codes [8] is proposed for collaborative two-hop relay networks [5], [6]. Using a fountain code, a transmitting node continuously sends packets until a receiver obtains the sufficient number of error-free packets to recover the message. It implies that it may potentially occur for a receiving node to indefinitely wait for the message recovery. Also, in the protocols proposed in [5], [6], feedback channels from the receiver to the transmitter or intra-channels connecting the relay nodes are inevitable in order to control packet transmissions at a transmitting node. Note that in [4]–[6], channels between nodes are modeled by erasure channel at packet level, so that a received packet with one or more channel errors is treated as packet erasure. Therefore, there will be still possibilities for further improvement by incorporating the error-correcting capability of the code employed, if we consider random error channels. To this end, the authors have proposed the use of MDS (Maximum Distance Separable) codes to multi-route cooperative multi-hop relay networks and evaluated the outage probability [9]. In the protocol proposed in [9], the number of relay nodes at each hop stage should coincide with the inverse of the coding rate of the MDS code.

In this paper, we propose random relaying of an MDS codeword block at each relay node and evaluate its link-level performance over random error channels by means of a nonhomogeneous absorbing Markov chain [10]. The proposed scheme is generalization of the scheme in [9], so that it can relax the restriction on the number of relay nodes located at each hop stage. A message encoded by an MDS code of coding rate $1/L$ is partitioned into L codeword blocks, where L is a positive integer. When each relay node receives codeword blocks of the MDS code, it aggregates the blocks, corrects channel errors with the MDS code, encodes the message by the MDS code, and relays one randomly selected codeword block. It should be emphasized here that the protocols in this paper and in [9] require no feedback channels nor intra-channels among relay nodes in contrast with ones in [5], [6].

Manuscript received November 2, 2011. A part of this work was supported by Japan Society for the Promotion of Science under Grant-in-Aid for Scientific Research (C) (KAKENHI no. 21560408).

Katsumi Sakakibara is with the Department of Information and Communication Engineering, Okayama Prefectural University, Soja, 719-1197 Japan. (phone: +81-866-94-2109; Fax: +81-866-94-2199; e-mail: sakaki@c.okapu.ac.jp).

Shuji Kobayashi is with the Department of Information and Communication Engineering, Okayama Prefectural University, Soja, 719-1197 Japan. (e-mail: s-koba@c.oka-pu.ac.jp).

Jumpei Taketsugu is with the Department of Information and Communication Engineering, Okayama Prefectural University, Soja, 719-1197 Japan. (e-mail: taketugu@c.oka-pu.ac.jp).

The rest of the present paper is organized as follows: Section II briefly reviews some useful properties of MDS codes and describes the protocol with the system model. In Section III, the expression of the outage probability is derived after constructing a non-homogeneous absorbing Markov chain. Numerical results are presented in Section IV. Finally, Section V concludes the present paper.

II. SYSTEM MODEL

A. MDS Codes

Denote a linear block code of length n and dimension k over a certain finite field by an $[n, k]$ code. An $[n, k]$ code is MDS if its minimum distance is $n-k+1$. A class of MDS codes, including Reed-Solomon codes, is known to be fruitful in advantageous properties [11]. Among them, the following two theorems; Theorems 8-4 and 8-6 in [11], are used afterward:

Theorem 1 *For an $[n, k]$ MDS code, a receiver can recover the encoded message, if it receives at least k code symbols with no errors.*

Theorem 2 *Punctured MDS codes are also MDS, that is, the minimum distance of an $[n-p, k]$ punctured MDS code is $n-p-k+1$, if $n-p \geq k$.*

Suppose an $[Ln, k]$ MDS code C, whose coding rate is $1/L$. Let $\boldsymbol{G}$ be a generator matrix of C. It is clear that $\boldsymbol{G}$ is a $k \times Lk$ matrix. Let

$$\boldsymbol{G} = [\,\underbrace{\boldsymbol{G}_1}_{k} \,|\, \underbrace{\boldsymbol{G}_2}_{k} \,|\cdots|\, \underbrace{\boldsymbol{G}_L}_{k}\,] \tag{1}$$

be the partition of $\boldsymbol{G}$ into L blocks of identical size, where $\boldsymbol{G}_\ell$ is a square matrix of order k for $\ell = 1, 2, \cdots, L$. Similarly, a codeword of C can be also partitioned into L codeword blocks $\boldsymbol{c}_\ell$ of length k;

$$\boldsymbol{c} = \boldsymbol{m}\boldsymbol{G} = [\,\underbrace{\boldsymbol{c}_1}_{k} \,|\, \underbrace{\boldsymbol{c}_2}_{k} \,|\cdots|\, \underbrace{\boldsymbol{c}_L}_{k}\,] \tag{2}$$

where $\boldsymbol{m}$ is a message block of length k and $\boldsymbol{c}_\ell = \boldsymbol{m}\boldsymbol{G}_\ell$ for $\ell = 1, 2, \cdots, L$. Then, from Theorem 1 and Theorem 2, the following corollary holds when a relay node receives one or more codeword blocks $\boldsymbol{c}_\ell$:

