Two Stages Time Delay and Frequency Estimator for Multiple Sinusoids

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***Abstract*- The problem of jointly estimating time delay and frequency of a signal received at two separated sensors is addressed in this paper. These methods require eigenvalue decomposition (EVD) of the covariance matrix of received data. Thus, the computational load and complexity is high especially when the number of samples is large. In addition, the noises field is assumed to be uncorrelated with constant variance. The objective of this paper is to develop an algorithm for estimating time delay and frequency without using any spectral decomposition techniques such as eigenvalue decomposition (EVD) or singular value decomposition (SVD) of the received data matrix. Proposed method only uses a linear transformation of the received data. Furthermore, this method accommodates more practical noise cases where the noise variance is not constant or in unknown correlated noise field. The proposed method can therefore is implemented with a reduced complexity compared to reference Y.Wu's method. Computer Simulation results demonstrate that the proposed method can estimate both the frequencies and time delay efficiently and accurately even at low signal to noise ratio (SNR).**

***Index Terms*- Time Delay and Frequency estimation, ESPRIT, Propagator Method, MUSIC, root-MUSIC**

I. Introduction

The problem of frequency estimation of multiple sinusoidal signals in the presence of noise has received a significant amount of attention in signal processing literature [1-4]. It has also played an important role in many application areas such as radar, sonar, radio astronomy, speech analysis, control theory, and communication systems. However, these methods rely on the singular value decomposition (SVD) of the received data or the eigenvalue decomposition (EVD) of the covariance data matrix which increase the computational complexity of the algorithm. It becomes a time consuming especially when implementation focusing on hard real time signal processing requirement. On the other hand, the problem of estimating the time delay in the signal at two separated sensors has been studied by many researchers [5-7]. These problems widely addressed in application areas such as radar, sonar, biomedical, and geophysics.

Recently, Y.Wu et al. studied the problem of jointly estimation of time delay and frequency of multiple sinusoidal [8-11] based on the measured data from two separated sensors. However, Y.Wu 's method [12] has some drawbacks; This method is separating the received data matrix or the covariance matrix into noise and signal space. This separation requires either EVD or SVD which is computationally intensive task. It has become even more time consuming process when the number of snapshots N is larger than the number of signals which is always the. In addition, Y.Wu's methods are applicable only when the noise is spatially white and the covariance noise matrices are known. But in practical situation, the covariance noise matrix may be difficult to obtain because the noise covariance are often measured experimentally.

In this paper, we developed a computationally efficient algorithm to estimate the time delay and frequencies of multiple sinusoids based on measurements from two separated sensors. Our algorithm performs better for low SNR and is suitable for real-time implementation. To achieve this, we first construct the data matrices (Toeplitz or Hankel) from measurements taken from two separated sensors. Second, we modify and extend the method of bearing estimation without EVD or SVD [13-15]. It has been developed for direction of arrival estimation to track moving sources. We utilized the Propagator method (PM) which has been previously developed for direction of arrival estimation problem [13] to estimate the time delay between the two separated sensors. The proposed method for frequency and delay estimation does not require SVD or EVD for the data covariance matrix. It applies only a linear operation on the received data matrix or the sample covariance matrix. In addition, the proposed method can handle more general noise cases such as spatial or unknown correlated noise field.

In Section II, we first formulate the problem of frequency and time delay estimation followed by derivation of the proposed algorithm. Section III presents the simulation results, and Section IV concludes the research work.

II. Proposed Method For Time Delay And Frequency Estimation Of Multiple Sinusoids

Consider that the discrete time of sinusoidal signals $x(n)$ and $y(n)$ are the two sensor measurements satisfying

$$x(n) = s(n) + e_1(n) \quad (1)$$

$$y(n) = s(n-D) + e_2(n) \quad (2)$$

where,

$$s(n) = \sum_{k=1}^{K} A_k \exp(j\omega_k n), \; n = 0,1,\dots,N-1$$

Also, A_k and ω_k represent the amplitude and the frequency of the k-th complex sinusoid, N denotes the number of data samples, $e_1(n)$ and $e_2(n)$, $n = 0,1,\dots,N-1$ are sequence of complex Gaussian noise variables. The parameter D is the differential time delay between the signals at the two sensors. The objective is to estimate both time difference D and the frequencies ω_k of multiple sinusoid given a data record at two separated sensors $x(n)$ *and* $y(n)$, *where* $n = 0,1,\dots,N-1$. Following steps illustrate the formulation of proposed method.

