Bistable Region of Backoff Algorithms with Contention Window in Slotted ALOHA Systems

Katsumi Sakakibara and Jumpei Taketsugu

***Abstract*—It has been widely recognized that bistable behavior of random access protocols may suddenly deteriorate the performance of the system due to the unexpected slip into an undesired stable operating point. The catastrophe theory succeeds in presenting the bistable region of random access protocols, where packet retransmissions occur in a memoryless manner. In this correspondence, the stability of slotted ALOHA systems using backoff algorithms with the contention window is analyzed without converting the retransmission with the contention window into the memoryless retransmissions. The analysis is based on the catastrophe theory. We first construct a two-dimensional Markovian model, which is equivalent to the model devised for IEEE 802.11 DCF by Bianchi. Then, the balance function of the system is formulated whose zeros provide equilibrium operating points. Finally, we prove that the system is mono-stable for any set of contention window sizes, if the number of allowable retransmissions is less than or equal to eight. Equivalently, the existence of the bistable region is proved if nine or more retransmissions of backlogged packet are permitted.**

***Index Terms*—Backoff algorithm, Bistable region, Catastrophe theory, Contention window, Slotted ALOHA**

I. Introduction

BISTABLE behavior observed in random access protocols has been extensively investigated, since Carleial and Hellman analyzed the stability of ALOHA-type protocols in terms of the expected drift [1]. The system possesses two stable operating points, if it exhibits bistable behavior; one of which offers comparatively high throughput, whereas the other produces considerably low throughput [2]. To make matters worse, the system operating at the "good and desired" stable point may suddenly slip into a catastrophic "air pocket", which contingently occurs as a result of an unexpected avalanche of offered load. In order to mitigate undesired bistable behavior of random access protocols, backoff algorithms and retransmission cutoff are effective countermeasures [3]-[6]. With backoff algorithms, a user defers its packet retransmission for a randomly selected interval. In general, each user independently keeps its own contention window (CW) size. The random interval is determined according to the current CW of the user. The binary exponential backoff algorithm is widely employed in wired and wireless local area networks, where the CW size is doubled upon every packet transmission failure [3][4]. On the other hand, with retransmission cutoff a user drops a packet which experiences excessive transmission failures. By dropping packets, an avalanche of traffic can be relaxed at the cost of an increase of the packet loss rate [5][6].

In conventional analyses of random access protocols with backoff algorithms [1][2][6], packet retransmissions with random backoff interval according to the current CW are converted into those with the retransmission probability. This conversion enables us to carry out tractable mathematical analysis due to the memoryless property. In this context, the catastrophe theory has succeeded in explicitly revealing the bistable region of random access protocols such as slotted ALOHA [6]-[8], CSMA (Carrier Sense Multiple Access) [9] and PRMA (Packet Reservation Multiple Access) [10]. However, in [6]-[10] the constant retransmission probability is assumed. Moreover, it has been mathematically proved that a slotted ALOHA with constant retransmission probability is mono-stable, if the number of retransmissions is limited to eight or less [6].

Recently, IEEE 802.11 DCF (Distributed Coordination Function) [11] has attracted researchers' attention to the use not only in wireless local area networks but also wireless sensor/ad-hoc networks. Bianchi proposed a two-dimensional Markovian model with respect to the backoff algorithm adopted in IEEE 802.11 DCF [4]. The model elaborately takes into account a CW-version of backoff algorithm, rather than a retransmission probability-version. Hence, it is a challenging issue to analyze the stability of random access protocols with a two-dimensional Markovian model without converting the CW into the retransmission probability

In this correspondence, we analyze the stability of slotted ALOHA systems using backoff algorithms with the CW in connection with the catastrophe theory. In particular, we prove that the system is mono-stable for any set of the CW sizes, if the permitted number of retransmissions is less than or equal to eight. Equivalently, the existence of the bistable region is proved if nine or more retransmissions of backlogged packet are permitted.

Manuscript received March 31, 2012. A part of this word was supported by Japan Society for the Promotion of Science under Grant-in-Aid for Scientific Research (C) (KAKENHI no. 21560408).