Corollary 1 *Assume that u distinct codeword blocks, $\boldsymbol{c}_{\ell_1}$, $\boldsymbol{c}_{\ell_2}$, $\cdots$, $\boldsymbol{c}_{\ell_u}$, are received $(u \leq L)$ and that a receiver can identify the received codeword block number, ℓ_1, ℓ_2, $\cdots$, ℓ_u, Then, a k-symbol message $\boldsymbol{m}$ can be recovered, if either of the following conditions is satisfied:*

1) At least one codeword block $\boldsymbol{c}_\ell$ is error-free;

2) The total number of errors occurred in the u codeword blocks is less than or equal to

$$t_u = \left\lfloor \frac{(u-1)k}{2} \right\rfloor \tag{3}$$

where $\lfloor x \rfloor$ is the maximum integer not greater than x.

Proof: Since every codeword block $\boldsymbol{c}_\ell$ consists of k symbols, it apparently follows from Theorem 1 that a receiver can recover the message $\boldsymbol{m}$ from one or more error-free codeword blocks. This leads to the first condition.

Aggregation of the u distinct received codeword blocks results in a codeword of a $[uk, k]$ punctured MDS code. Thus, t_u or less errors can be corrected according to Theorem 2, which provides the second condition.

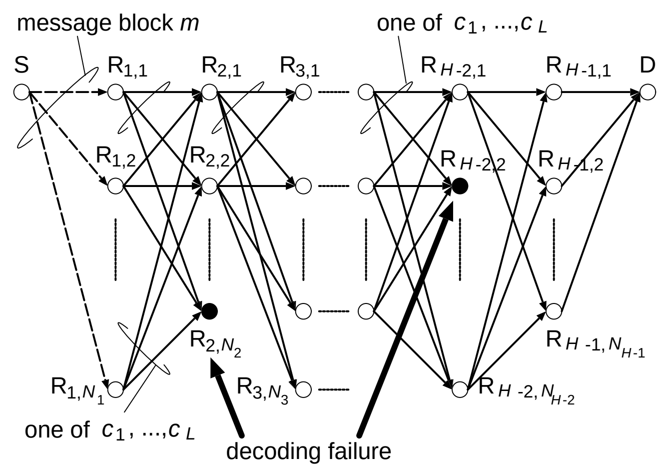

Fig. 1. Topology of cooperative *H*-hop networks.

B. Cooperative Multi-Hop Networks

Consider a cooperative H-hop network, where nodes are orderly aligned on a two-dimensional plane as shown in Fig. 1. The h th hop stage consists of N_h relay nodes, $\mathrm{R}_{h,1}$, $\mathrm{R}_{h,2}$, $\cdots$, R_{h,N_h}, for $h = 1, 2, \cdots, H-1$. A relay node $\mathrm{R}_{h,n}$ can receive blocks from the N_{h-1} nodes in the previous hop stage, $\mathrm{R}_{h-1,1}$, $\mathrm{R}_{h-1,2}$, $\cdots$, $\mathrm{R}_{h-1,N_{h-1}}$, for $h = 2, 3, \cdots, H-1$. However, in contrast with topology in [5], [6], no links exist among relay nodes in the identical hop stage. Note that topology shown in Fig. 1 can be viewed as generalization of that in [3], [4], [9].

Source node S broadcasts the message $\boldsymbol{m}$ of k-symbol length to N_1 first relay nodes $\mathrm{R}_{1,1}$, $\mathrm{R}_{1,2}$, $\cdots$, R_{1,N_1} together with FCS (Frame Check Sequence) for error detection. Each of the first relay nodes $\mathrm{R}_{1,n}$ for $n = 1, 2, \cdots, N_1$ can retrieve $\boldsymbol{m}$, if no symbol errors occur. Then, a successful relay node $\mathrm{R}_{1,n}$ encodes $\boldsymbol{m}$ by C and broadcasts one codeword block $\boldsymbol{c}_j$ which is randomly selected among L blocks $\boldsymbol{c}_1$, $\boldsymbol{c}_2$, $\cdots$, $\boldsymbol{c}_L$ to N_2 second relay nodes $\mathrm{R}_{2,1}$, $\mathrm{R}_{2,2}$, $\cdots$, R_{2,N_2}. FCS is also appended to $\boldsymbol{c}_j$. The codeword block number j is included in the block header, as assumed in Corollary 1.