Step 1: Frequency estimation for multiple sinusoidal

To derive the proposed method for frequency estimation we construct the square Henkel matrix $\boldsymbol{X}$ with dimension $\left(\frac{N}{2}\times\frac{N}{2}\right)$ from the data samples $x(0), x(1), \dots\dots\dots x(N-1)$ as follow

$$\boldsymbol{X} = \begin{bmatrix} x(0) & x(1) & \dots & x(\frac{N}{2}-1) \\ x(1) & x(2) & \dots & x(\frac{N}{2}) \\ \vdots & \vdots & \ddots & \vdots \\ x(\frac{N}{2}-1) & x(\frac{N}{2}) & \dots & x(N-1) \end{bmatrix} \quad (3)$$

The elements of the *i*-th column can be rewritten as follows

$$\boldsymbol{r}_i = \begin{bmatrix} x(i) \\ x(i+1) \\ \vdots \\ x(i+\frac{N}{2}-1) \end{bmatrix} = \boldsymbol{B}(\omega)(\boldsymbol{\varphi}(\omega))^i\boldsymbol{S} + \boldsymbol{e}_i^1 \quad (4)$$

$$i = 0,1,\dots\dots\frac{N}{2}-1$$

where, $\boldsymbol{S} = [S_1\ S_2 \dots S_K]^T$, T represent the matrix transpose, and the Vandermonde matrix $\boldsymbol{B}(\omega)$ is given by

$$\boldsymbol{B}(\omega) = \begin{bmatrix} 1 & 1 & \dots & 1 \\ e^{j\omega_1} & e^{j\omega_2} & \dots & e^{j\omega_K} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j(\frac{N}{2}-1)\omega_1} & e^{j(\frac{N}{2}-1)\omega_2} & \dots & e^{j(\frac{N}{2}-1)\omega_K} \end{bmatrix} \quad (5)$$

and

$$\boldsymbol{\varphi}(\omega) = diag(\, e^{j\omega_1}\ \ e^{j\omega_2} \dots\dots\ e^{j\omega_K}) \quad (6)$$

is $(K\times K)$ diagonal matrix whose diagonal elements contain the information about the frequencies of the complex sinusoids, and $\boldsymbol{e}_i^1$ is the gaussian noise vector. In general, the data matrix in (3) can be rewritten in terms of $\boldsymbol{B}(\boldsymbol{\omega})$, $\boldsymbol{S}$, and $\boldsymbol{\varphi}(\omega)$ as follow

$$\boldsymbol{X} = \left[\boldsymbol{B}(\omega)\boldsymbol{S}\ \ \boldsymbol{B}(\omega)\boldsymbol{\varphi}(\omega)\boldsymbol{S} \dots\dots \boldsymbol{B}(\omega)\left(\boldsymbol{\varphi}(\omega)\right)^{\frac{N}{2}-1}\boldsymbol{S}\right] + [\boldsymbol{e}_0^1\ \ \boldsymbol{e}_1^1 \dots\dots\ \boldsymbol{e}_{(\frac{N}{2}-1)}^1] \quad (7)$$