Katsumi Sakakibara is with the Department of Information and Communication Engineering, Faculty of Computer Science and System Engineering, Okayama Prefectural University, Soja, 719-1197 Japan. (phone: +81-866-94-2109; Fax: +81-866-94-2199; e-mail: sakaki@c.oka-pu.ac.jp).

Jumpei Taketsugu is with the Department of Information and Communication Engineering, Faculty of Computer Science and System Engineering, Okayama Prefectural University, Soja, 719-1197 Japan. (e-mail: taketugu@c.oka-pu.ac.jp).

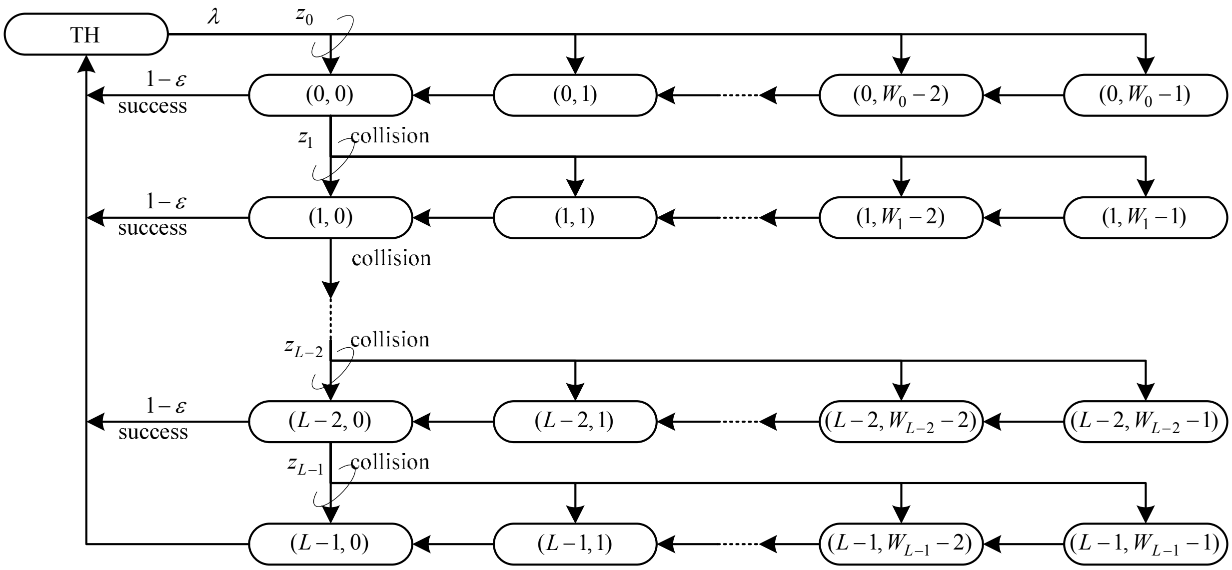

Fig. 1. Markovian model with respect to the user state with the CW profile $\boldsymbol{W} = (W_0, W_1, \ldots, W_{L-1})$ and retransmission cutoff L.

In Section II, we describe a two-dimensional Markovian model for the system using backoff algorithm with CW. Section III presents the main result with the proof. Numerical results of the bistable region of the binary exponential backoff algorithm are shown in Section IV. Section V concludes the present correspondence.

II. System Model

We consider a slotted ALOHA system for its simplicity and mathematical tractability, which enables us to focus on a backoff algorithm itself. Let N be the number of users contending a shared common channel. Each user with empty buffer generates a packet to transmit with probability λ at the beginning of each time slot. The capacity of a user buffer is assumed to be limited to one packet, so that a backlogged user can generate no new packets. A packet retransmission is cut off and the packet is dropped, if it experiences consecutive L transmission failures. Let W_ℓ denote the ℓ th CW size, where ℓ is the number of transmission failures experienced by the packet in the buffer for $\ell = 0, 1, \ldots, L-1$. Here, we refer to an L-tuple $\boldsymbol{W} = (W_0, W_1, \ldots, W_{L-1})$ as the *CW profile*. A user inserts a random backoff interval of k slots before packet (re)transmission, where k is a non-negative integer in $[0, W_\ell - 1]$. The value of k is decreased every time slot and a packet is (re)transmitted when k reaches to zero. Here, we assume that a packet (re)transmission succeeds, if no other simultaneous packet (re)transmission occurs and that a packet collision results in transmission failure for all the packet involved.