Let us denote by i the number of the first relay nodes which have transmitted a codeword block $(i = 0, 1, \cdots, N_1)$. If we assume that no transmitted blocks are lost on the channels, each of the second relay nodes $\mathrm{R}_{2,n}$ for $n = 1, 2, \cdots, N_2$ receives i codeword blocks. Let u be the number of different codeword blocks included in the i received codeword blocks. Clearly the relation $u = 1, 2, \cdots, \min[i, L]$ holds, since duplication may occur due to random selection of a transmitted codeword block among L blocks. Also, i and u are identical for all relay nodes in the second hop stage. Aggregation of the u different codeword blocks out of the i received blocks results in a codeword of a $[uk, k]$ punctured MDS code of C.

Then, each second relay node $\mathrm{R}_{2,n}$ can recover the message block $\boldsymbol{m}$ according to Corollary 1. In this sense, it is possible that all the N_2 second relay nodes can recover the message $\boldsymbol{m}$ by decoding of C or its punctured version, even when all the received blocks from the first relay nodes suffer from channel errors. This particular property may enhance the performance of cooperative multi-hop networks. Similarly to the first relay node, a node $\mathrm{R}_{2,n}$ which has successfully recovered the message $\boldsymbol{m}$ encodes $\boldsymbol{m}$ by C and broadcasts one randomly selected codeword block. The identical procedure proceeds hop-by-hop, until destination node D receives codeword blocks or all the relay nodes at a certain hop stage fail to recover the message $\boldsymbol{m}$. In Fig. 1, two relay nodes R_{2,N_2} and $\mathrm{R}_{H-2,2}$ fail to recover $\boldsymbol{m}$, so that they do not broadcast any codeword blocks further. Note that the number of divided codeword blocks L is constant for all the nodes.

III. Analysis

A. Assumptions

We model each channel between neighboring nodes by a random error channel of the symbol error rate ε, which is identical and independent for mathematical tractability. It is also assumed that no blocks are lost on the channels and that no retransmission mechanisms are employed, that is, no feedback channels are required in contrast with [5], [6]. Furthermore, we ignore both the undetected error probability of FCS and the probability of erroneous reception of the block header in order to make the analysis tractable and to reveal the potential capabilities of the proposed scheme.

B. Probability of Successful Decoding

Denote by $r_i^{(h)}$ the probability that a node at the h th hop stage can successfully retrieve the message block $\boldsymbol{m}$ upon i -packet reception for $i = 0, 1, \cdots, N_{h-1}$ and $h = 1, 2, \cdots, H$, where $N_0 = 1$. It is apparent that for $i = 1$, no errors are allowable for the message to be recovered. Thus,

$$r_1^{(h)} = r_1 = (1-\varepsilon)^k \tag{4}$$

for $h = 1, 2, \cdots, H$, since no error-correcting capability can be available.

For $i \geq 2$, we can take advantage of the error-correcting capability of C or its punctured version, unless all the i received blocks are the identical codeword block. From the first condition in Corollary 1, a node can recover the message $\boldsymbol{m}$ from one or more error-free codeword blocks among the i received ones, which occurs with probability $1-(1-r_1)^i$.

Next, assuming that all the i received blocks include symbol errors, we derive the probability for the second condition in Corollary 1 to be satisfied. Let $q(u \mid i)$ be the conditional probability that there exist u different codeword blocks in the i received blocks $(u = 1, 2, \cdots, \min[i, L])$. This probability can be given by

$$q(u \mid i) = \binom{L}{u} \sum_{j=0}^{u} (-1)^j \binom{u}{j} \left(\frac{u-j}{L} \right)^i \tag{5}$$

since the probability is equivalent to the classical occupation problem [12].[1] Suppose aggregation of u distinct codeword blocks out of the i received blocks, which is a codeword of a $[uk, k]$ punctured MDS code of C. Let e_j be the number of symbol errors occurred in the j th aggregated codeword block for $j = 1, 2, \cdots, u$. Then, $e_j \in \{1, 2, \cdots, k\}$, since all the received blocks are assumed to be erroneous. Define a u -tuple of positive integers by $\boldsymbol{E}_j = (e_1, e_2, \cdots, e_u) \in \mathbb{N}^u$ for $u = 1, 2, \cdots, \min[i, L]$, where $\mathbb{N}^u$ is a set of direct product of u positive integers. For given u ($u = 1, 2, \cdots, \min[i, L]$), we then denote by Γ_u a subset of $\{\boldsymbol{E}_u\}$ whose norm is less than or equal to t_u;

$$\Gamma_u = \{\boldsymbol{E}_u \mid \boldsymbol{E}_u \in \mathbb{N}^u, e_1 + e_2 + \cdots + e_u \leq t_u\} \tag{6}$$

Note that Γ_u represents a set of the error distribution which is correctable by a $[uk, k]$ punctured MDS code of C, provided that all the u distinct codeword blocks include errors. Let δ_u denote the probability of correct decoding of the $[uk, k]$ MDS code, when all the u aggregated codeword blocks include channel errors. It follows from the i.i.d. assumption of the channel error process that

$$\delta_u = \sum_{\boldsymbol{E}_u \in \Gamma_u} \prod_{j=1}^{u} \binom{k}{e_j} \varepsilon^{e_j} (1-\varepsilon)^{k-e_j} \tag{7}$$

for $u = 1, 2, \cdots, \min[i, L]$. The rest of $i-u$ received codeword blocks should be erroneous, whose probability is given by $(1-r_1)^{i-u}$.