In the proposed method we introduce the following partition on the array response matrix $\boldsymbol{B}(\omega)$

$$\boldsymbol{B}(\omega) = [\boldsymbol{B}_1^T(\omega)\ \ \ \boldsymbol{B}_1^T(\omega)\ \ \ \boldsymbol{B}_1^T(\omega)\ \ \ \boldsymbol{B}_1^T(\omega)]^T \quad (8)$$

where, $\boldsymbol{B}_1, \boldsymbol{B}_2$ and $\boldsymbol{B}_3$ are matrices of dimension $(K\times K)$ and $\boldsymbol{B}_4$ has a dimension of $(N/2-3K)\times K$, respectively. Since $\boldsymbol{B}_1$ is nonsingular and has full rank of K, we can define uniquely three matrices Ψ_1, Ψ_2, and Ψ_3 with dimension $K\times(N/2-3K)$ using the parathion of array response vector $\boldsymbol{B}(\omega)$ as follows

$$\boldsymbol{\psi}_1 = -\boldsymbol{B}_1(\omega)^{-H}\boldsymbol{B}_4(\omega)^H \quad (9)$$
$$\boldsymbol{\psi}_2 = -\boldsymbol{B}_2(\omega)^{-H}\boldsymbol{B}_4(\omega)^H \quad (10)$$
$$\boldsymbol{\psi}_3 = -\boldsymbol{B}_3(\omega)^{-H}\boldsymbol{B}_4(\omega)^H \quad (11)$$

Let us define matrix $\boldsymbol{U}$ with dimension $N/2\times 3(N/2-2K)$ matrix using $\boldsymbol{\psi}_1$, $\boldsymbol{\psi}_2$, and $\boldsymbol{\psi}_3$ as

$$\boldsymbol{U} = \begin{bmatrix} \boldsymbol{\psi}_1 & 0 & 0 \\ 0 & \boldsymbol{\psi}_1 & 0 \\ 0 & 0 & \boldsymbol{\psi}_1 \\ I & I & I \end{bmatrix} \quad (12)$$

where, $\boldsymbol{0}$ is a zeros matrix with dimension $K\times(N/2-3K)$ and $\boldsymbol{I}$ is an identity matrix with dimension $(N/2-3K)\times(N/2-3K)$. We show that the matrix $\boldsymbol{U}$ span the null space of $\boldsymbol{B}(\omega)^H$ as follow

$$\boldsymbol{B}_4(\omega)^H\boldsymbol{U} = \boldsymbol{0} \quad (13)$$

Equation (13) implies that the subspace spanned by the columns of $\boldsymbol{U}$ is orthogonal to the columns of $\boldsymbol{B}(\omega)$. Since the basis of $\boldsymbol{U}$ is not orthonormal we can introduce the orthogonal projection as $\boldsymbol{O} = \boldsymbol{U}(\boldsymbol{U}^H\boldsymbol{U})^{-1}\boldsymbol{U}^H$.The matrix $\boldsymbol{U}$ can also be written in terms of $\boldsymbol{\psi}_1$, $\boldsymbol{\psi}_2$, and $\boldsymbol{\psi}_3$ as

$$\boldsymbol{U} = [\boldsymbol{\psi}_1^T\ \ \boldsymbol{\psi}_2^T\ \ \boldsymbol{\psi}_3^T\ \ 3I]^T \quad (14)$$

Now, we show here how to find the elements of the $\boldsymbol{U}$ matrix $\boldsymbol{\psi}_1, \boldsymbol{\psi}_2$, and $\boldsymbol{\psi}_3$ from the covariance data matrix. The covariance matrix $\boldsymbol{R}$ of the square data Hankel matrix can be written as

$$\boldsymbol{R} = E[\boldsymbol{X}\boldsymbol{X}^H] = \begin{bmatrix} \boldsymbol{R}_{11} & \boldsymbol{R}_{12} & \boldsymbol{R}_{13} & \boldsymbol{R}_{14} \\ \boldsymbol{R}_{21} & \boldsymbol{R}_{22} & \boldsymbol{R}_{23} & \boldsymbol{R}_{24} \\ \boldsymbol{R}_{31} & \boldsymbol{R}_{32} & \boldsymbol{R}_{33} & \boldsymbol{R}_{34} \\ \boldsymbol{R}_{41} & \boldsymbol{R}_{42} & \boldsymbol{R}_{43} & \boldsymbol{R}_{44} \end{bmatrix} \quad (15)$$