Based on the above assumptions, we can construct a Markovian system model with respect to the user state, as shown in Fig. 1, which is equivalent to the system model in [4]. In Fig. 1, TH represents the empty state.

III. Analysis

First, we derive equations in equilibrium for backlogged users. Then, considering the flow balance for users in State TH, we formulate the balance function, whose zeros provide equilibrium operating points [6]. Finally, the main theorem is proved.

A. Equations in Equilibrium

Let $b_{\ell,k}$ be the average number of users in State (ℓ, k) at equilibrium for $\ell = 0, 1, \ldots L-1$ and $k = 0, 1, \ldots W_\ell - 1$. The average number of backlogged users is then given by

$$n_B = \sum_{\ell=0}^{L-1} \sum_{k=0}^{W_\ell - 1} b_{\ell,k}. \tag{1}$$

In equilibrium, the average in-flow and out-flow balance at each state. Then, the following equations hold at State (ℓ, k);

$$b_{\ell,k} = \begin{cases} b_{\ell,k+1} + \dfrac{z_\ell}{W_\ell}, & \text{for } \ell = 0, 1, \ldots, W_\ell - 2, \\ \dfrac{z_\ell}{W_\ell}, & \text{for } \ell = W_\ell - 1, \end{cases} \tag{2}$$

where z_ℓ represents the total average in-flow to States (ℓ, k) for $k = 0, 1, \ldots W_\ell - 1$, as shown in Fig.1, that is,

$$z_\ell = \begin{cases} (N - n_B)\lambda, & \text{for } \ell = 0, \\ \varepsilon b_{\ell-1,0}, & \text{for } \ell = 1, 2, \ldots L-1, \end{cases} \tag{3}$$

where ε is the probability of packet transmission failure. Recursively solving (2), we obtain

$$b_{\ell,k} = \frac{W_\ell - k}{W_\ell} z_\ell \quad (4)$$

for $\ell = 0,1,\dots L-1$ and $k = 0,1,\dots W_\ell - 1$. In particular, for $k = 0$, a trivial relation

$$b_{\ell,0} = z_\ell \quad (5)$$

can be derived, which implies that any in-flow to State (ℓ, k) departs from State $(\ell, 0)$. Moreover, it follows from (3) and (5) that

$$b_{\ell,0} = \varepsilon b_{\ell-1,0} = \cdots = \varepsilon^\ell b_{0,0} = \varepsilon^\ell (N - n_B)\lambda \quad (6)$$

for $\ell = 0,1,\dots L-1$. Substituting (4)-(6) into (1), we can derive

$$n_B = (N - n_B)\lambda \sum_{\ell=0}^{L-1} \frac{W_\ell + 1}{2} \varepsilon^\ell, \quad (7)$$

which provides us

$$n_B = \frac{N\lambda \sum_{\ell=0}^{L-1} \frac{W_\ell + 1}{2} \varepsilon^\ell}{1 + \sum_{\ell=0}^{L-1} \frac{W_\ell + 1}{2} \varepsilon^\ell} \quad (8)$$

Here, let us denote traffic to the channel by

$$G = \sum_{\ell=0}^{L-1} b_{\ell,0}, \quad (9)$$

since only the users in State $(\ell, 0)$ are allowed to transmit a packet. With the aid of Poisson approximation to the binomial distribution [6]-[10], the probability of packet transmission failure ε is evaluated by

$$\varepsilon = 1 - e^{-G}. \quad (10)$$

B. Balance Function

The in-flow to State TH consists of successful users and users who drop a packet in buffer due to L consecutive transmission failures. The Poisson approximation provides us the average number of successful users as

$$\sum_{\ell=0}^{L-2} (1-\varepsilon) b_{\ell,0} = (1-\varepsilon) G = G e^{-G}. \quad (11)$$