[1] According to [12], the probability that there are exactly $L-u$ empty cells when i balls are randomly distributed in L cells is given by

$$p_{L-u}(i, L) = \binom{L}{L-u} \sum_{j=0}^{u} (-1)^j \binom{u}{j} \left(1 - \frac{L-u+j}{L} \right)^i$$

which leads us to (5).

As a result, the probability $r_i^{(h)}$ is given by

$$r_i^{(h)} = 1-(1-r_1)^i + \sum_{u=1}^{\min[i,L]} \delta_u \cdot (1-r_1)^{-u} q(u \mid i) \tag{8}$$

for $i = 2,3.\cdots, N_{h-1}$ and $h = 2,3,\cdots,H$. Here, we neglect the probability of decoder error of the MDS codes [11], that is, the probability of erroneous decoding when more than t_u symbol errors occur, since it is considerably small.[2]

C. Markovian Model

The decoding procedure at a relay node is dependent only on the number of received blocks i , equivalently, the number of relay nodes in the previous hop stage which succeed in recovering the message $\boldsymbol{m}$, since no transmitted blocks are lost on channels from the assumption. Let $S^{(h)}$ be a random variable representing the number of successful relay nodes at the h th hop stage for $h = 1,2,\cdots,H-1$. Apparently,

$$S^{(h)} \in \Omega^{(h)} = \{0,1,\cdots,N_h\} \tag{9}$$

Then, we can construct a non-homogeneous Markovian model with respect to the evolution of $S^{(h)}$, as shown in Fig.2. The state $S^{(h)}$ evolves in a hop-by-hop manner. However, once no relay nodes are successful at a certain hop stage, the message block $\boldsymbol{m}$ can not be delivered to the next hop stage. It implies that it is impossible for the system to escape from $S^{(h)} = 0$, so that it is an absorbing state.

Let $p_{i,j}^{(h)}$ represent a transition probability from State i at the ($h-1$)th hop to State j at the h th hop; $S^{(h-1)} = i \in \Omega^{(h-1)}$ and $S^{(h)} = j \in \Omega^{(h)}$. From the assumption that no blocks are lost on the channels, each relay node at the h th hop stage receives i codeword blocks if $S^{(h-1)} = i$. The i.i.d. assumption on the channels error process provides us

$$\begin{aligned} p_{i,j}^{(h)} &= \Pr[S^{(h)} = j \mid S^{(h-1)} = i] \\ &= \binom{N_h}{j} \left\{r_i^{(h)}\right\}^j \left\{1-r_i^{(h)}\right\}^{N_h - j} \end{aligned} \tag{10}$$

for $i \in \Omega^{(h-1)} - \{0\}$, $j \in \Omega^{(h)}$ and $h = 2,3,\cdots$, since every relay node decodes the i received codeword blocks in an independent manner. Clearly, for $i = 0$, we have

$$p_{0,j}^{(h)} = \begin{cases} 1 & \text{for } j = 0 \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

for any h , since State 0 is an absorbing state.

Then, using the transition probabilities $p_{i,j}^{(h)}$, we derive the probability distribution $\pi^{(h)}$ with respect to the number of successful relay nodes $S^{(h)}$ at the h th hop stage.

First, consider the N_1 relay nodes at the first hop stage. Each of these nodes receives the original uncoded message $\boldsymbol{m}$ transmitted by the source node S. Thus, they can obtain no benefit from the MDS code C . The probability of successful recovery of the message $\boldsymbol{m}$ at each first relay node is r_1 . It follows from the independent operation assumption of relay nodes that $\pi^{(1)}$ is subject to the binomial distribution;

$$\begin{aligned} \pi^{(1)} &= \left[\Pr[S^{(1)} = 0], \Pr[S^{(1)} = 1], \cdots, \Pr[S^{(1)} = N_1] \right] \\ &= \left[(1-r_1)^{N_1}, \binom{N_1}{1} r_1 (1-r_1)^{N_1 - 1}, \cdots, r_1^{N_1} \right] \end{aligned} \tag{12}$$