Now, consider the partition of the covariance data matrix $\boldsymbol{R}$ similar to the partition of the array response vector $\boldsymbol{B}(\omega)$. The matrices $\boldsymbol{\psi}_1, \boldsymbol{\psi}_2$, and $\boldsymbol{\psi}_3$ can be found form the partition blocks of the covariance data matrix in (15) as follow

$$\boldsymbol{\psi}_1 = -{\boldsymbol{R}_{21}}^{-1}{\boldsymbol{R}_{42}}^H \quad (16)$$
$$\boldsymbol{\psi}_2 = -{\boldsymbol{R}_{12}}^{-1}{\boldsymbol{R}_{41}}^H \quad (17)$$
$$\boldsymbol{\psi}_3 = -{\boldsymbol{R}_{13}}^{-1}{\boldsymbol{R}_{41}}^H \quad (18)$$

here, $\boldsymbol{R}_{21}$, $\boldsymbol{R}_{12}$, and $\boldsymbol{R}_{13}$ have a dimension of $(K\times K)$ while block matrices $\boldsymbol{R}_{42}$ and $\boldsymbol{R}_{41}$ have dimension of $\left(\frac{N}{2}-3K\right)\times K$. Note that the proposed method does not require any EVD or SVD in estimating the matrix $\boldsymbol{U}$.

Estimation of the frequencies of the multiple sinusoids is a search for the peaks which maximize the power spectrum.

These peaks in the spectrum corresponding to the true frequencies are examined by

$$P(e^{j\omega}) = \frac{1}{\boldsymbol{B}(\omega)^H \boldsymbol{O}\boldsymbol{B}(\omega)} \quad (19)$$

Since we have considered uniformly sampled data, the proposed method can also utilize the root MUSIC algorithm [15] to find the roots of polynomial which reveal the frequencies of multiple sinusoids. Note that the proposed method does not require any EVD or SVD in estimating the orthonormalized version matrix $\boldsymbol{O}$, whereas the original Root MUSIC, and MUSIC algorithm does. The frequency estimates may be taken to be the angles of the K roots of the polynomial $D(z)$ that are closest to the unit circle

$$D(z) = \sum_{i=0}^{N-1} V_i(z) V_i^*(1/z^*) \quad (20)$$

where, $V_i(z)$ is the z-transform [5] of the ***i***-th column of the projection matrix $\boldsymbol{O}$.

Step 2: Time delay estimation by proposed method

The estimate ω obtained in first step is used to estimate the time delay *D*. From the given data record at the second sensor $\{y(n),\ n = 0,1,\dots,N-1\}$, we can construct a square Henkel matrix $\boldsymbol{Y}$ with dimension $\left(\frac{N}{2} \times \frac{N}{2}\right)$ as

$$\boldsymbol{Y} = \begin{bmatrix} y(0) & y(1) & \dots & y(N/2-1) \\ y(1) & y(2) & \dots & y(N/2) \\ \vdots & \vdots & \ddots & \vdots \\ y(N/2-1) & y(N/2) & \dots & y(N-1) \end{bmatrix} \quad (21)$$

In (21), the ***i***-th column can be written as

$$\boldsymbol{q}_i = \begin{bmatrix} y(i) \\ y(i+1) \\ \vdots \\ y(i+\frac{N}{2}-1) \end{bmatrix} = \boldsymbol{B}(\omega)\boldsymbol{\Omega}(\omega, D)\left(\boldsymbol{\varphi}(\omega)\right)^i \boldsymbol{S} + \boldsymbol{e}_i^2 \quad (22)$$

$$i = 0,1,\dots\dots\frac{N}{2}-1$$

where, $\boldsymbol{e}_i^2$ is the complex noise vector of size $\left(\frac{N}{2} \times 1\right)$ and $\boldsymbol{\Omega}(\omega, D) = diag(e^{-jD\omega_1}\ \ e^{-jD\omega_2} \dots e^{-jD\omega_K})$ is a $(K \times K)$ diagonal matrix which contains an information about the time delay. The received data from the two separated sensors $\boldsymbol{X}$ and $\boldsymbol{Y}$, respectively can be grouped as