The average number of users dropping a packet is $eb_{L-1,0}$. Then, subtracting the average out-flow from the average in-flow, we define the balance function as

$$A(G \mid N\lambda, N\beta, L, \boldsymbol{W}) = Ge^{-G} + \varepsilon b_{L-1,0} - (N - n_B)\lambda. \quad (12)$$

From (6), (8), and (10), the balance function (12) can be rewritten as

$$\begin{aligned} A(G \mid N\lambda, N\beta, L, \boldsymbol{W}) &= Ge^{-G} - (N - n_B)\lambda(1 - \varepsilon^L) \\ &= Ge^{-G} - \frac{N\lambda(1-\varepsilon^L)}{1 + \sum_{\ell=0}^{L-1} \frac{W_\ell + 1}{2} \varepsilon^\ell} \\ &= Ge^{-G} - \frac{N\lambda\{1 - (1 - e^{-G})^L\}}{1 + \sum_{\ell=0}^{L-1} \frac{W_\ell + 1}{2} (1 - e^{-G})^\ell} \\ &= Ge^{-G} - \frac{N\lambda\{1 - (1 - e^{-G})^L\}}{1 + \frac{N\lambda}{N\beta} \sum_{\ell=0}^{L-1} \alpha_\ell (1 - e^{-G})^\ell}, \end{aligned} \quad (13)$$

where

$$\beta = \frac{2}{W_0 + 1} \quad \text{and} \quad \alpha_\ell = \frac{\beta(W_\ell + 1)}{2}. \quad (14)$$

Note that the balance function is equivalent to the first derivative of the potential function of some dynamic system. The roots of $A(G \mid N\lambda, N\beta, L, \boldsymbol{W}) = 0$ provide equilibrium operating points for given $N\lambda$, $N\beta$, L and $\boldsymbol{W}$. The system is mono-stable, if $A(G \mid N\lambda, N\beta, L, \boldsymbol{W}) = 0$ has the unique root, which is the globally stable operating point. The system is bistable, if $A(G \mid N\lambda, N\beta, L, \boldsymbol{W}) = 0$ has three roots, two of which correspond to locally stable operating points and one to an unstable operating point [2].

C. Cusp Catastrophe and Bifurcation Sets

According to the catastrophe theory, the cusp catastrophe may exist in our slotted ALOHA system, if

$$A(G \mid N\lambda, N\beta, L, \boldsymbol{W}) = \frac{\partial A}{\partial G} = \frac{\partial^2 A}{\partial G^2} = 0 \quad \text{and} \quad \frac{\partial^3 A}{\partial G^3} \neq 0 \quad (15)$$

have roots $(G, N\lambda, N\beta)$ for given L and $\boldsymbol{W}$ [7][8]. The cusp point is provided by the root of (15) and the bifurcation sets, B_A^+ and B_A^-, are defined as

$$\begin{cases} B_A^+ = \left\{ (N\lambda, N\beta) \middle| A(G \mid N\lambda, N\beta, L, \boldsymbol{W}) = \frac{\partial A}{\partial G} = 0, \right. \\ \qquad \left. \frac{\partial^2 A}{\partial G^2} > 0, N\lambda > 0, N\beta > 0 \right\} \\ B_A^- = \left\{ (N\lambda, N\beta) \middle| A(G \mid N\lambda, N\beta, L, \boldsymbol{W}) = \frac{\partial A}{\partial G} = 0, \right. \\ \qquad \left. \frac{\partial^2 A}{\partial G^2} < 0, N\lambda > 0, N\beta > 0 \right\} \end{cases} \quad (16)$$

Solving $A = \partial A / \partial G = 0$, we can obtain from (13)

$$N\lambda = \frac{N\beta G e^{-G}}{N\beta\{1 - (1 - e^{-G})^L\} - Ge^{-G} \sum_{\ell=0}^{L-1} \alpha_\ell (1 - e^{-G})^\ell} \quad (17)$$

and

$$N\beta = \frac{(Ge^{-G})^2 \sum_{\ell=0}^{L-1} \ell \alpha_\ell (1 - e^{-G})^\ell}{(G-1)\{1 - (1 - e^{-G})^L\} - Ge^{-G} L (1 - e^{-G})^{L-1}} \quad (18)$$

which must be positive to be the valid bifurcation sets.