Next, the probability distribution of $S^{(h)}$ for $h = 2,3,\cdots$ can be obtained in a recursive manner:

$$\begin{aligned} \pi^{(h)} &= \left[\Pr[S^{(h)} = 0], \Pr[S^{(h)} = 1], \cdots, \Pr[S^{(h)} = N_1] \right] \\ &= \pi^{(h-1)} \begin{bmatrix} 1 & 0 & \cdots & 0 \\ p_{1,0}^{(h)} & p_{1,1}^{(h)} & \cdots & p_{1,N_h}^{(h)} \\ \vdots & \vdots & & \vdots \\ p_{N_{h-1},0}^{(h)} & p_{N_{h-1},1}^{(h)} & \cdots & p_{N_{h-1},N_h}^{(h)} \end{bmatrix} \\ &= \pi^{(h-1)} \boldsymbol{P}^{(h)} = \pi^{(h-2)} \boldsymbol{P}^{(h-1)} \boldsymbol{P}^{(h)} = \cdots \\ &= \pi^{(1)} \boldsymbol{P}^{(2)} \boldsymbol{P}^{(3)} \cdots \boldsymbol{P}^{(h)} \end{aligned} \tag{13}$$

where $\boldsymbol{P}^{(h)}$ is the transition matrix from the $(h-1)$ th hop stage to the h th hop stage for $h = 2,3,\cdots,H-1$.

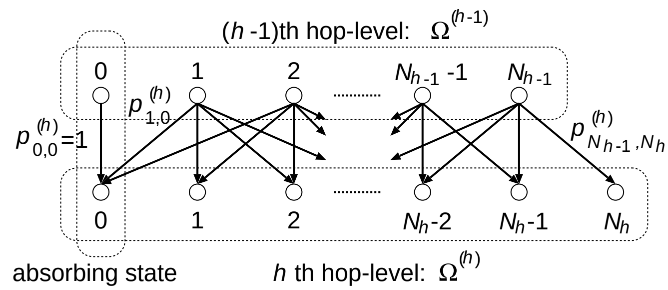

Fig. 2. Non-homogeneous Markovian model with respect to the number of successful nodes at the hop stage $S^{(h)}$.

D. Outage Probability

Destination node D, which is located at H hops away from the source node S, receives $S^{(H-1)}$ blocks. We define the outage probability as the probability that the destination node D can not recover the message $\boldsymbol{m}$. The outage probability is then evaluated as

[2] For example, according to [13], the probability of decoder error of a $[64u, 64]$ MDS code over GF(2^8), $k = 64$, is at most 1.12×10^{-47} for $u = 2$, 4.22×10^{-103} for $u = 3$, and 8.14×10^{-160} for $u = 4$.

$$
\begin{aligned}
\eta_H &= \sum_{i=0}^{N_{H-1}-1} \left\{1 - r_i^{(H-1)}\right\} \Pr[S^{(H-1)} = i] \\
&= \pi^{(H-1)} \left[1, 1 - r_1^{(H-1)}, 1 - r_2^{(H-1)}, \cdots, 1 - r_{N_{H-1}}^{(H-1)}\right]^{\mathrm{T}} \\
&= \pi^{(1)} \boldsymbol{P}^{(2)} \boldsymbol{P}^{(3)} \cdots \boldsymbol{P}^{(H-1)} \\
&\quad \times \left[1, 1 - r_1^{(H-1)}, 1 - r_2^{(H-1)}, \cdots, 1 - r_{N_{H-1}}^{(H-1)}\right]^{\mathrm{T}}
\end{aligned} \tag{14}
$$

where a superscript T is transpose of a vector. Note that, if $S^{(H-1)} = 0$, the destination node can never recover the message; $r_0^{(H-1)} = 0$.

IV. Numerical Examples and Discussions

We numerically examine the derived expressions for a $[64L, 64]$ MDS code C $(k = 64)$. The channel symbol error rate is assumed to be $\varepsilon = 10^{-2}$. For these values, we have $r_1 = 1 - \eta_1 = (1 - 10^{-2})^{64} \approx 0.526$. It should be emphasized that the above analysis can be also applied to evaluation of the outage probability without MDS codes [4], when we impose $t_u = 0$ instead of (3), which results in $r_i^{(h)} = 1 - (1 - r_1)^i$ for any hop stage h. Furthermore, if the relation $N_1 = N_2 = \cdots = L$ holds, the previous analysis with substitution of

$$
q(u \mid i) = \begin{cases} 1 & \text{for } u = i \\ 0 & \text{otherwise} \end{cases} \tag{15}
$$

for (5) provides the outage probability of the protocol in [9]. In the protocol in [9], a relay node $\mathrm{R}_{h,\ell}$ at the h th hop stage broadcasts the ℓ th codeword block $\boldsymbol{c}_\ell$ according to its node number ℓ in each hop stage for $\ell = 1, 2, \cdots, L$. It means that all the received codeword blocks are different in the protocol in [9].

A. Outage Probability

First, we suppose three types of topology for $N_1 = N_2 = \cdots = N_9 = 2, 3, 4$ and $N_{10} = N_{11} = \cdots = 3$.