$$\boldsymbol{Z} = \begin{bmatrix} \boldsymbol{X} \\ \boldsymbol{Y} \end{bmatrix} = \begin{bmatrix} \boldsymbol{BS} & \boldsymbol{B\varphi S} & \dots\dots & \boldsymbol{B\varphi}^{(N/2-1)}\boldsymbol{S} \\ \boldsymbol{B\Omega S} & \boldsymbol{B\Omega\varphi S} & \dots\dots & \boldsymbol{B\Omega\varphi}^{(N/2-1)}\boldsymbol{S} \end{bmatrix} + \begin{bmatrix} \boldsymbol{e}^1 \\ \boldsymbol{e}^2 \end{bmatrix} \quad (23)$$

The objective here is to find the time delay information matrix $\boldsymbol{\Omega}(\omega, D)$. In proposed method, we utilize the PM (Propagator Method) which is originally proposed for classical direction of arrival estimation problem. The estimated frequencies ω_k in step one is used to estimate the time delay from $\boldsymbol{\Omega}(\omega, D)$**.** For sake of simplicity, we dropped time delay and frequency index.

We introduce the following partition on the matrix *B* as $B = [B_1^T\ B_2^T]^T$. where, B_1 and B_2 are sub-matrices with dimension$(K \times K)$ and $(N/2 - K) \times K$; Similarly, we can define matrix $\boldsymbol{Q}$ as

$$\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{Q}_1 \\ \boldsymbol{Q}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{B}_1 & \boldsymbol{B}_1\boldsymbol{\varphi} & \dots\dots & \boldsymbol{B}_1\boldsymbol{\varphi}^{(N/2-1)} \\ \hline \boldsymbol{B}_2 & \boldsymbol{B}_2\boldsymbol{\varphi} & \dots\dots & \boldsymbol{B}_2\boldsymbol{\varphi}^{(N/2-1)} \\ \boldsymbol{B}_1\boldsymbol{\Omega} & \boldsymbol{B}_1\boldsymbol{\Omega\varphi} & \dots\dots & \boldsymbol{B}_1\boldsymbol{\Omega\varphi}^{(N/2-1)} \\ \boldsymbol{B}_2\boldsymbol{\Omega} & \boldsymbol{B}_2\boldsymbol{\Omega\varphi} & \dots\dots & \boldsymbol{B}_2\boldsymbol{\Omega\varphi}^{(N/2-1)} \end{bmatrix} \quad (24)$$

Under the hypothesis that $\boldsymbol{B}_1$ is $(K \times K)$ non-singular matrix, the propagator matrix $\boldsymbol{P}$ is a unique linear operator which can be written as $\boldsymbol{P}^H\boldsymbol{Q}_1 = \boldsymbol{Q}_2$. Here, $\boldsymbol{Q}_1$ and $\boldsymbol{Q}_2$ contain the first K and the last $(N-K)$ rows of $\boldsymbol{Q}$. The propagator estimation can be found from the data matrix $\boldsymbol{Z}$ or from the data covariance matrix $\boldsymbol{R} = [\boldsymbol{ZZ}^H]$. Partitioning $\boldsymbol{R}$ as

$$\boldsymbol{R}_{zz} = [\boldsymbol{R}_1\ \ \boldsymbol{R}_2] \quad (25)$$

where, matrices $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ contain the first K and the last $(N-K)$ columns of Q . The propagator estimate matrix can be obtained by minimizing the following cost function:

$$\xi(\boldsymbol{P}_{data}) = \|\boldsymbol{Q}_2 - \boldsymbol{P}_{data}^H\boldsymbol{Q}_1\|_F \quad (26)$$

$$\xi(\boldsymbol{P}_{CM}) = \|\boldsymbol{R}_1 - \boldsymbol{R}_2\boldsymbol{P}_{CM}^H\|_F \quad (27)$$

where, $\boldsymbol{P}_{data}$ and $\boldsymbol{P}_{CM}$ are propagator estimate from the data matrix and the covariance matrix respectively. $\|\ \|_F$ denotes the frobenius norm. Following above argument, the propagator estimate matrix can be found from (26) and (27) as

$$\boldsymbol{P}_{data} = (\boldsymbol{Q}_1\boldsymbol{Q}_1^H)^{-1}\boldsymbol{Q}_1\boldsymbol{Q}_2^H \quad (28)$$

$$\boldsymbol{P}_{CM} = (\boldsymbol{R}_1\boldsymbol{R}_1^H)^{-1}\boldsymbol{R}_1\boldsymbol{R}_2^H \quad (29)$$