D. No Bistable Region if $L \le 8$

This subsection is devoted to the proof of the following theorem.

Theorem 1: No bistable region exists for any CW profile $\boldsymbol{W} = (W_0, W_1, \ldots, W_{L-1})$, if $L \le 8$. □

Proof: In order for (17) and (18) to offer the valid bifurcation sets in our slotted ALOHA system, both $N\lambda$ in (17) and $N\beta$ in (18) must be positive. We prove the theorem by showing that for $L \le 8$, no valid bifurcation sets can be obtained for any positive G.

First, the numerators in (17) and (18) are clearly positive for any positive traffic G.

Secondly, the denominator in (18) is equivalent to (8) in [6]. Thus, there exists the fold catastrophe in a certain dynamic system whose balance function is equal to the denominator in (18). According to Fig. 2 in [6], the fold point is $(G, L) \approx (1.443, 8.300)$, so that the denominator in (18) is negative for any positive G, if $L < 8.300$ and it can be positive for some G if $L > 8.300$.

Thirdly, it can be proved that the denominator in (17) is positive as follows. Let $\delta(G)$ be the denominator in (17) for given $N\lambda$, $N\beta$, L and $\boldsymbol{W}$. Then,

$$\begin{aligned}\delta(G) &= N\beta\{1-(1-e^{-G})^L\} - Ge^{-G}\sum_{\ell=0}^{L-1}\alpha_\ell(1-e^{-G})^\ell \\ &= N\beta\sum_{\ell=0}^{L-1}(1-e^{-G})^\ell - Ge^{-G}\sum_{\ell=0}^{L-1}\alpha_\ell(1-e^{-G})^\ell \\ &= e^{-G}\sum_{\ell=0}^{L-1}(N\beta - G\alpha_\ell)\varepsilon^\ell\end{aligned} \tag{19}$$

where ε is given by (10). From (6), (8) and (9), we have

$$\begin{aligned}G &= \sum_{m=0}^{L-1} b_{m,0} = (N-n_B)\lambda\sum_{m=0}^{L-1}\varepsilon^m \\ &= \frac{N\lambda\sum_{m=0}^{L-1}\varepsilon^m}{1+\lambda\sum_{m=0}^{L-1}\frac{W_m+1}{2}\varepsilon^m} = \frac{N\lambda\sum_{m=0}^{L-1}\varepsilon^m}{1+\lambda\sum_{m=0}^{L-1}\frac{\alpha_m}{\beta}\varepsilon^m},\end{aligned} \tag{20}$$

where β and α_ℓ are given by (14). Substituting (20) into (19), we can express the denominator $\delta(G)$ as

$$\begin{aligned}\delta(G) &= e^{-G}\sum_{\ell=0}^{L-1}\frac{N\beta + N\lambda\sum_{m=0}^{L-1}(\alpha_m-\alpha_\ell)\varepsilon^m}{1+\lambda\sum_{m=0}^{L-1}\frac{\alpha_m}{\beta}\varepsilon^m}\varepsilon^\ell \\ &= e^{-G}\frac{N\beta\sum_{\ell=0}^{L-1}\varepsilon^\ell + N\lambda\sum_{\ell=0}^{L-1}\sum_{m=0}^{L-1}(\alpha_m-\alpha_\ell)\varepsilon^{m+\ell}}{1+\lambda\sum_{m=0}^{L-1}\frac{\alpha_m}{\beta}\varepsilon^m}.\end{aligned} \tag{21}$$

Here, let us consider the double summation in the numerator in (21). Let

$$f(\varepsilon) = \sum_{\ell=0}^{L-1}\sum_{m=0}^{L-1}(\alpha_m-\alpha_\ell)\varepsilon^{m+\ell}. \tag{22}$$