1) For $N_1 = N_2 = \cdots = N_9 = 2$, the number of relay nodes at a hop stage increases at the 10th hop.

2) For $N_1 = N_2 = \cdots = N_9 = 3$, the number of relay nodes at a hop stage is homogeneous.

3) For $N_1 = N_2 = \cdots = N_9 = 4$, the number of relay nodes at a hop stage decreases at the 10th hop.

For a half-rate [128, 64] MDS code, $L = 2$, the outage probability η_H is shown in Fig. 3. In Fig. 3, solid lines indicate the results of the proposed scheme and dotted lines are the results with no MDS codes [4]. Note that in this case, the protocol in [9] can not be applied, since $N_h \neq L$ for some h.

Apparently, an increment of the number of relay nodes at a hop stage can improve the outage probability with or without the use of an MDS code, since for a large N_h the possibility that a relay node receives two or more different codeword blocks is enhanced. It is clearly observed that the use of the MDS code can successfully achieve significant reduction of the outage probability. In comparison with the case of no MDS code, an MDS code affords performance improvement by δ_u in (7) due to its error-correcting capability t_u. Particularly, for a half-rate MDS code $(L = 2)$, the performance can be greatly improved when three or more relay nodes are located at a hop stage, $N_h \geq 3$. For $N_h = 2$ and $L = 2$, the conditional probability of two different codeword block reception is $q(2 \mid 2) = 1/2$, given that two relay nodes in the previous hop stage randomly transmit

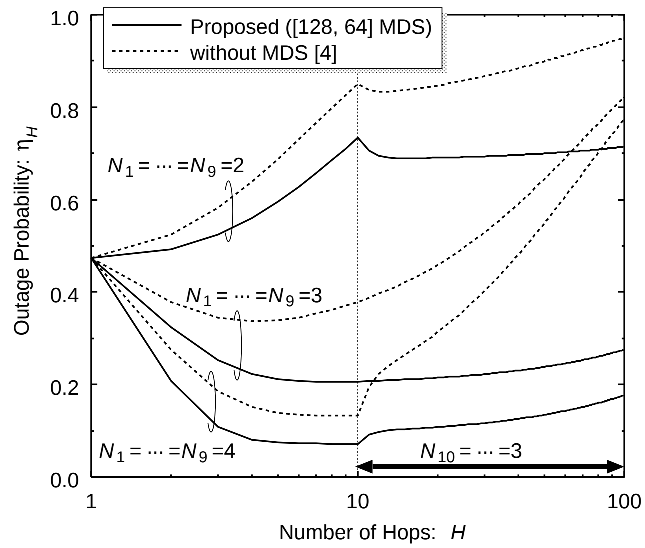

Fig. 3. Outage probability of 64-symbol message block for [128, 64] MDS code ($L = 2$), $N_{10} = N_{11} = \cdots = 3$ and $\varepsilon = 10^{-2}$.

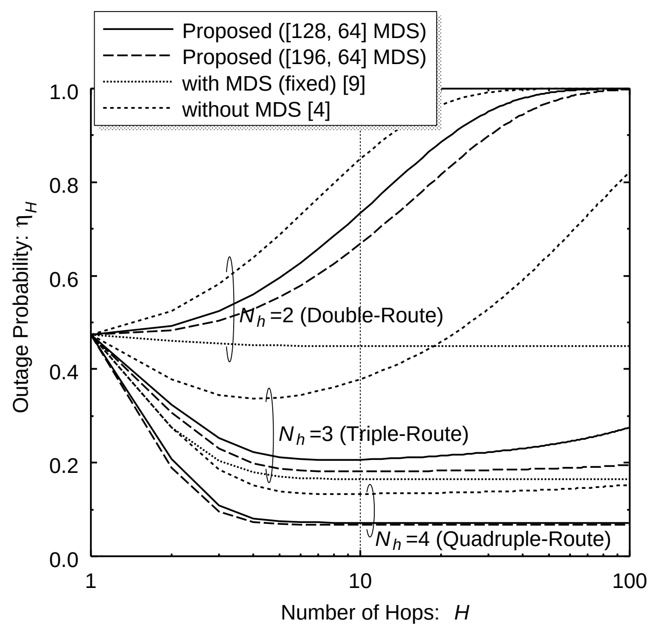

Fig. 4. Outage probability of 64-symbol message block for homogeneous topology $(N_1 = N_2 = \cdots = N_{H-1})$ and $\varepsilon = 10^{-2}$.

a codeword block. For $N_h = 3$ and $L = 2$, however, the conditional probability that two or more different codeword blocks are received is rising up to $q(2|3) = 2/3$ and $q(3|3) = 2/9$, if three relay nodes at the previous hop stage transmit a codeword block. This results in enlargement of possibility for a relay node to enjoy the error-correcting capability of the MDS code.