We can partition either $\boldsymbol{P}_{data}$ or $\boldsymbol{P}_{CM}$ as follows:

$$\boldsymbol{P}_{data} = [\boldsymbol{P}_1^T\ \boldsymbol{P}_2^T\ \boldsymbol{P}_3^T]^T \quad (30)$$

$$\boldsymbol{P}_2\left[\boldsymbol{B}_1\ \ \boldsymbol{B}_1\boldsymbol{\Phi} \dots \boldsymbol{B}_1\boldsymbol{\Phi}^{(N/2-1)}\right] = \left[\boldsymbol{B}_1\boldsymbol{\Omega}\ \ \boldsymbol{B}_1\boldsymbol{\Omega\Phi}\ \dots\ \boldsymbol{B}_1\boldsymbol{\Omega\Phi}^{(N/2-1)}\right] \quad (31)$$

From (31), the K eigenvalues of the propagator $\boldsymbol{P}_2$ are corresponding to the K diagonal elements of $\boldsymbol{\Omega}$. Let the angle of K diagonal elements of $\boldsymbol{P}_2$ equals to $\boldsymbol{Q}_0 = diag(\angle\boldsymbol{P}_2)$. This implies that the trace of $\boldsymbol{Q}_0$is equal the trace of $\boldsymbol{\Omega}$. Then the time delay estimation can be found as

$$D = \frac{trace(\boldsymbol{Q}_0)}{\sum_{k=1}^{K}\omega_k} \quad (32)$$

III. Simulation Results

In the first part, we test the proposed method using the power spectrum plots for multiple sinusoids *(K=3)* in presence of AWGN noise and the time delay estimation for 50 independent trials. In the second part, the performance of the proposed method verifies at two different noise cases: 1) non-uniform noise power and 2) unknown correlated noise field. Finally in the third part, we compared the performance of the proposed method with the ESPRIT algorithm for estimating the frequencies of two sinusoids *(K=2)* and the time delay estimation. We considered the number of data samples, *N=40*. The signal to noise ratio (SNR) values tested from 0 to 20 dB, and 200 independent trials.

For Fig. 1 and Fig 2, we assume three sinusoid signals with frequencies at $[0.51\pi \;\; 0.636\pi \;\; 0.764\pi]$, time delay at D=0.9, the number of recorded samples is N=40 for each sensors, and SNR of 0 dB for all signals. The complex attenuation coefficients of the three signals are (.4+.8*i*), (–.5–.7*i*), and (–.3+.8*i*). Fig. 1 shows the power spectrum versus the frequency estimation. It is become clear that our proposed algorithm gives accurate frequency estimation for all the unknown signals at exact frequencies. In Fig. 2 we show the time delay estimation for fifty independent trials. It is observed that the proposed method give accurate estimation for the time delay even at low SNR.

For Fig 3, we assume two sinusoid signals at $[0.6\pi \;\; 0.8\pi]$, time delay at D =1.4, the number of recorded samples is N=20 for each sensors, and the SNR of 5 dB for all signals. The attenuation coefficient of the two signals are (–.7–.7*i*), and (.2+5*i*). In this Figure, we considered two cases of noise field; In first case, the non-uniform covariance noise matrix whose diagonal elements for the first sensor is diag[1 2 2 3 4 6 6 8 7 8] and for the second sensor is diag[1 3 3 2 4 4 5 8 7 9] and in second case, unknown correlated noise filed whose covariance matrix in the form of $Q_{k,l} = \sigma^2 0.9^{(k-l)} e^{j(\pi/2)(k-l)}$ where k, l =1,2,…,10 and σ^2 is the variance. It is become clear that our proposed algorithm gives accurate frequency estimation for all signals even the noise filed is non-uniform or correlated. Also, the proposed method deliver accurate estimation for the time delay for the non-uniform noise field (estimated $\hat{D} = 1.4011$) and for unknown correlated noise field (estimated $\hat{D} = 1.4130$).