It can be derived that $f(\varepsilon) = 0$ as follows;

$$\begin{aligned}f(\varepsilon) &= \sum_{\ell=m}(\alpha_m-\alpha_\ell)\varepsilon^{m+\ell} + \sum_{\ell<m}(\alpha_m-\alpha_\ell)\varepsilon^{m+\ell} \\ &\quad + \sum_{\ell>m}(\alpha_m-\alpha_\ell)\varepsilon^{m+\ell} \\ &= \sum_{\ell<m}(\alpha_m-\alpha_\ell)\varepsilon^{m+\ell} + \sum_{m>\ell}(\alpha_\ell-\alpha_m)\varepsilon^{m+\ell} \\ &= 0.\end{aligned} \tag{23}$$

Therefore, $\delta(G)$ is positive;

$$\delta(G) = e^{-G}\frac{N\beta\sum_{\ell=0}^{L-1}\varepsilon^\ell}{1+\lambda\sum_{m=0}^{L-1}\frac{\alpha_m}{\beta}\varepsilon^m} > 0, \tag{24}$$

which proves that the denominator in (17) is also positive.

In consequence, according to the sign of the denominator in (18), $N\beta < 0$ for any positive G, if $L \le 8$ and $N\beta > 0$ for some G, if $L \ge 9$. It completes the proof. (Q.E.D.)

IV. Numerical Example

We examine the derived expressions for the binary exponential backoff algorithm;

$$W_\ell = \min[16\times 2^\ell, 1024] \tag{25}$$

for $\ell = 0, 1, \ldots L-1$, whose CW profile is $\boldsymbol{W} = (16, 32, 64, \cdots, 512, 1024, \cdots, 1024)$. In this case, we have

$$\beta = \frac{2}{17} \quad \text{and} \quad \alpha_\ell = \min\left[\frac{16\times 2^\ell + 1}{17}, \frac{1025}{17}\right]. \tag{26}$$

From Theorem 1, the system with CW operates with the unique globally stable point, if $L \le 8$. Note that the stability analysis in the previous section can be applied to any random access protocols, if they operate in a synchronous manner similarly to slotted ALOHA systems. Hence, Theorem 1 ensures that IEEE 802.11 DCF operates with globally stable equilibrium point, since the retransmission is cut off after 4 consecutive transmission failures for DotShortRetryLimit and 7 for DotLongRetryLimit as the nominal values [11].

The bistable region for $L = 9, 10, 15, 20, 50, 100$ is shown in Fig. 2, where the bifurcation sets; B_A^+ and B_A^-, are drawn by red lines and their intersection represents the cusp point. The horizontal axis $N\lambda$ is the average traffic when no backlogged users exist and the vertical axis $N\beta$ is the maximum average traffic when all the users are backlogged, that is, all the users are in States (0,0) to $(0, W_0 - 1)$. From Fig. 2, for given Nb, the slotted ALOHA system possesses two stable equilibrium operating points, if Nl is between B_A^+ and B_A^-. Otherwise, the system can operate with the unique globally stable