Next, for a [128, 64] MDS ($L = 2$) and a [196, 64] MDS codes ($L = 3$), Fig. 4 shows the outage probability of homogeneous topology, where each hop stage consists of $N_h = 2,3,4$ relay nodes for any h. The outage probability of the protocol proposed in [9] is also depicted in Fig. 4. The protocol in [9] can be viewed as an asymptotic version of the proposed scheme for sufficiently large L, since the probability that two or more relay nodes randomly select the identical codeword block can be negligible for large L, that is, every relay node transmits a different codeword block.

It is evident that the use of long MDS code can improve the performance. In particular, for $N_h = 3$, the outage probability of [196, 64] MDS code ($L = 3$) is close to the protocol in [9]. Thus, if we can locate three or more relay nodes at each hop, the use of [196, 64] MDS code suffices. However, only two relay nodes are permitted to locate, much longer MDS codes are required in order to obtain further gains.

B. Average Number of Hops to Enter the Absorbing State

From Fig. 4, it appears that for large H, the outage probability with an MDS code C converges into a certain value less than unity. However, this is not true, since the multi-hop network shown in Fig. 1 is modeled by the absorbing Markov chain in Fig. 2. It implies that for sufficiently large H, the state enters into State 0 and never leaves it, so that the outage probability must reach unity for large H. In such a case, it is of importance to estimate the average number of hops for the system with homogeneous topology to enter into State 0.

Let us suppose that each hop stage consists of N relay nodes, $N_1 = N_2 = \cdots = N_H = N$. Then, the non-homogeneous Markov chain in Fig. 2 is regenerated to the homogeneous Markov chain, so that the transition matrix $\boldsymbol{P}^{(h)}$ in (13) and the state space $\Omega^{(h)}$ in (9) are independent of the hop number h; $\boldsymbol{P}^{(h)} = \boldsymbol{P}$ and $\Omega^{(h)} = \Omega$. Based on the argument of absorbing Markov chains [10], the fundamental matrix is defined by $\boldsymbol{I} - \boldsymbol{Q}$, where $\boldsymbol{I}$ is the identity matrix of order N and $\boldsymbol{Q}$ is a submatrix of the transition matrix $\boldsymbol{P}$ which represents transition probabilities among the transient states, $\Omega - \{0\}$, and is given by

$$\boldsymbol{Q} = \begin{bmatrix} p_{1,1}^{(h)} & p_{1,2}^{(h)} & \cdots & p_{1,N}^{(h)} \\ p_{2,1}^{(h)} & p_{2,2}^{(h)} & \cdots & p_{1,N}^{(h)} \\ \vdots & \vdots & & \vdots \\ p_{N,1}^{(h)} & p_{N,2}^{(h)} & \cdots & p_{N,N}^{(h)} \end{bmatrix} \tag{16}$$

Then, the conditional average number of hops τ_n for the system to be absorbed in State 0, given that it starts at State n for $n = 1,2,\cdots,N$, is defined by [10]

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_N \end{bmatrix} = (\boldsymbol{I} - \boldsymbol{Q})^{-1} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \tag{17}$$

As a result, taking into account the first hop from the source node, we can evaluate the average number of hops to enter the absorbing state, State 0, as

$$h_{\text{av}} = 1 + \pi^{(1)}[0, \tau_1, \tau_2, \cdots, \tau_N]^{\text{T}} \tag{18}$$

since $\pi^{(1)}$ provides the initial distribution of the Markovian model in Fig.2. Table I provides the calculated values of the average number of hops h_{av} for $N = 2,3,4$ with and without [128, 64] MDS ($L = 2$) and [196, 64] MDS codes ($L = 3$).

TABLE I
AVERAGE NUMBER OF HOPS h_{av} TO ENTER THE ABSORBING STATE FOR $k = 64$ AND $\varepsilon = 10^{-2}$

(a) $L = 2$ ([128, 64] MDS code)

No. of Nodes per Hop	$N = 2$	$N = 3$	$N = 4$
With MDS (proposed)	9.53	8.28×10^{2}	2.41×10^{6}
With MDS (fixed) [9]	2.66×10^{18}	---	---

(b) $L = 3$ ([196, 64] MDS code)

No. of Nodes per Hop	$N = 2$	$N = 3$	$N = 4$
With MDS (proposed)	1.24×10^{1}	4.30×10^{3}	1.04×10^{8}
With MDS (fixed) [9]	---	1.41×10^{38}	---

(c) without MDS code [4]

No. of Nodes per Hop	$N = 2$	$N = 3$	$N = 4$
Without MDS [4]	6.50	6.22×10^{1}	3.69×10^{3}

Let us consider the case of $N = 2$ and $L = 2$, as an example. If no MDS code is employed, the message can be delivered to the destination node which is six or seven hops away from the source node on average, since $h_{\text{av}} = 6.50$, and no further delivery can be expected. Incorporation of the MDS code with random codeword block selection, that is, the proposed scheme, can enlarge h_{av} to 9.53 or 12.4 hops. Furthermore, the fixed codeword block assignment of the MDS code, proposed in [9], can augment h_{av} in orders of magnitude, which results in $h_{\text{av}} = 2.66\times10^{18}$. This considerable improvement stems from the fact that the conditional probability of successful decoding, given that two codeword blocks are received, is $r_2^{(h)} \approx 1.00$ if two blocks are always different [9], while $r_2^{(h)} \approx 0.89$ if two blocks are randomly selected.