Fig. 4, 5 and 6 show the standard deviation of frequency and time delay estimation versus SNR while considering K=2, and N=40 for each sensors. The proposed estimation scheme is utilizing the root MUSIC to find the accurate frequencies and compared with ESPRIT algorithm. It is observed in Fig. 4 and Fig. 5 that the proposed method can produce accurate estimation of sinusoids for the two sources and perform quite alike the ESPRIT algorithm. However, the ESPRIT algorithm requires the EVD or SVD which is computationally intensive. Also, it is observed in Fig. 6 that the proposed method can estimate accurate estimation for time delay with lower complexity.

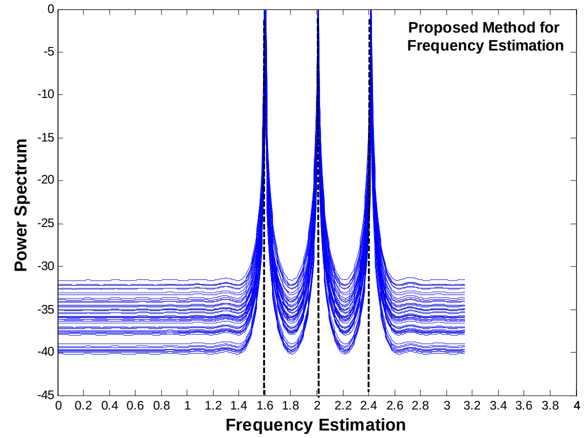

Fig.1. Power spectrum of Frequency estimation.

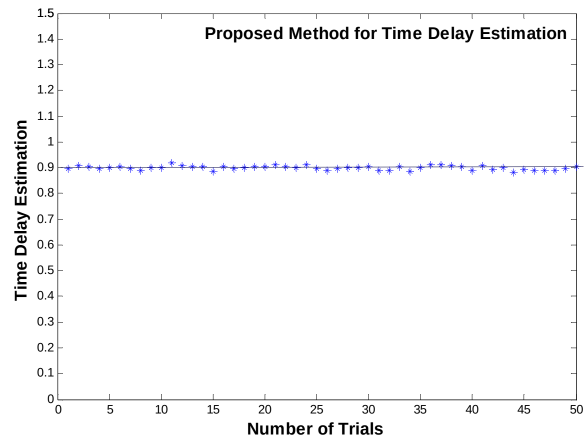

Fig.2. Time delay estimation for D=0.9.

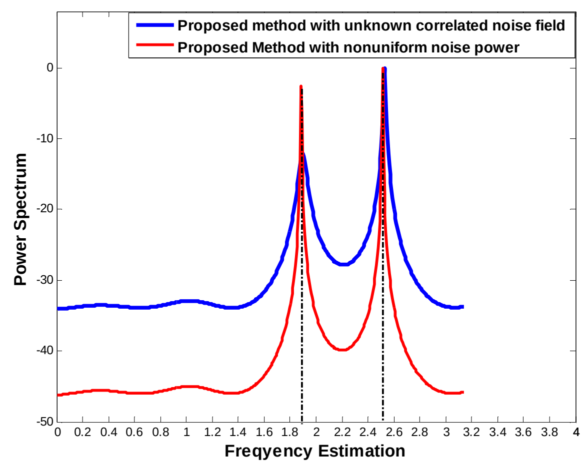

Fig.3. Power spectrum of Frequency estimation obtained by the proposed method for using correlated noise and non-uniform noise power.