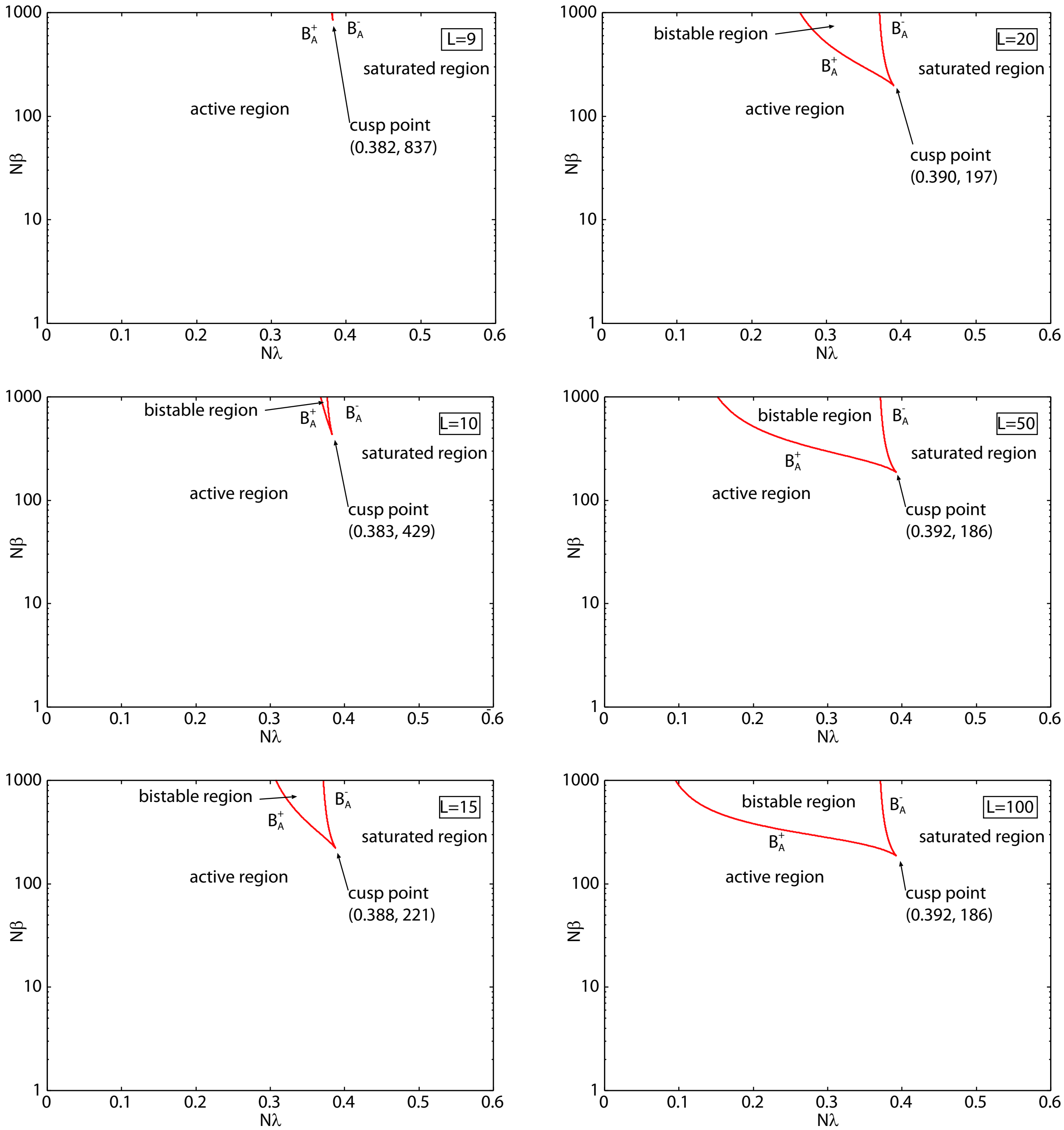

Fig. 2. Bifurcation sets and bistable region of slotted ALOHA with binary exponential backoff algorithm $W_\ell = \min[16\times 2^{\ell}, 1024]$, $\ell = 0,1,\ldots L-1$, for $L = 9,10,15,20,50,100$.

equilibrium point. In Fig. 2, mono-stable region with light (resp. heavy) traffic is referred to as the *active region* (resp. *saturated region*). If the system operates in the active region, there exists a unique globally stable equilibrium point which provides comparatively low traffic. The system also possesses a unique globally stable equilibrium point even if it operates in the saturated region. However, the system suffers from heavy traffic in contrast. It can be observed how bistable region grows according to an increment of L. Similarly to Fig. 3 in [6], one of the bifurcation sets, B_A^-, is stable with respect to the increment of L, whereas the other, B_A^+, moves to smaller $N\lambda$. Therefore, the bistable region expands by curtailing the active region rather than the saturated region. Comparing to Fig. 3 in [6], which corresponds to the constant CW profile $W_0 = W_1 = \cdots = W_{L-1}$, the cusp point of the slotted ALOHA with the binary exponential backoff algorithm appears at larger $N\beta$. This implies that the system with binary exponential backoff algorithm can achieve stable operation for heavier traffic, compared to the system with the constant CW profile. For example, the cusp point is $(N\lambda_{\text{cusp}}, N\beta_{\text{cusp}}) = (0.3903, 197.0)$ for $L = 20$ from Fig. 2. Hence, the slotted ALOHA operates with the global stable point for any packet generation probability λ, if the number of users satisfies

$$N < \frac{N\beta_{\text{cusp}}}{\beta} = \frac{197.0}{2/17} \approx 1674.5. \quad (27)$$