For $N \geq 3$ $, the proposed scheme can offer larger gains over the case without MDS codes [4]. Further gains can be expected through the fixed codeword block assignment [9], although the number of relay nodes N should be equal to L.

V. Conclusion

We have proposed the random relaying of an MDS codeword block at each relay node in cooperative multi-hop networks. In the proposed scheme, the message block is encoded by an MDS code of coding rate $1/L$ and the codeword is partitioned into L blocks. A relay node broadcasts one randomly selected codeword block. A receiving node aggregates received codeword blocks and corrects channel errors by the MDS decoder. The proposed scheme is generalization of the scheme proposed in [9], where a relay node broadcasts one pre-assigned codeword block according to its node number. Then, we have analyzed the outage probability of the proposed scheme over random error channels by means of a non-homogeneous absorbing Markov chain. The expression of the outage probability has been derived. Numerical results with a $[64L, 64]$ MDS code for $L = 2, 3$ have indicated that significant improvement can be achieved by incorporating an MDS code. When a relay node receives two or more different codeword blocks, the powerful error-correcting capability of an MDS code results in significant reduction of the outage probability. In particular, when a half rate MDS code, $L = 2$, is employed, a multi-hop network with three or more relay nodes at each hop stage can achieve good performance.

Further study includes the consideration of, for example, retransmission mechanisms between adjacent relay nodes and the physical layer issues.

References

[1] R. Pabst, et al., ``Relay-based deployment concepts for wireless and mobile broadband radio," *IEEE Commun. Mag.*, vol. 42, no. 9, pp. 80-89, Sep. 2004.

[2] T. Miyano, H. Murata and K. Araki, ``Cooperative relaying scheme with space time code for multihop communications among single antenna terminals," in *Proc. IEEE GLOBECOM 2004*, Dallas, TX, Nov./Dec. 2004.

[3] T. Koike, M. Tanaka and S. Yoshida, ``Capacity improvement of multihop inter-vehicle communication networks by STBC cooperative relaying," *IEICE Trans. Commun.*, vol. E88-B, no. 9, pp. 3546-3553, Sep. 2005.

[4] Y. Oishi, H. Murata, K. Yamamoto and S. Yoshida, ``Theoretical FER performance of multi-hop wireless cooperative networks using transit diversity," in *Proc. IEEE VTC 2008-Spring*, Singapore, Singapore, May 2008.

[5] A. F. Molisch, N. B. Mehta, J. S. Yedidia and J. Zhang, ``Performance of fountain codes in collaborative relay networks," *IEEE Trans. Wireless Commun.*, vol. 6, no. 11, pp. 4108-4119, Nov. 2007.

[6] W. Lei, X. Xie and G. Li, ``Performance analysis of wireless dynamic cooperative relay networks using fountain codes," *J. Commun.*, vol. 5, no. 4, pp. 307-316, Apr. 2010.

[7] S. M. Alamouti, ``A simple transmit diversity technique for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16, no. 8, pp. 1451-1458, Oct. 1998.

[8] J. W. Byers, M. Luby and M. Mitzenmacher, ``A digital fountain approach to asynchronous reliable multicast," *IEEE J. Select. Areas Commun.*, vol. 20, no. 8, pp. 1528-1540, Oct. 2002.

[9] K. Sakakibara, D. Ito and J. Taketsugu, ``Effect of an MDS code on the outage probability in cooperative multi-hop relaying networks," in *Proc. IEEE MICC 2009*, Kuala Lumpur, Malaysia, Dec. 2009.

[10] K. S. Trivedi, *Probability and Statistics with Reliability, Queuing and Computer Science Applications, 2nd Ed.*, pp. 392-402, John Wiley & Sons, New York, NY, 2002.

[11] S. B. Wicker, *Error Control Systems for Digital Communication and Storage*, Prentice-Hall, Englewood Cliffs, NJ, 1995.

[12] W. Feller, *An Introduction to Probability Theory and Its Application, 3rd Ed. Volume 1*, pp. 101-103, John Wiley & Sons, New York, NY, 1968.

[13] T. Kasami and S. Lin, ``On the probability of undetected error for the maximum distance separable codes," *IEEE Trans. Commun.*, vol. COM-32, no. 4, pp. 998-1006, Sep. 1984.