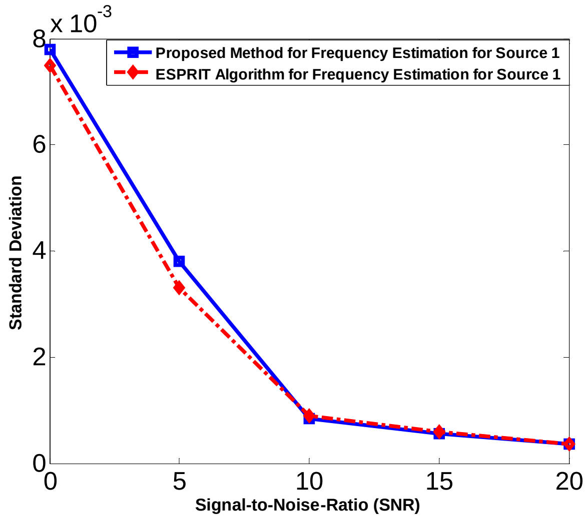

Fig.4. Standard deviation Vs SNR for the frequency estimation for source 1 at frequency at $w_1 = 0.6\pi$, assuming *N*=40 snapshots

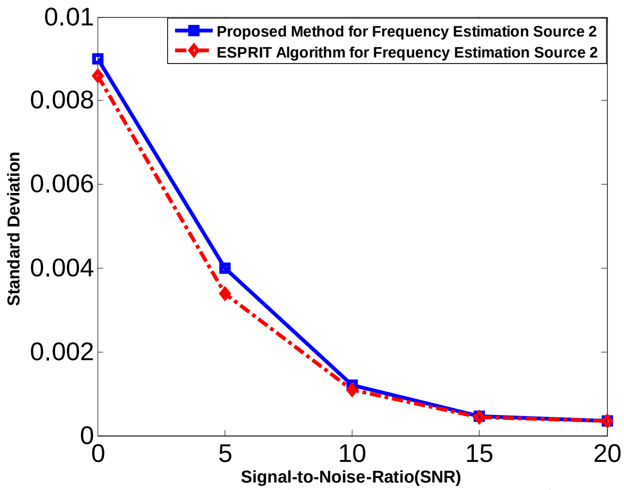

Fig.5. Standard deviation Vs SNR for the frequency estimations for source 2 at frequency at $w_2 = 0.65\pi$, assuming *N*=40 snapshots

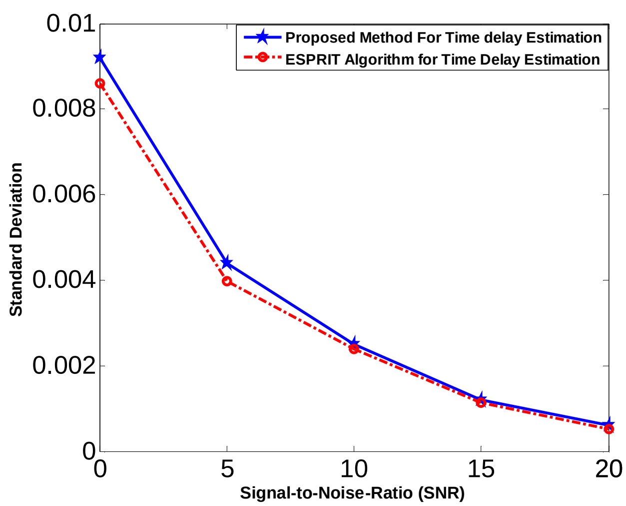

Fig.6. Standard deviation Vs SNR for time delay estimation with time delay *D*=1.4 sec, assuming N=*40* snapshots.

IV. Conclusion

In this paper we proposed a method for estimating time delay and frequency for multiple sinusoids in the presence of different noise filed. The proposed method does not require any spectral decomposition techniques such as EVD or SVD. Thus, the proposed scheme has much lower computational complexity and cost compared with peer algorithms such that MUSIC, ESPRIT and the methods [10-11]. The proposed method is more appropriate when concerning non-uniform noise power or unknown spatially correlated noise field whereas the eigenvalue analysis based methods [10-11]. In addition, the proposed method is accurately estimating the time delay and frequencies even at lower SNR.

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