On the other hand, it follows from [6] that the cusp point of the slotted ALOHA system with the constant CW profile for $L = 20$ is $(N\lambda_{\text{cusp}}, N\beta_{\text{cusp}}) = (0.4945, 4.714)$. Thus, the system is mono-stable for any packet generation probability λ, if the number N of users is less than

$$\frac{N\beta_{\text{cusp}}}{\beta} = \frac{4.714}{2/17} \approx 40.069. \quad (28)$$

Comparing the above results, we can find that the binary exponential backoff algorithm can operate mono-stably for more number of users than the backoff algorithm with the constant CW profile.

V. Conclusion

The stability of slotted ALOHA systems using backoff algorithms with the CW has been analyzed. The analysis has been based on the catastrophe theory. We have first constructed a two-dimensional Markovian model. Then, the balance function of the system has been formulated. Finally, we have proved that the system is mono-stable for any CW profiles, if the number of allowable retransmissions is less than or equal to eight. Equivalently, the bistable region exists if nine or more retransmissions of backlogged packet are permitted. This main result implies that no static CW profiles can eliminate the bistable region of the system with nine or more retransmissions of backlogged packet, so that some adaptive CW profiles are desired to stabilize the system. Numerical results of the bistable region have revealed that the binary exponential binary backoff algorithm can accommodate more users with the globally stable equilibrium operating point, compared to the backoff algorithm with the constant CW profile.

References

[1] A. B. Carleial and M. E. Hellman, "Bistable behavior of ALOHA-type systems," *IEEE Trans. Commun.*, vol. COM-23, no. 4, pp. 401–410, Apr. 1975.

[2] G. E. Keiser, *Local Area Networks*, McGraw-Hill, New York, NY, 1989.

[3] J. Goodman, A. G. Greenberg, N. Madras and P. March, "Stability of binary exponential backoff," *J. ACM*, vol. 35, no. 3, pp. 579–602, July 1988.

[4] G. Bianchi, "Performance analysis of the IEEE 802.11 distributed coordination function," *IEEE J. Select. Areas Commun.*, vol. 18, no. 3, pp. 535–547, Mar. 2000.

[5] J. H. Sarker and S. J. Halme, "An optimum retransmission cut-off scheme for slotted ALOHA," *Wireless Personal Commun.*, vol. 13, no. 1-2, pp. 185–202, May 2000.

[6] K. Sakakibara, H. Muta and Y. Yuba, "The effect of limiting the number of retransmission trials on the stability of slotted ALOHA systems," *IEEE Trans. Veh. Tech.*, vol. 49, no. 4, pp. 1449–1453, July 2000.

[7] N. D. Birrell, "Application of catastrophe theory to a slotted ALOHA communication system," *IEE Proc.-F*, vol. 130, no. 4, pp. 337–342, June 1983.

[8] Y. Onozato and S. Noguchi, "On the thrashing cusp in slotted ALOHA systems," *IEEE Trans. Commun.*, vol. COM-33, no. 11, pp. 1171–1182, Nov. 1985.

[9] T. Yokohira, T. Nishida and H. Miyahara, "Analysis of dynamic behavior in p-persistent CSMA/CD using cusp catastrophe," *Comput. Networks & ISDN Sys.*, vol. 12, no. 5, pp. 277–289, 1986.

[10] R. Fantacci and F. Innocenti, "Performance evaluation of a modified PRMA protocol for joint voice and data packet wireless networks, " *IEEE Trans. Commun.*, vol. 47, no. 12, pp. 1837–1848, Dec. 1999.

[11] *IEEE Standard Part 802.11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications*, Nov. 1997.